

**Electromagnetics and Signal Processing for Spaceborne Applications – EM part**  
**September 11<sup>th</sup>, 2024**

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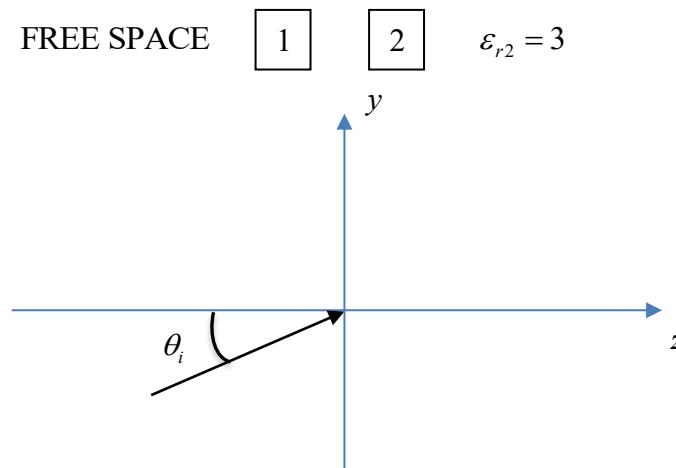
**Problem 1**

A plane sinusoidal EM wave propagates from free space into a medium with electric permittivity  $\epsilon_{r2} = 3$  ( $\mu_{r2} = 1$ ), with incidence angle  $\theta_i$ . The expression for the electric field in the first medium is:

$$\vec{E}_i(z, y) = \left[ \left( \frac{1}{2} \vec{\mu}_y - \frac{\sqrt{3}}{2} \vec{\mu}_z \right) + j2 \vec{\mu}_x \right] e^{-j104.72z} e^{-j181.38y} \text{ V/m}$$

For this wave:

- 1) Determine  $\theta_i$ .
- 2) Determine the frequency.
- 3) Determine the polarization of the incident and reflected EM wave.
- 4) Write the expression of the TE component of the refracted field.



**Solution:**

1) The incident angle can be obtained from the TM component of the field. For example:

$$|\vec{E}_{TM}| = \sqrt{(E_{TM}^y)^2 + (E_{TM}^z)^2} = 1$$

$$E_{TM}^y = \cos \theta_i |\vec{E}_{TM}| = \frac{1}{2} \rightarrow \theta_i = 60^\circ$$

2) The frequency of the incident EM wave can be derived from the phase constant. For example:

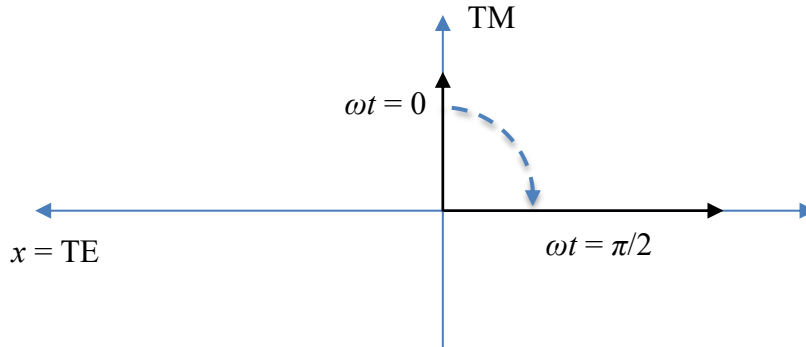
$$\beta_z = \beta \cos \theta_i = \frac{2\pi f}{c} \sqrt{\epsilon_{r1}} \cos \theta_i \rightarrow f = \frac{c\beta_z}{2\pi\sqrt{\epsilon_{r1}} \cos \theta_i} = 10 \text{ GHz}$$

3) The polarization of the incident wave is RHEP (right-hand elliptical polarization) because the two TE and TM components have different amplitudes and a phase shift of  $\pi/2$ . In fact, setting  $y$  and  $z$  to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re} \left\{ \left[ \left( \frac{1}{2} \vec{\mu}_y - \frac{\sqrt{3}}{2} \vec{\mu}_z \right) + j2\vec{\mu}_x \right] e^{j\omega t} \right\} = \cos(\omega t) \vec{\mu}_{TM} + 2 \cos\left(\omega t + \frac{\pi}{2}\right) \vec{\mu}_{TE} \text{ V/m}$$

Thus, making reference to the figure below that shows the reference system as seen from behind the wave, for  $t = 0 \rightarrow \vec{E}(0,0)|_{\omega t=0} = \vec{\mu}_{TM} \text{ V/m}$

Afterwards, for  $\omega t = \pi/2 \rightarrow \vec{E}(0,0)|_{\omega t=\pi/2} = -2\vec{\mu}_{TE} \text{ V/m}$



The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. As there is a TM component, it is worth checking the Brewster angle  $\theta_B$ :

$$\theta_B = \tan^{-1} \left( \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \right) = 60^\circ$$

As  $\theta_i = \theta_B$ , the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

4) First, we need to calculate the reflection coefficient for TE waves, which, in turn, requires calculating the refraction angle as:

$$\sqrt{\epsilon_{r1}\mu_{r1}} \sin \theta_i = \sqrt{\epsilon_{r2}\mu_{r2}} \sin \theta_t \rightarrow \theta_t = 30^\circ$$

$$\eta_1^{TE} = \frac{\eta_0}{\cos \theta_i \sqrt{\epsilon_{r1}}} = 754 \text{ } \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos \theta_t \sqrt{\epsilon_{r2}}} = 251.3 \text{ } \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.5$$

The full expression for the refracted TE component is given by:

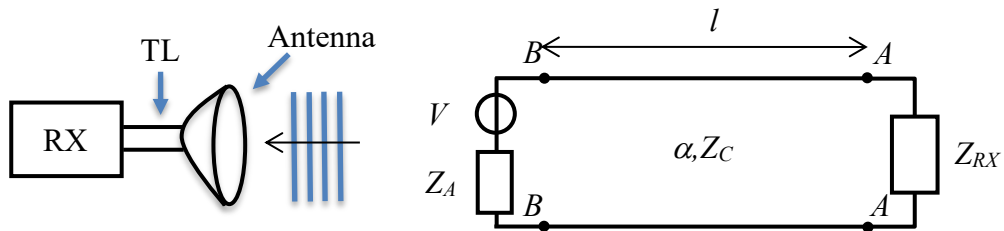
$$\vec{E}_t^{TE}(z, y) = \vec{E}_t^{TE}(0,0)(1 + \Gamma^{TE})e^{-j\beta_2 \cos \theta_t z} e^{-j\beta_2 \sin \theta_t y} = j\vec{\mu}_x e^{-j312.4z} e^{-j184.3y} \text{ V/m}$$

where  $\beta_2 = 362.76 \text{ rad/m}$ .

## Problem 2

A plane EM wave carries a power density of  $S = 10 \text{ mW/m}^2$  in front of an antenna. The power received by such antenna is conveyed into the receiver RX via a lossy coaxial cable ( $\alpha_{dB} = 30 \text{ dB/km}$ ), with intrinsic impedance  $Z_C = 75 \Omega$ . The antenna, whose gain is  $G = 10 \text{ dB}$ , acts as an equivalent generator with voltage  $V$  and internal impedance  $Z_A = 75 \Omega$ ; the RX, which acts as a load, has impedance  $Z_{RX} = 100 + j50 \Omega$ . The frequency is  $f = 600 \text{ MHz}$ . The line length is  $l = 40 \text{ m}$ .

1. Calculate the power received by the antenna.
2. Calculate the power absorbed by RX,  $P_{RX}$ .
3. With the aim of maximizing  $P_{RX}$ , is it better to have a capacitor-like or an inductor-like reactive part in  $Z_{RX}$ ?



## Solution

1) The power received by the antenna is the power available to the generator. The wavelength is  $\lambda = c/f = 0.5 \text{ m}$ , and the antenna effective area is:

$$A_E = G \frac{\lambda^2}{4\pi} = 0.1989 \text{ m}^2$$

The received power is:

$$P_{AV} = S A_E = 2 \text{ mW}$$

2) First, let us calculate the reflection coefficient at section AA:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0.2075 + j0.2264$$

The solution is simplified by the partial match at section BB, so the power absorbed by the load can be simply calculated as (only one reflection at the load section):

$$P_{RX} = P_{AV} e^{-2\alpha l} (1 - |\Gamma_L|^2) = 1.4 \text{ mW}$$

where  $\alpha = 3.5 \times 10^{-3} \text{ Np/m}$ .

3) It is easy to verify that any reactive part in  $Z_{RX}$  will generate an increase in  $|\Gamma_L|$ : the same absolute value of the reactive part will generate the same effect. To maximize the received power, such reactive part needs to be nullified.

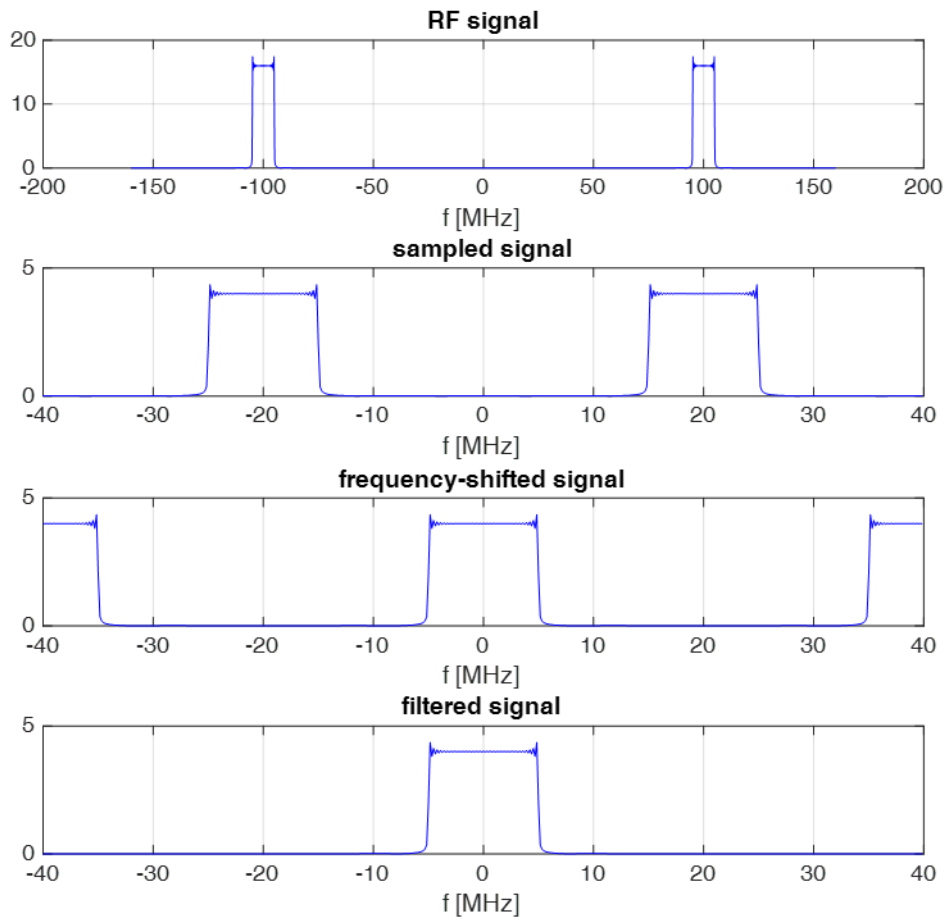
### Problem 3

Let  $s(t)$  be a RF signal:  $s(t) = g(t)\cos(2\pi f_0 t)$ , where  $g(t)$  is a short pulse with a (two-sided) bandwidth of 10 MHz and the carrier frequency is  $f_0 = 100$  MHz.

1. Write the expressions of the Fourier Transform of  $s(t)$  and draw a graph of the magnitude (you can represent  $|G(f)|$  as a rectangular pulse with bandwidth  $B$ ).
2. A discrete-time signal  $s_n$  is obtained by sampling  $s(t)$  with sampling frequency  $f_s = 80$  MHz. Write the expressions of the Fourier Transform of  $s_n$  and draw a graph of the magnitude.
3. Propose a procedure for the extraction of the complex envelope (I & Q components) of  $s(t)$  based on the sampled signal  $s_n$ .

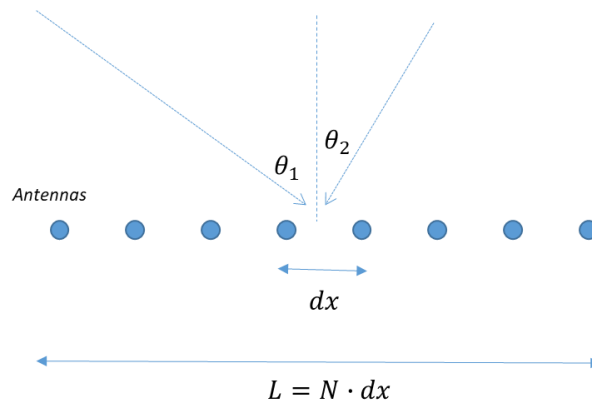
### Solution

- 1)  $S(f) = \frac{1}{2}G(f - f_0) + \frac{1}{2}G(f + f_0)$ , as resulting from the convolution between the Fourier Transform of  $g(t)$  ( $\Rightarrow G(f)$ ) and  $\cos(2\pi f_0 t)$  ( $\Rightarrow \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$ ). See the figure below, top row.
- 2) Replicating the signal with a period of  $f_s = 80$  MHz yields  $S(f) = \frac{f_s}{2}G(f - f_1) + \frac{f_s}{2}G(f + f_1)$ , with  $f_1 = 20$  MHz. See the second row in the figure below.
- 3) As the two replicas do not overlap, it is possible to correctly extract the complex envelope simply by shifting the signal by 20 MHz and applying a low pass filter. In the time domain this can be done as follows:  $z_n = h_n * (s_n e^{-j2\pi f_1 nT})$ , with  $T = \frac{1}{f_s}$ . See third and fourth rows.



### Problem 4

Multiple electromagnetic waves radiated from distant sources at the frequency  $f_0 = 435$  MHz impinge simultaneously on an antenna array as represented in the figure below:



Each antenna is equipped with its own circuitry to generate the complex envelope of the received signal.

1. Describe a procedure to measure the directions of arrival  $\theta_1, \theta_2, \dots$  based on the  $N$  signals (complex envelope) output by the array.
2. Discuss how the set antenna spacing  $dx$
3. After fixing antenna spacing  $dx$ , determine the number of antennas required to obtain an angular resolution  $\Delta\theta = 6^\circ$  (you can assume an incident direction of  $0^\circ$  as a reference in the calculation of resolution)
4. How would you change your answers at points 3 and 4 if you had knowledge that the direction of arrival of all impinging waves is limited in the interval  $(-30^\circ, +30^\circ)$ ?
5. The antenna array at point 4 is now used to transmit a wave by letting all antennas radiate simultaneously. Calculate the angular width of the transmitted beam and the angular position of secondary beams.

### Solution

- 1) A signal coming from direction  $\theta$  determines a phase variation across antenna elements equal to  $\varphi(x) = 2\pi \frac{\sin(\theta)}{\lambda} x$  with. Running a Fourier Transform along the array produces a peak at spatial frequency  $f_x = \frac{\sin(\theta)}{\lambda}$ , allowing for the detection of the direction of arrival of individual signals.
- 2) With no prior information on the range of angular direction of impinging signals one must choose  $dx \leq \frac{\lambda}{2}$ , to guarantee no ambiguous peak.
- 3) Angular resolution is obtained as  $\Delta\theta = \frac{\lambda}{Lx} = \frac{\lambda}{Ndx}$ , with  $Lx = Ndx$  array length. Hence  $N = \frac{\lambda}{\Delta\theta dx} = \frac{2}{\Delta\theta} = 19.1 \Rightarrow 20$  antennas
- 4) The prior knowledge about the angular interval converts into the minimum and maximum possible spatial frequencies to be  $\pm \frac{\sin(\theta)}{\lambda} = \pm \frac{1}{2\lambda}$ . The total spatial bandwidth is therefore  $Bx = \frac{1}{\lambda}$ , and by the sampling theorem this requires  $dx \leq \frac{1}{Bx} = \lambda$ . The number of antennas required to have a resolution of  $6^\circ$  is therefore  $N = \frac{\lambda}{\Delta\theta dx} = \frac{1}{\Delta\theta} = 9.5 \Rightarrow 10$  antennas

- 5) When used in transmission, the array will produce a main beam in the direction  $\theta = 0^\circ$  (orthogonal to the array) and beamwidth of  $6^\circ$ . The main beam corresponds to spatial frequency  $f_x=0$ . Ambiguous beams will be produced at spatial frequencies  $\frac{k}{dx} = \frac{k}{\lambda}$ , with k any integer (except 0, where we have the main beam). Those spatial frequencies are converted to angles as  $\theta = \text{asin}(f_x \lambda)$ , yielding  $\theta = [-90^\circ \ 90^\circ]$ .