Electromagnetics and Signal Processing for Spaceborne Applications – EM part
September 6th, 2022

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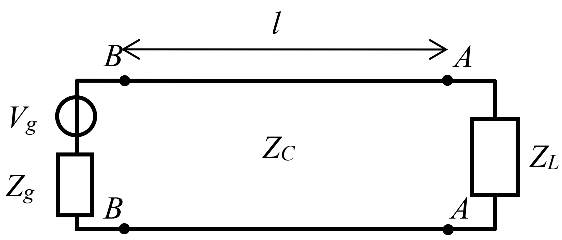
Problem 1

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a transmission line with characteristic impedance Z_C = 75 Ω, which terminates on a load Z_L = 50 Ω. The frequency is f = 300 MHz and the length of the line is l = 5.25 m.

1) Calculate the power absorbed by Z_L.
2) Calculate the voltage at the load section (V_{AA}) if Z_L becomes a short circuit due to a fault in the load; express V_{AA} in the time domain.



Solution

1) As there is no match at the generator section and at the load section, let us calculate the reflection coefficient at AA:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.2$$

The reflection coefficient at section BB is:

$$\Gamma_{BB} = \Gamma_L e^{-2j\beta l} = 0.2$$

As a result, the input impedance is:

$$Z_{BB} = Z_C \frac{1 + \Gamma_{BB}}{1 - \Gamma_{BB}} = 112.5\ \Omega$$

As the line length is a multiple of $\lambda/4$, the input impedance could have been easily calculated as:

$$Z_{BB} = \frac{Z_C^2}{Z_L}$$

The reflection coefficient at the generator section (left side) is:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = 0.3846$$

Therefore, the power crossing section BB is:

$$P_{BB} = P_L = \frac{|V_g|^2}{8\mathrm{Re}[Z_g]}\left(1 - |\Gamma_g|^2\right) = 0.213 \text{ W}$$

This is also the power absorbed by the load, as no other element in the circuit beyond section BB can absorb power.

2) If the load becomes a short circuit, there is no need to perform calculations: indeed, as $Z_L = 0\ \Omega \rightarrow \Gamma_L = -1$. As a result, whatever the progressive wave reaching the load, it will be totally reflected with a change in the sign. Therefore, the total voltage at the section (progressive+regressive) will always be 0 V (as expected from a short circuit) → $V_{AA}(t) = 0$ V.

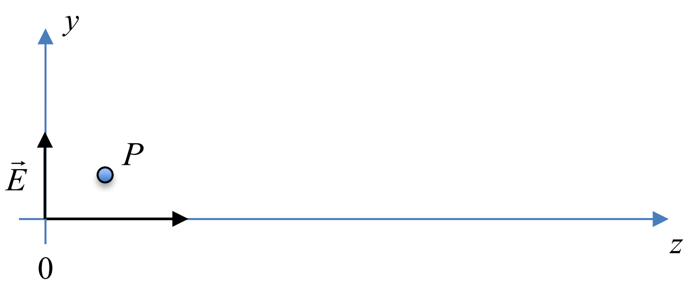
Problem 2

A uniform sinusoidal plane wave propagates in a medium characterized by relative electric permittivity $\varepsilon_r = 1$, magnetic permeability $\mu_r = 9$ and conductivity $\sigma = 0.1$ S/m. The expression of the electric field is ($E_0 = 1$ V/m):

$$\vec{E}(z,t) = E_0\, e^{-\alpha z} \cos(2\pi 10^9 t - \beta z)\vec{\mu}_y \text{ V/m}$$

For such a wave:

1) What is the wave polarization?
2) Calculate the phase velocity of the wave.
3) Calculate the power received by an isotropic antenna located at P(0.1λ,0.1λ,0.1λ), which has efficiency $\eta_A = 0.9$.



Solution

1) The wave is linearly polarized (vertical polarization).

2) Let us first check the loss tangent for the wave:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} \approx 1.8$$

No approximations can be applied; therefore, the propagation constant is calculated as:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 45.7 + j77.7 \text{ 1/m}$$

From β, the phase velocity is calculated as:

$$v = \frac{\omega}{\beta} = 8.1 \times 10^7 \text{ m/s}$$

3) The wavelength is given by:

$$\lambda = \frac{2\pi}{\beta} = 0.0808 \text{ m}$$

Therefore P is in (0.00808 m, 0.00808 m, 0.00808 m).

The power received at P by the antenna is:

$$P_R = SA_e = \frac{1}{2}\frac{|\vec{E}(P)|^2}{|\eta|}\cos(\measuredangle\eta)\, A_e = \frac{1}{2}\frac{|E_0|^2}{|\eta|} e^{-2\alpha z_P}\cos(\measuredangle\eta)\, \frac{\lambda^2}{4\pi} D\eta_A$$

where:

$D = 1$ (isotropic antenna with directivity 1)

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = 679 + j\,399\ \Omega$$

Therefore:

$P_R = 0.12$ μW.

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Problem 3

The signal received from a distant source is modeled as

$$s(t) = A_- \cdot cos\left(2\pi\left(f_0 - \tfrac{\Delta f}{2}\right)t\right) + A_+ \cdot cos\left(2\pi\left(f_0 + \tfrac{\Delta f}{2}\right)t\right),$$

where $f_0 = 1$ GHz is the carrier frequency and $\Delta f = 1$ KHz

1. Write the expression of the complex received signal. ***Tip***: *remember that the Fourier Transform of the complex signal is the same as the FT of the real-valued signal for positive frequencies, whereas it is identically zero for negative frequencies*.
2. Write the expression of the complex envelope (after demodulation by f_0)
3. Describe a procedure to measure the two constants A_- and A_+ based on the complex envelope.
4. For how long a time should you observe the received signal to be able to make a good measurement of A_- and A_+ ?
5. Write a short pseudo-code to implement the procedure at point 3.

Solution

The FT of s(t) is

$$S(f) = \frac{A_-}{2}\delta\left(f - \left(f_0 - \frac{\Delta f}{2}\right)\right) + \frac{A_+}{2}\delta\left(f - \left(f_0 + \frac{\Delta f}{2}\right)\right) +$$
$$+\frac{A_-}{2}\delta\left(f + \left(f_0 - \frac{\Delta f}{2}\right)\right) + \frac{A_+}{2}\delta\left(f + \left(f_0 + \frac{\Delta f}{2}\right)\right)$$

The lower line represents pulses found at negative frequencies, so it vanishes if only positive frequencies are considered. When returning in the time domain each δ becomes a complex exponential, thus:

$$s_c(t) = A_- \cdot exp\left(j2\pi\left(f_0 - \tfrac{\Delta f}{2}\right)t\right) + A_+ \cdot exp\left(j2\pi\left(f_0 + \tfrac{\Delta f}{2}\right)t\right),$$

The complex envelope is obtained by multiplication by $exp(j2\pi f_0 t)$, hence

$$s_{ce}(t) = A_{-} \cdot exp\left(-j2\pi \frac{\Delta f}{2} t\right) + A_{+} \cdot exp\left(j2\pi \frac{\Delta f}{2} t\right),$$

The amplitudes can be measured by taking the FT of the complex envelope. If the FT is evaluated over an infinite observation time we get:

$$s_{ce}(f) = A_{-} \cdot \delta\left(f + \frac{\Delta f}{2}\right) + A_{+} \cdot \delta\left(f - \frac{\Delta f}{2}\right),$$

So one has to evaluate the area of the two delta.
More realistically, we will get

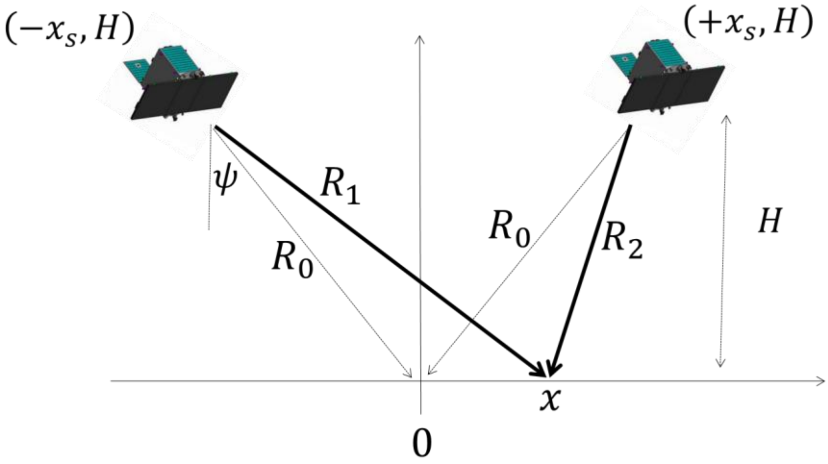
$$s_{ce}(f) = A_{-} \cdot T_o sinc\left(\left(f + \frac{\Delta f}{2}\right) T_o\right) + A_{+} \cdot T_o sinc\left(\left(f - \frac{\Delta f}{2}\right) T_o\right),$$

So A_{-} and A_{+} are obtained by taking the peak of the two sinc divided by T_o

The condition to see two separate peaks is that $T_o > \frac{1}{\Delta f}$. Otherwise the two peaks interfere.

Pragmatically, one would like to ensure that $T_o \gg \frac{1}{\Delta f}$

Problem 4



Consider two satellites at a height H = 1000 Km that transmit a signal at the frequency f_0 = 1 GHz toward a receiver placed at *x* = 0. The positions of the two satellites along the x-axis are $+x_s$ and $-x_s$, with $x_s = 500$ Km.

1. Assuming that the two satellites transmit simultaneously, derive the graph of the field amplitude along the x-axis. ***Tip****: assume small values of x, so that you can – as always – linearize the expression of the distances w.r.t. the reference position x = 0.*
2. Repeat point 1 assuming that the satellite on the right transmits with a delay of Δt seconds w.r.t. the one on the left. Where should you place the receiver to maximize the intensity of the received signal?
3. Imagine now to place a few receivers on the ground (in the neighborhood of x=0) to measure the angle ψ. Discuss the role of the spatial sampling between any two nearby receivers and of the total number of receivers.

Solution

1) The field in the neighborhood of the receiver is:

$$E(x) = \frac{1}{R_1(x)} exp\left(-j\frac{2\pi}{\lambda}R_1(x)\right) + \frac{1}{R_2(x)} exp\left(-j\frac{2\pi}{\lambda}R_2(x)\right)$$

The distances are approximated as:

$$R_1(x) = \sqrt{H^2 + (x + x_s)^2} \cong R_0 + sin\psi \cdot x$$
$$R_2(x) = \sqrt{H^2 + (x - x_s)^2} \cong R_0 - sin\psi \cdot x$$

Where the angle for the two satellites is defined such that $sin\psi > 0$

Hence:

$$E(x) \cong \frac{1}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)\left[exp\left(-j\frac{2\pi}{\lambda}sin\psi \cdot x\right) + exp\left(+j\frac{2\pi}{\lambda}sin\psi \cdot x\right)\right]$$
$$\cong \frac{2}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right) cos\left(\frac{2\pi}{\lambda}sin\psi \cdot x\right)$$

2) If the second transmission is delayed one has:

$$E(x) \cong \frac{1}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)\left[exp\left(-j\frac{2\pi}{\lambda}sin\psi\cdot x\right) + exp\left(+j\frac{2\pi}{\lambda}sin\psi\cdot x\right)exp(-j2\pi f_0\Delta t)\right]$$

$$\cong \frac{1}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)exp(-j\pi f_0\Delta t) \cdot\left[exp\left(-j\frac{2\pi}{\lambda}sin\psi\cdot x\right)exp(j\pi f_0\Delta t) + exp\left(+j\frac{2\pi}{\lambda}sin\psi\cdot x\right)exp(-j\pi f_0\Delta t)\right]$$

$$\cong \frac{1}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)exp(-j\pi f_0\Delta t) \cdot\left[exp\left(-j\pi\left(\frac{2}{\lambda}sin\psi\cdot x - f_0\Delta t\right)\right) + exp\left(+j\left(\frac{2}{\lambda}sin\psi\cdot x - f_0\Delta t\right)\right)\right]$$

$$\cong \frac{1}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)exp(-j\pi f_0\Delta t) \cdot\left[exp\left(-j\pi\left(\frac{2}{\lambda}sin\psi\cdot x - f_0\Delta t\right)\right) + exp\left(+j\left(\frac{2}{\lambda}sin\psi\cdot x - f_0\Delta t\right)\right)\right]$$

$$E(x) \cong \frac{2}{R_0} exp\left(-j\frac{2\pi}{\lambda}R_0\right)exp(-j\pi f_0\Delta t)\cdot cos\left(\pi\left(\frac{2}{\lambda}sin\psi\cdot x - f_0\Delta t\right)\right)$$

Field intensity peaks at $x = \frac{\lambda}{2sin\psi} f_0\Delta t$

From point 1, the field is proportional to $cos\left(\frac{2\pi}{\lambda}sin\psi\cdot x\right)$. Accordingly, receivers must be placed so as to estimate the frequency of the cosine ($f_x = \frac{sin\psi}{\lambda}$) unambiguously, which yields $\Delta x \le \frac{\lambda}{2}$.
The total number of receivers determines frequency resolution, so with *N* receivers one has $\Delta f_x = \frac{1}{N\Delta x}$, hence $\Delta\psi = \frac{d\psi}{df_x}\Delta f_x = \frac{\lambda}{cos\psi}\frac{1}{N\Delta x}$.

Notice that the analysis yields two peaks at $f_x = \pm\frac{sin\psi}{\lambda}$ (one peak per satellite)