FUNDAMENTALS OF ELECTROMAGNETIC FIELDS – June 29th, 2018

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Name and surname
Identification number
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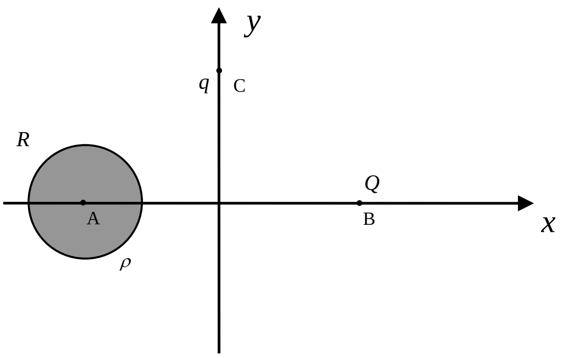
Problem 1 - A uniform charge density $\rho = 10^{-10}$ C/m^3 is embedded in a dielectric sphere with radius R = 2 m (grey part in the figure); the center of the sphere is fixed in A(-4,0). A point charge, with unknown value Q, is fixed in point B(4,0). Another charge q = 1 nC, free to move, is placed in C(0,4). Determine the sign and the value of the charge Q in B such that the charge q in C is subject to a force $\vec{F} = k\vec{\mu}_x$ (only component along x). Also, determine k.



Solution

The electric field in C due to the charged sphere is given by the Gauss' theorem:

$$\int_S \vec{D} \cdot d\vec{S} = \int_V \rho \, dV$$

Solving the equation, we obtain:

$$\vec{E}_1 = \frac{Q_T}{4\pi\varepsilon_0 AC^2}\vec{\mu}_{AC} = 0.94\vec{\mu}_{AC} \text{ V/m}$$

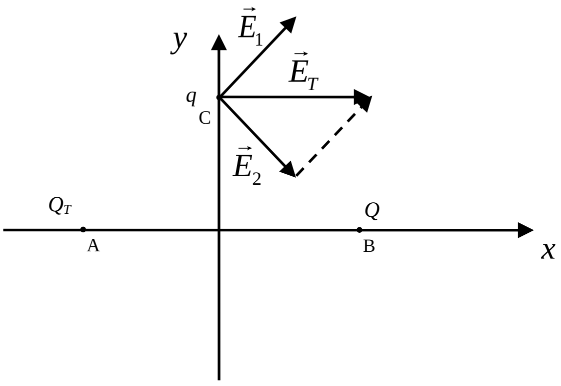
where $Q_T = \rho 4/3\pi R^3 = 3.35$ nC.

As both Q_T and q are positive, the latter will tend to move in the direction of the electric field $\vec{E}_1$ (see figure below). In order to have a total electric field $\vec{E}$, composition of $\vec{E}_1$ (given by Q_T) and of $\vec{E}_2$ (given by Q), directed as x, Q must be negative and its absolute value be that of Q_T, i.e.:

$Q = -Q_T = -3.35$ nC.

As a result:

$$\vec{E}_T = 2\left|\vec{E}_1\right|\cos\frac{\pi}{4}\vec{\mu}_x = 1.33\vec{\mu}_x \text{ V/m}$$

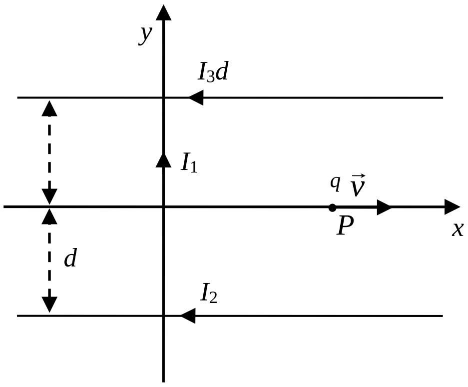


Finally, the force acting on q, due both to Q_T and Q, is:

$$\vec{F} = q\vec{E}_T = 1.33\vec{\mu}_x \text{ nN}$$

Problem 2 - A constant current flows through the wires reported in the picture below: wire 1 lies on the y axis and its current is $I_1 = 1$ A, wire 2 ad 3 are parallel to x axis and their currents are $I_2 = I_3 = 2$ A ($d = 1$ m). An electron with charge $q = -1.6 \cdot 10^{-19}$ C is in P(2,0), travelling with velocity $\vec{v} = c\,\vec{\mu}_x$ m/s, being c the speed of light. Calculate:

1) The magnetic field in P.
2) The force which the electron in P is subject to (amplitude and direction).



Solution

The magnetic field in P will be given by the contributions coming from all the wires. Specifically, wire 1 will give the following contribution:

$$\vec{H}_1(P) = -\frac{I_1}{2\pi x_P}\vec{\mu}_z = -79.6\,\vec{\mu}_z \text{ mA/m}$$

The contributions from wires 2 and 3 in P cancel out: the two wires have the same currents, they are at the same distance from P and the two associated magnetic fields have opposite direction ($-\vec{\mu}_z$ for I_2 and $\vec{\mu}_z$ for I_3). As a result:

$$\vec{H}(P) = \vec{H}_1(P) + \vec{H}_2(P) + \vec{H}_3(P) = -79.6\,\vec{\mu}_z \text{ mA/m}$$

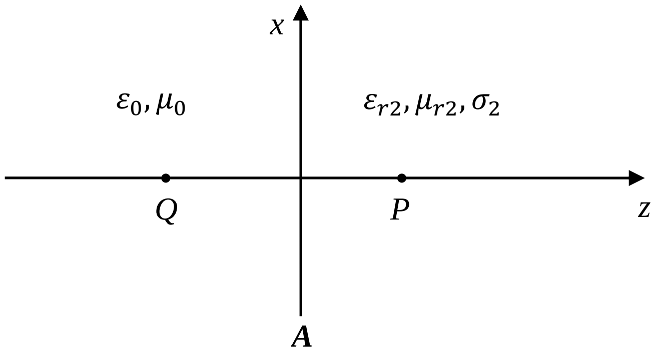
The electron is subject to Lorentz's force:

$$\vec{F}(P) = q\vec{v} \times \mu_0 \vec{H}(P)$$

Considering the vector product operation and the negative sign for the electron charge, the resulting force is directed along $-\vec{\mu}_y$:

$$\vec{F}(P) = q\vec{v} \times \mu_0 \vec{H}(P) = q\mu_0 \left|\vec{v}\right| \left|\vec{H}(P)\right| \vec{\mu}_y = -4.8 \cdot 10^{-18}\,\vec{\mu}_y \text{ N}$$

Problem 3 - A uniform plane wave (frequency f = 1 GHz) propagates in free space and impinges on a dielectric material ($\varepsilon_{r2} = 9, \mu_{r2} = 3, \sigma_2 = 2 \cdot 10^{-4}$ S/m). The power density of the wave in P(0,0,10λ_2) is 1 μW/m^2 (λ_2 is the wavelength in the second medium). Calculate the power density of the incident wave in Q(0,0,λ_1) (λ_1 is the wavelength in the first medium).



Solution

First, let us calculate the intrinsic impedance for the two media. For the first one:

$$\eta_1 = \eta_0 \approx 377\ \Omega$$

For the second one, the loss tangent is $\sigma/\omega\varepsilon \approx 4 \cdot 10^{-4} << 1$. Therefore the second medium can be considered as a good dielectric:

$$\eta_2 \approx \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\eta_0 = 217.5\ \Omega$$

The reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.268$$

Also, let us calculate the propagation constant for the second medium (using the approximation defined above):

$$\gamma_2 = \alpha_2 + j\beta_2 \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\frac{2\pi f}{c}\sqrt{\varepsilon_{r2}\mu_{r2}} = 0.0251 + j108.8 \quad \text{1/m}$$

The wavelength is obtained from β_2 as:

$$\lambda_2 = \frac{2\pi}{\beta_2} \approx 0.0577 \quad \text{m}$$

Therefore P has coordinates (0.577 m,0).

The power density in P is:

$$S(P) = S_i(1-|\Gamma|^2)e^{-2\alpha_2 x_P}$$

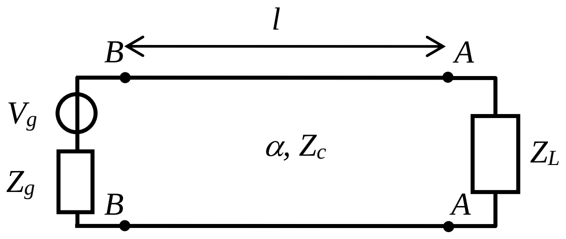
where S_i is the incident power density: as there are no losses in medium 1, S_i is constant throughout medium 1, including in Q. Inverting such equation, we obtain:

$$S(Q) = \frac{S(P)}{(1-|\Gamma|^2)e^{-2\alpha_2 x_P}} = 1.11\ \mu\text{W/m}$$

Problem 4 - A source with voltage V_g = 100 V and internal impedance Z_g = 50 Ω is connected to a load Z_L = 100 Ω by a transmission line with characteristic impedance Z_C = 100 Ω and attenuation constant α = 20 dB/km. The line length is l = 30 m and the frequency is f = 600 MHz.

Calculate:

a) The power absorbed by the load
b) The power dissipated along the line
c) The absolute value of the voltage at the load section (section AA), V_A



Solution

a) As the load is matched to the line ($Z_L = Z_C$), the input impedance is simply $Z_B = Z_L$. Therefore the reflection coefficient at the generator section is:

$$\Gamma_g = \frac{Z_L - Z_g}{Z_L + Z_g} = \frac{1}{3}$$

As a result, the fraction of the power crossing section BB is:

$$P_{BB} = P_d\left(1 - \left|\Gamma_g\right|^2\right) = \frac{V_g^2}{8Z_g}\left(1 - \left|\Gamma_g\right|^2\right) = 22.2 \text{ W}$$

Such power will be attenuated by the line and will afterwards reach the load. Therefore, the power absorbed by the load is:

$$P_L = P_{BB}\, e^{-2\alpha l} = 19.4 \text{ W}$$

where α = 20/(1000·8.686) = 0.0023 Np/m.

b) The power dissipated along the line is simply given by:

$$P_l = P_{BB} - P_L = 2.9 \text{ W}$$

c) The absolute value of the voltage at the load section can be obtained by inverting the following equation:

$$P_L = \frac{1}{2}\frac{|V_A|^2}{Z_L}$$

Thus:

$$|V_A| = \sqrt{2P_L Z_L} = 62.2 \quad \text{V}$$