Electromagnetics and Signal Processing for Spaceborne Applications – EM part
July 19th, 2024

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SURNAME AND NAME ______________________________

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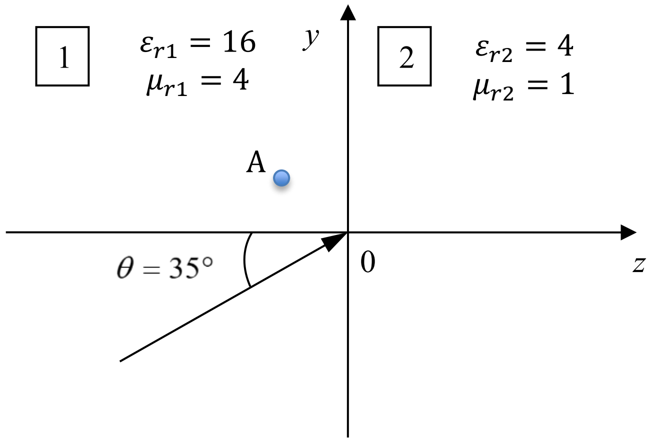
Problem 1

Consider the plane sinusoidal wave below, with f = 1 GHz and incident electric field given by:

$$\vec{E}_i(0,0,0) = j\mu_x \quad \text{(V/m)}$$



Calculate the electric field in point A(x = -5 m, y = 1 m, z = -1 m).

Solution

The total electric field is given by the summation between the incident field and the reflected field (if any). To determine the reflected field, it is first necessary to calculated the refraction angle:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\mu_{r1}\varepsilon_{r1}}{\mu_{r2}\varepsilon_{r2}}}\right) \approx \sin^{-1}(2.294)$$

This result indicates an evanescent wave, which means total reflection. For the complete expression of the reflected field though, it is necessary to calculate the reflection coefficient. To this aim, let us calculate β_{2z}:

$$\beta_{2z} = \beta_2\cos(\theta_2) = \beta_2\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{\mu_{r2}\varepsilon_{r2}}\sqrt{1-[\sin(\theta_2)]^2} = \pm j4.13\beta_0 = -j4.13\beta_0$$

It is necessary to pick the negative sign to obtain a physical solution. The reflection coefficient can now be calculated:

$$\eta_1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}} \frac{1}{\cos(\theta)}} = 230.1\ \Omega$$

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}} \frac{1}{\cos(\theta_2)}} = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}} \frac{1}{\cos(\theta_2)}} = j91.3\ \Omega$$

The choice of the negative sign for $\cos(\theta_2)$ in η_2 is consistent with the one in β_{2z}.

$$\Gamma = \frac{\eta_2 - \eta_2}{\eta_2 + \eta_1} = -0.728 + j0.686$$

The expression of the total electric field in medium 1 is therefore:

$$\vec{E}(y,z) = \vec{E}_i(y,z) + \vec{E}_r(y,z) = j\mu_x e^{-j\beta_1 \cos(\theta) z} e^{-j\beta_1 \sin(\theta) y} + \Gamma j\mu_x e^{j\beta_1 \cos(\theta) z} e^{-j\beta_1 \sin(\theta) y}\ \text{V/m}$$

The value of the electric field in A is:

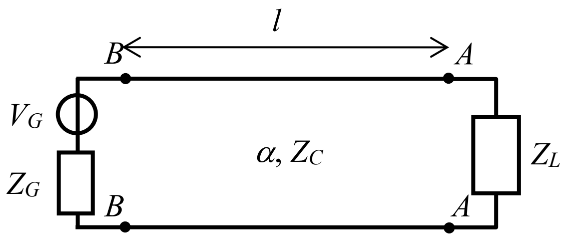
$\vec{E}(A) = (-0.697 - j0.893)\mu_x$ V/m

with $\beta_1 = \beta_0 \sqrt{\mu_{r1}\varepsilon_{r1}} = \frac{2\pi}{\lambda_0}\sqrt{\mu_{r1}\varepsilon_{r1}} = 167.6\ \text{rad/m}$

Problem 2

A source with voltage V_G = 100 V and internal impedance Z_G = 60 Ω is connected to a load Z_L = 60 Ω through a lossy transmission line with characteristic impedance Z_C = 100 Ω and specific attenuation α = 30 dB/km. The line length is l = 20 m, the frequency is f = 300 MHz.
For this circuit:

1) Calculate the power absorbed by Z_L.
2) Calculate the absolute value of the voltage at section, $|V_{AA}|$.
3) Calculate the trend in time of the voltage at section, $V_{AA}(t)$.



Solution

1) First, let us calculate the reflection coefficient at AA:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.25$$

The reflection coefficient at BB is:

$$\Gamma_{BB} = \Gamma_L e^{-j2\beta l} e^{-2\alpha l} = -0.2177$$

where $\beta = 6.283 \text{ rad/m}$

The equivalent input impedance is:

$$Z_{BB} = 64.24\ \Omega$$

Let us now calculate:

$$\Gamma_G = 0.034$$

The power absorbed by what is beyond section BB (load plus line) is:

$$P_{BB} = P_{AV}(1 - |\Gamma_G|^2) = 20.8 \text{ W}$$

with

$$P_{AV} = \frac{|V_G|^2}{8Z_G} = 20.83 \text{ W}$$

This is not yet the power reaching the load, as we need to discriminate what part of the power is absorbed by the line. As question 3 requires the calculation of $V_{AA}(t)$, we can proceed by using voltage transfer. To this aim:

$$V_{BB} = V_G \frac{Z_{BB}}{Z_G + Z_{BB}} = 51.7 \text{ V}$$

Then, the progressive wave at BB is:

$$V_{BB}^{+} = \frac{V_{BB}}{1 + \Gamma_{BB}} = 66.1 \text{ V}$$

Finally:

$$V_{AA} = V_{BB}^{+} e^{-j\beta l} e^{-\alpha l}(1 + \Gamma_L) = 46.3 \text{ V}$$

The power absorbed by the load is:

$$P_L = \frac{1}{2}\frac{|V_{AA}|^2}{Z_L} = 17.8 \text{ W}$$

2) From V_{AA} → $|V_{AA}| = 46.3$ V

3) Finally, as $\measuredangle V_{AA} = 0$ rad, $V_{AA}(t) = 46.3\cos(1.88 \times 10^9 t)$ V