# A Rain Cell Model for the Simulation and Performance Evaluation of Site Diversity Schemes

L. Luini and C. Capsoni

***Abstract*—This letter presents a rain cell model for the simulation and performance evaluation of site diversity schemes, useful for the design of Satellite Communication (SatCom) systems operating at very high frequency (e.g. Ka band and above) for which the use of effective Fade Mitigation Techniques (FMTs) is mandatory. The model takes advantage of MultiEXCELL for the synthesis of the local rainfall environment, whose interaction with the SatCom system is simulated to estimate the diversity gain for site separation distances up to 250 km. Tests against the data included in the diversity experiment database maintained by the ITU-R indicate a very good prediction accuracy. The proposed model represents a synthesis, in terms of applicability, of the two methodologies currently recommended by ITU-R, and, in addition, it is more flexible, enabling the analysis of diversity schemes of any order, configuration and complexity.**

***Index Terms*—Rain attenuation, Fade Mitigation Techniques, Site diversity, Satellite Communications.**

## I. Introduction

In the near future, Satellite Communication (SatCom) systems are expected to deliver advanced services such as worldwide high data rate connectivity to the Internet. The need of very large bandwidths, necessary to support this kind of services, is pushing towards the employment of high operational frequencies. Whilst other advantages originate from the use of frequencies higher than 20 GHz (e.g. smaller and lighter terminals, reduced channel congestion, limited interference issues), the main drawback comes from the increasingly detrimental effects induced by atmospheric constituents, mainly by rain, on electromagnetic waves [1]. In this scenario, the design of reliable SatCom systems requires the introduction of smart solutions known as Fade Mitigation Techniques (FMTs) to optimize the system resources (e.g. dynamic reconfiguration of the available onboard power [2]) or to guarantee high-availability of the system even under extreme atmospheric conditions. Site diversity, which consists in the simultaneous use of more than one station to augment the connectivity between the satellite and the ground [3], is one of the most effective FMTs for the mitigation of the fades induced by rain, which quickly decorrelates as the site separation distance $D$ enlarges [4], [5]. Huge efforts have been devoted so far to developing methods for the performance prediction of site diversity systems (see [3] and [6] just as examples), most of them making reference to the classical two-site (balanced) configuration (with some few exceptions considering more stations, e.g. [7] and [8]). As the frequency increases, the system design becomes more and more critical, which triggers the need of models with high prediction accuracy and increased flexibility.

This work proposes a new model for the simulation and performance evaluation of site diversity schemes offering the chance to consider any kind of complex scheme (e.g. more than two stations, different baseline orientation, unbalanced systems). First, the rationale of the method, which takes advantage of MultiEXCELL for the representation of the local rainfall environment [9], is briefly recalled in section II. After its performance assessment against the global database made available by the ITU-R (International Telecommunication Union, Radio communication sector) in section III, an example of the advantages offered by the model (simulation of a multiple-site diversity scheme) is shown in section IV. Finally, section V draws some conclusions.

## II. Site diversity gain prediction

The site diversity prediction method proposed here relies on the MultiEXCELL (Multi EXponential CELL) model described in detail in [9]. MultiEXCELL allows to simulate the interaction between radio systems and the precipitation environment by generating a set of synthetic rain fields, whose ensemble preserves the local Complementary Cumulative Distribution Function (CCDF) of the rain rate (with 1-minute integration time) [10], commonly known as $P(R)$, and reproduces the correct rainfall spatial correlation. Synthetic rain fields, whose spatial resolution is 1 km×1 km and whose dimension is of the order of 250 km×250 km, originate from the arrangement of multiple synthetic rain cells, whose exponential shape is inherited from the EXCELL (EXponential CELL) model [11], according to their natural aggregative process observed in a real rain environment and to the local statistics of fractional rainy area [9]. The occurrence of a given type of cell (i.e. with specific dimension and peak rain intensity) depends on the local $P(R)$, the only input to the model. The key advantage of MultiEXCELL is that, in force of its mixed physical/statistical nature, a relatively small set of synthetic rain fields (approximately 400/500) is sufficient to

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reliably represent the local rainfall process and, thus, to allow the efficient simulation of the interaction between precipitation and radio systems [12], [13].

The method devised in this work to evaluate the performance of site diversity systems relies on the physically-based integration of the specific rain attenuation along each Earth-space link, starting from the synthetic rain maps generated by MultiEXCELL and from the knowledge of the system geometric/electrical characteristics. Taking into account the classical geometry of an Earth-space link undergoing rain attenuation, the prediction procedure involves the calculation of the slant path affected by rain, $L_S$, and of its projection to the ground, $L_H$:

$$L_S = \frac{(h_R - h_S)}{\sin(\theta)} \text{ (km)} \quad \text{and} \quad L_H = L_S \cos(\theta) \text{ (km)} \tag{1}$$

In (1), $\theta$ is the link elevation angle, $h_S$ is the station altitude, whilst $h_R$ is the local mean yearly rain height which, according to recommendation ITU-R P. 839-3 [14], is calculated as $h_R = h_{ITU} + 0.36$ km, being $h_{ITU}$ derived from the global maps of the 0 °C isotherm height provided in [14] and the second addend above introduced to equivalently take into account the extra attenuation caused by the melting layer.

The specific rain attenuation is estimated from synthetic rain maps using the customary power-law relationship $\gamma_R = kR^\alpha$ (dB/km), where $k$ and $\alpha$, derived from recommendation ITU-R P. 838-3 [15], are function of the frequency $f$, link elevation $\theta$ and wave polarization. Afterwards, making reference to Fig. 1, which shows the calculation geometry for a sample position of the diversity scheme within the rain map, the total rain attenuation $A$ affecting each radio link is calculated according to the classical assumption that the rain rate is invariant from the ground up to $h_R$:

$$A = \int_{L_S} \gamma_R(s)\,ds = \int_{L_H} \gamma_R(l)\frac{dl}{\cos(\theta)} = \sum_{i=1}^{N} k(R_i)^\alpha \frac{\Delta l_i}{\cos(\theta)} \text{ (dB)} \tag{2}$$

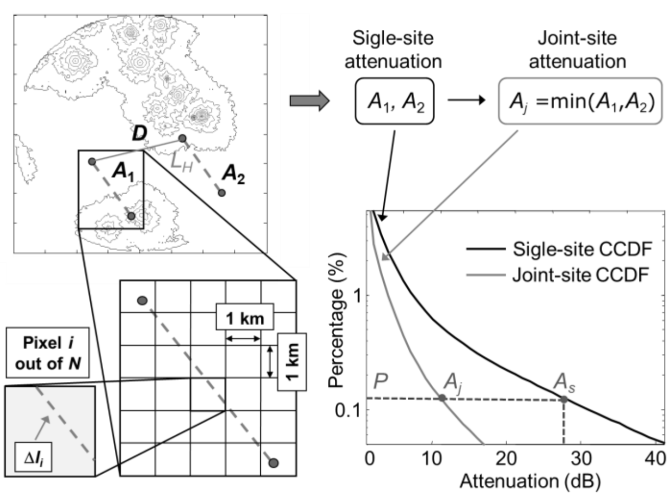


Fig. 1. Reference geometry for the calculation of simultaneous rain attenuation statistics using the synthetic rain maps

For a given position of the scheme within the rain map (which therefore implies site separation distance $D \leq 250$ km), by applying (2), it is thus possible to estimate the rain attenuation simultaneously affecting all the links ($A_1$ and $A_2$), as well as the attenuation $A_j$ that would impair an ideal site diversity system, calculated as $A_j = \min\{A_1, A_2\}$. As a result, by considering different positions of the diversity scheme within the rain field, which considerably increases the statistical significance of the results, and by cumulating all the results obtained from all the MultiEXCELL maps, the single- and joint-site rain attenuation statistics are obtained (refer to the right side of Fig. 1).

## III. Assessment of the Model's Accuracy

The accuracy of the proposed model has been tested against the measurements included in database of two-site diversity experiments maintained by the ITU-R Study Group 3 (DBSG3). Such dataset includes measured single- ($P_s(A)$) and joint-site ($P_j(A)$) rain attenuation statistics, as well as the electrical (e.g. frequency, wave polarization, …) and geometrical (e.g. receiver site, satellite orbital position, …) characteristics of the links and of the diversity scheme (e.g. baseline orientation and length). The database contains 59 entries associated mostly to experiments with 1-year duration performed in many countries characterized by subtropical or temperate climate (latitude ranging from 27.6° N for Tampa, USA, to 55.7° N for Albertslund, Denmark); in the testing activity, only 46 site diversity experiments have been retained because some of the $P_s(A)$s and/or $P_j(A)$s included in the database consist of less than 3 samples and, therefore, are not appropriate for the task. In the selected set of experiments, frequency covers the 11.4-42 GHz band, whilst the site separation distance ranges between 1.7 and 213 km.

As a preliminary step, we have derived the site diversity gain from the available measurements, which is defined as (refer to the right side of Fig. 1) [7]:

$$G(D, A_s) = A_s(P) - A_j(D, P) \text{ (dB)} \tag{3}$$

where $A_s$ and $A_j$ are the attenuation values extracted from $P_s(A)$ and $P_j(A)$ (both for the same probability level $P$), respectively, and $D$ is the site separation distance. Afterwards, in the tests, the following error figure has been adopted [7]:

$$\varepsilon(D, A_s) = 100\left[G_e(D, A_s) - G_m(D, A_s)\right]/A_s \text{ (dB)} \tag{4}$$

where $G_m(D, A_s)$ and $G_e(D, A_s)$ are the site diversity gain, both correspondent to the reference attenuation $A_s$, calculated respectively from the measured and model-predicted rain attenuation CCDFs. Only probability levels equal to or higher than $5\times10^{-3}$% have been considered in the tests so as to maintain a suitable degree of statistical stability in the results. Since only a few measured rainfall rate statistics are available for the site diversity experiments included in the DBSG3, for all the tests, the $P(R)$ predicted using recommendation ITU-R P. 837-6 (Annex 1) has been provided as input to

MultiEXCELL for the generation of the synthetic rain maps [16].

Fig. 2 and Fig. 3 respectively show the average (E$\varepsilon$) and root mean square (RMS$\varepsilon$) values of the error figure $\varepsilon$ in (4) separately for each of the 46 selected experiments. As a reference, the figures also include the scores obtained by the one of the two site diversity prediction models adopted in recommendation ITU-R P. 618-10 (section 2.2.4.1), hereinafter referred to as the "ITU-R model", which relies on the assumption that rainfall, and the associated attenuation, follows a lognormal process both in time and space [17].

Notwithstanding the overall similar scores reported in the figures' legend ($M_E$ and $M_{RMS}$ are the mean values of E$\varepsilon$ and RMS$\varepsilon$, respectively, calculated over all the 46 experiments), it is evident that the performance variation of MultiEXCELL from experiment to experiment is more limited than the one associated to the ITU-R model: whilst similar values are obtained for the standard deviation of E$\varepsilon$ ($\sigma_E$ = 12.2% and 12.9% for MultiEXCELL and the ITU-R model, respectively), a clear difference emerges in the standard deviation of RMS$\varepsilon$ ($\sigma_{RMS}$ = 6.8% and 9.2% for the two models). This finding indicates that, overall, the prediction accuracy delivered by MultiEXCELL is more stable, which, in turn, guarantees an increased reliability in its use.

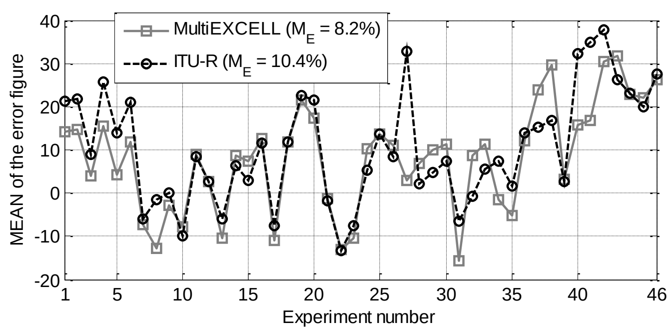


Fig. 2. Results of the testing activity on site diversity models: mean of the error figure (E$\varepsilon$) for all the 46 selected experiments

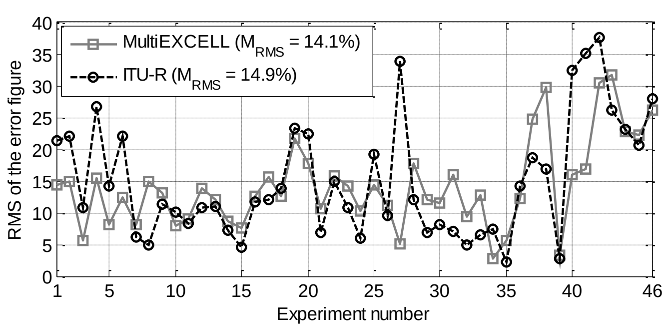


Fig. 3. Results of the testing activity on site diversity models: root mean square of the error figure (RMS$\varepsilon$) for all the 46 selected experiments

A further insight into the models' performance is given in Fig. 4 and Fig. 5, which show the $M_{RMS}$ conditioned to the site separation distance $D$ and the operational frequency $f$, respectively. In both cases, experiments have been classified so as to include approximately the same number of samples in each class, i.e. 9 and 12 in Fig. 4 and Fig. 5, respectively (the $x$-axis reports the central value of $D$ or $f$ for each class). Results in Fig. 4 show that, for both models, the prediction performance degrades as $D$ increases; moreover MultiEXCELL proves to perform better than the ITU-R model for approximately $D$ > 15 km, i.e. for distances (gain values) of interest in practical systems. Advantages in the use of the proposed model emerge also from Fig. 5: a better accuracy than the one of the ITU-R model is achieved for approximately $f$ > 12 GHz, which are indeed the target frequencies for the application of future advanced SatCom systems.

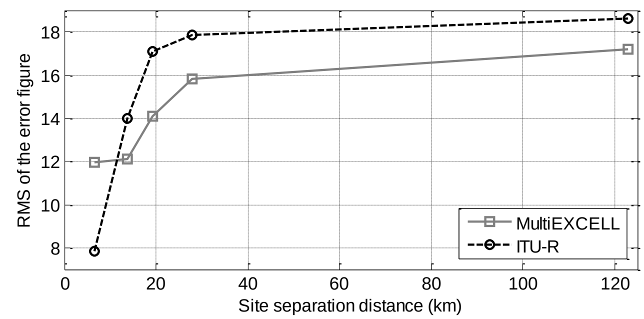


Fig. 4. Results of the testing activity on site diversity models: mean root mean square of the error figure (RMS$\varepsilon$) conditioned to site separation distance (approximately 9 experiments per class)

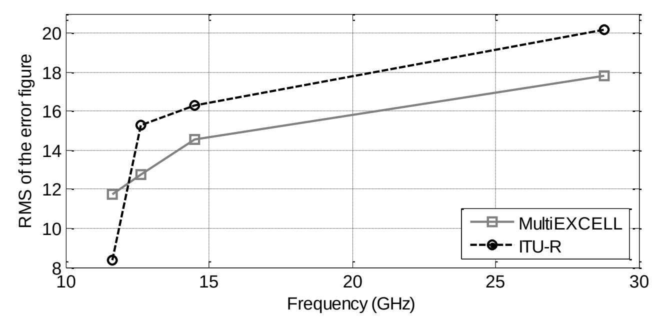


Fig. 5. Results of the testing activity on site diversity models: mean root mean square of the error figure (RMS$\varepsilon$) conditioned to frequency (approximately 12 experiments per class)

## IV. Advantages of the Models

Besides the good prediction accuracy discussed in section III, the proposed model, although far more laborious than other analytical/empirical methodologies, offers additional advantages over some more classical approaches (e.g. [6] and [17]). In fact, whatever the complexity of the radio system, its interaction with the rainfall environment can be simulated quite conveniently: as a result, also unbalanced systems and/or configurations with more than two stations and with different baselines (both in terms of length and orientation) can be considered. As an example, the two reference schemes reported in Fig. 6 (top side) have been investigated ($D$ is the site separation distance; master and slave links are depicted in black and gray, respectively): *scheme A*, which averages the results from the horizontal and the vertical baselines; *scheme B*, which takes advantage of three stations to increase even more the system gain.

Fig. 6 shows the results obtained for a system operating at

30 GHz whose stations are located near Rome NY, USA (latitude = 43.22° N, longitude = -75.41° E, $h_S$ = 150 m, satellite orbital position = 100° W, $\theta$ = 34.4°, $h_R$ = 3.851 km, vertical polarization), using recommendation ITU-R P. 837-6 to estimate the input $P(R)$; only rain attenuation has been considered here (which the effectiveness of diversity is most sensitive to), though, for proper system design, also the effects induced by clouds and gases must be added. The bottom side of the figure shows the trend with distance of the estimated site diversity gain for scheme A ($G_A$, left) and scheme B ($G_B$, right) for different reference attenuation levels 10 dB ≤ $A_s$ ≤ 70 dB (roughly correspondent to $10^{-3}\% \leq P_s(A_s) \leq 10^{-1}\%$).

Results clearly indicate the advantage of multiple-site diversity in mitigating rain attenuation. On the average, considering low outage probabilities (roughly 0.01%), approximately 70%-75% of the rain attenuation can be compensated using two stations, whilst this percentage increases to 80%-85% in the case of three stations.

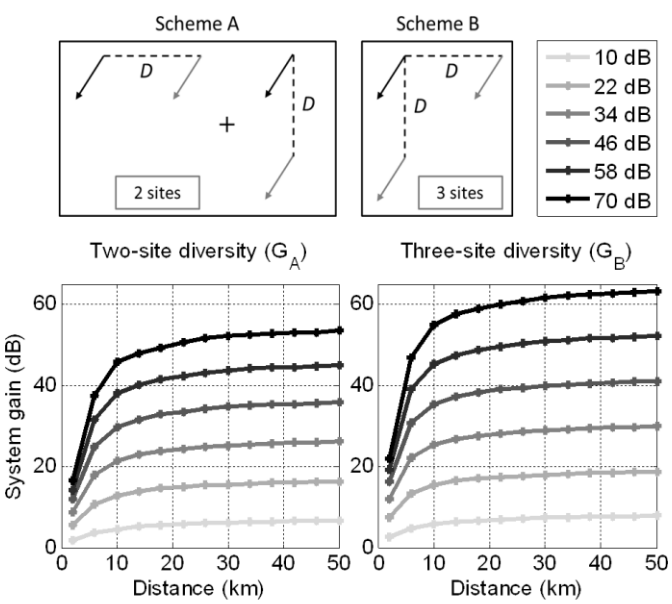


Fig. 6. Top side: reference site diversity schemes considered in the simulations (Rome NY, USA, $f$ = 30 GHz). Bottom side: estimated site diversity gain for schemes A (left) and B (right)

## V. Conclusion

A rain cell model for the simulation and performance evaluation of diversity systems with site separation distances up to 250 km is presented here. The advantage of the proposed model, whose inputs are the local rainfall statistics and the system electrical and geometric characteristics, lies in the possibility to simulate the interaction between precipitation and diversity schemes, whichever the number of stations and the complexity, by using only the limited (yet fully representative of the local rainfall environment) set of synthetic rain maps (roughly 400/500) generated by MultiEXCELL. When tested against the global data included in the site diversity experiments database maintained by ITU-R (sites characterized by subtropical or temperate climate), the proposed model shows a good accuracy in predicting the site diversity gain (overall root mean square of the error, $M_{RMS}$, equal to 14.1%), comparable with the one delivered by the model currently recommended by ITU-R ($M_{RMS}$ = 14.9%), but more stable from experiment to experiment as well as with the operational frequency and site separation distance. This finding, in turn, provides an increased reliability in the use of the proposed model, which, as an example of its potentialities and extended applicability, has been applied for the simulation of a SatCom system operating at 30 GHz with a three-site diversity scheme.

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