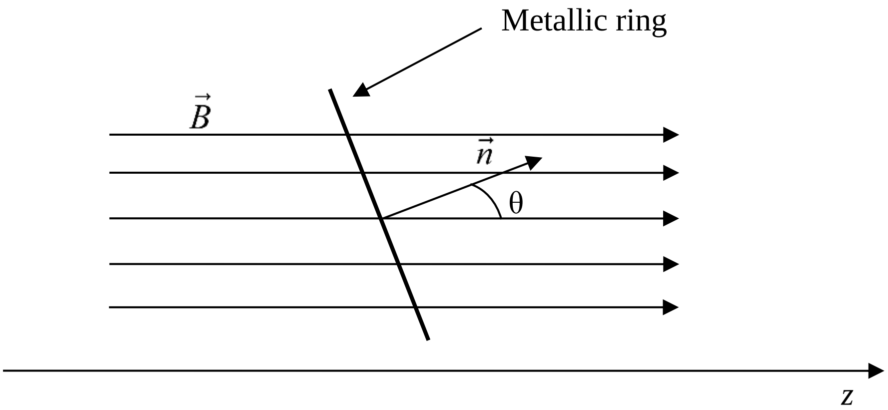
Problem 1

A metallic square ring with lateral dimension a = 5 cm is subject to a uniform magnetic filed such that the normal to the ring is at $\theta = 30°$ from the flux lines of the magnetic field, which is given by:

$$\vec{B}(t) = B_0 \sin\left(2\pi ft\right)e^{-0.01t}\vec{\mu}_z \quad [\text{Wb/m}^2]$$

Knowing that f = 50 Hz, $B_0 = 10^{-6}$ Wb/m^2, calculate for $t \geq 0$:

1) the magnetic flux through the ring;
2) the electromotive force.



Solution

1) Flux:

$$\phi_m\left(\vec{B}\right) = \int_S \vec{B}\, d\vec{S} = \int_S \vec{B}\,\vec{n}\, dS = \left|\vec{B}\right| \cos\theta \int_S dS = a^2 \left|\vec{B}\right| \cos\theta = 2.165 \cdot 10^{-9} e^{-0.01t} \sin\left(314.16\, t\right) \quad [\text{Wb}]$$

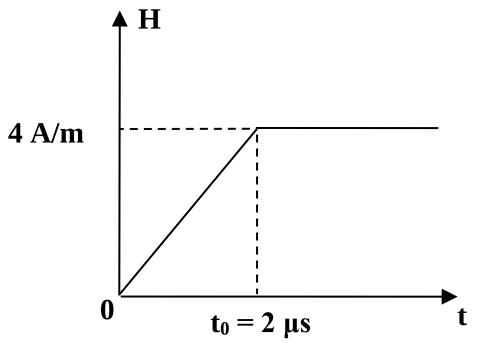
2) Electromotive force:

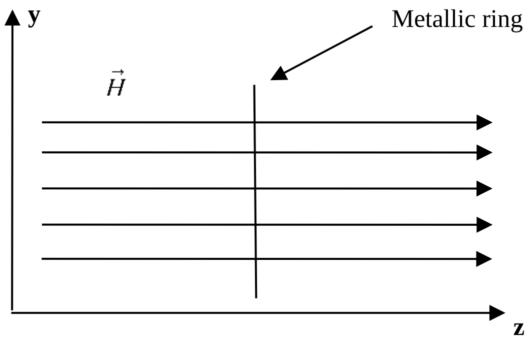
$$fem = -\frac{\partial \phi_m}{\partial t} = -B_0 a^2 \cos\theta \frac{\partial}{\partial t}\left(\sin(2\pi f\, t)e^{-0.01t}\right) = -B_0 a^2 \cos\theta\left[2\pi f\, e^{-0.01t} \cos(2\pi f\, t) - 0.01\sin(2\pi f\, t)e^{-0.01t}\right] =$$

$$= 2.165 \cdot 10^{-11} e^{-0.01t} \sin(314.16\, t) - 6.08 \cdot 10^{-7} e^{-0.01t} \cos(314.16\, t) \quad [\text{V}]$$

Problem 2

A metallic square ring with lateral dimension a = 5 cm is subject to a uniform magnetic filed such that the normal to the ring is parallel to the flux lines of the magnetic field, which varies in time as depicted below:





For $t \geq 0$:

1) write the expression of the equation giving the variation of the magnetic field in time;
2) calculate the flux of the magnetic field through the ring;
3) calculate the electromotive force;
4) calculate the magnitude and direction of the current flowing in the ring, assuming that it is characterized by resistance R = 50 Ω.

Solution

1) Magnetic field:

$$\vec{H} = \begin{cases} \dfrac{4}{2 \cdot 10^{-6}} t \vec{\mu}_z = 2 \cdot 10^6 t \vec{\mu}_z & 0 \leq t \leq t_0 \\ 4\vec{\mu}_z & t > 0 \end{cases} \quad \text{[A/m]}$$

2) Flux:

$$\phi_m\left(\vec{B}\right) = \int_S \vec{B}\, d\vec{S} = \int_S \vec{B}\,\vec{n}\, dS = \left|\vec{B}\right| \int_S dS = a^2 \left|\vec{B}\right| = a^2 \left|\vec{H}\right| \mu_0 = \begin{cases} 2 \cdot 10^6 t\, \mu_0 & 0 \leq t \leq t_0 \\ 4\mu_0 & t \geq t_0 \end{cases} \quad \text{[Wb]}$$

where $a^2 = 1$ m^2.

3) Electromotive force:

$$\text{EMF} = -\frac{\partial \phi_m}{\partial t} = \begin{cases} -\dfrac{\partial}{\partial t}\left(2 \cdot 10^6 t\, \mu_0\right) & 0 \leq t \leq t_0 \\ -\dfrac{\partial}{\partial t}\left(4\mu_0\right) & t \geq t_0 \end{cases} = \begin{cases} -2 \cdot 10^6 \mu_0 \approx -2.5 & 0 \leq t \leq t_0 \\ 0 & t \geq t_0 \end{cases} \quad \text{[V]}$$

4) The current flows from the bottom to the top considering the side of the ring shown in the picture above. Its value is:

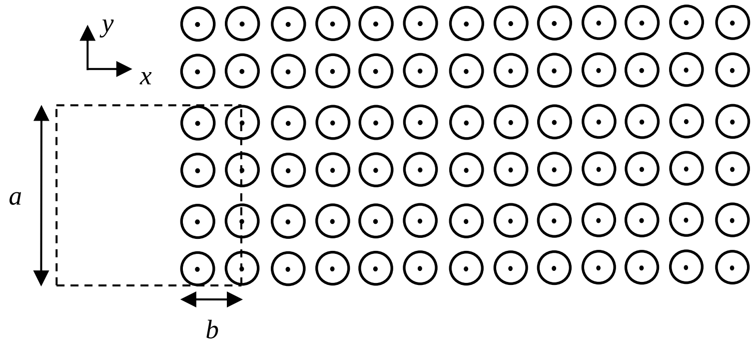
$$I = \frac{V}{R} = 50 \quad \text{[mA]}$$

Problem 3

Consider a square metallic ring with lateral dimension a = 1 m partially immersed in a spatially homogeneous magnetic field (see figure below). The magnetic field is given by:

$$\vec{H} = 100\sin\left(100\pi t\right) + 5\,\vec{a}_z \text{ A/m} \quad \text{for } t \geq 0 \text{ s}$$

Calculate the electromotive force for $t \geq 0$ s. Assuming then that the metallic ring is associated to a resistance R = 100 Ω, calculate the value and the direction of the current flowing in the wire.



Solution

The magnetic flux is:

$$\phi = \int_S \vec{B}\cdot d\vec{S} = \mu_0 \vec{H}\,A = \mu_0 \vec{H}\,a\,b \qquad \text{(Wb)}$$

So the electromotive force is:

$$V(t) = \left|-\frac{\partial\phi}{\partial t}\right| = \begin{cases} \mu_0\left[100\cos\left(100\pi t\right)100\pi\right]ab = 9.9\cos\left(100\pi t\right) & t \geq 0\,s \\ 0 & t < 0\,s \end{cases} \qquad \text{(mV)}$$

Thus the current is:

$I(t) = V(t)/R = 0.99\cos\left(100\pi t\right)$ mA

The flowing direction of the current is such that it counteracts the increase and the decrease in the magnetic flux. Making reference to the figure below, the positive sign for the current corresponds to the counter clockwise direction in the wire.