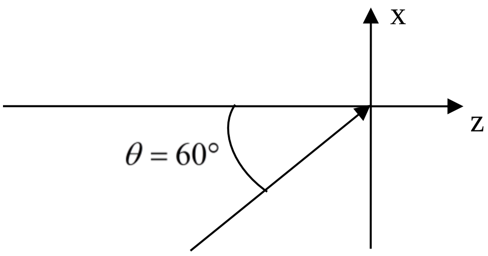
Exercize 1

Let's consider a plane sinusoidal wave whose propagation vector lies on the *xz* plane and identifies an angle of θ = 60° with the *z* axis (see figure below). The wave propagates in a lossy dielectric (ε_r = 2, μ_r = 1 e σ = 0.1 S/m) at *f* = 800 MHz. The electric field is perpendicular to the *xz* plane and its absolute value is 3 V/m (assume the wave phase is zero for *z* = 0 m).



For this wave:

a) Determine the expression of the propagation vector $\vec{\gamma}$.
b) Calculate the phase velocity along the propagation direction.
c) Write the full expression of the electric and magnetic fields as a function of *x*, *y* e *z*.

Solution:

a) $\tan\delta = \sigma/(\omega\varepsilon) = 1.1235$ → no approximations hold

$$|\vec{\gamma}| = \alpha + j\beta = \sqrt{j\omega\mu(j\omega\varepsilon + \sigma)} = 11.9 + j26.5 \quad \text{1/m}$$

$$\vec{\gamma} = (11.9 + j26.5)(\vec{\mu}_z \cos(\theta) + \vec{\mu}_x \sin(\theta)) = \frac{1}{2}(11.9 + j26.5)\vec{\mu}_z + \frac{\sqrt{3}}{2}(11.9 + j26.5)\vec{\mu}_x \quad \text{1/m}$$

b) $v_f = \frac{\omega}{\beta} = 1.89 \times 10^8 \quad \text{m/s}$

c) $\vec{E}(x,y,z) = 3\vec{\mu}_y e^{-j\vec{\gamma}\cdot\vec{r}} = 3\vec{\mu}_y e^{-j\left[\frac{1}{2}(11.9+j26.5)z + \frac{\sqrt{3}}{2}(11.9+j26.5)x\right]} \quad \text{V/m}$

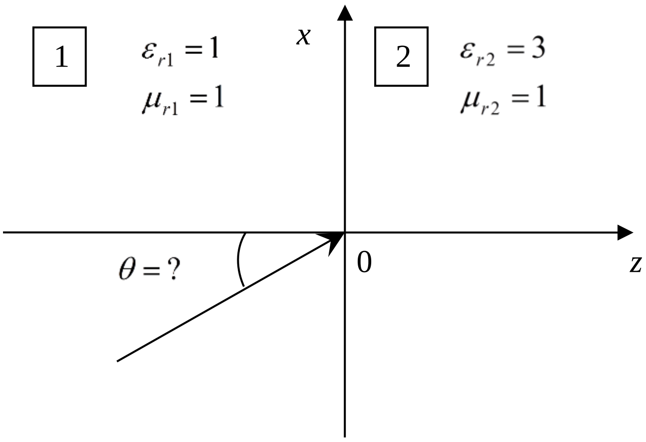
$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon + \sigma}} = 198.2 + j8.9 \quad \Omega$$

$$\vec{H}(x,y,z) = \frac{3}{\eta}(\sin(\theta)\vec{\mu}_z - \cos(\theta)\vec{\mu}_x)e^{-j\vec{\gamma}\cdot\vec{r}} = (\text{-}12.6 + j5.7)\left(\frac{\sqrt{3}}{2}\vec{\mu}_z - \frac{1}{2}\vec{\mu}_x\right)e^{-j\left[\frac{1}{2}(11.9+j26.5)z + \frac{\sqrt{3}}{2}(11.9+j26.5)x\right]}$$

mA/m

Exercize 2

A sinusoidal plane wave at 1 GHz impinges on the discontinuity depicted in the figure below.



The incident wave is circularly polarized, it carries a power density S = 10 W/m^2, and the incident angle is the Brewster angle.

Determine:

a) The incidence, reflection and refraction angles.
b) The TE and TM reflection coefficients at the interface.
c) The polarization of the reflected and transmitted wave.

Solution:

a) Brewster angle (incidence and reflection angles):

$$\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}\right) = 60°$$

Refraction angle:

$$n_1 \sin(\theta_B) = n_2 \sin(\theta_T) \quad \Rightarrow \quad \sin(\theta_T) = \sin(\theta_B)/\sqrt{3} \quad \Rightarrow \quad \theta_T = 30°$$

b) $\Gamma_{TM} = 0$

$$\eta_1^{TE} = \frac{\eta_0}{\cos(\theta_B)} = 754\ \Omega$$

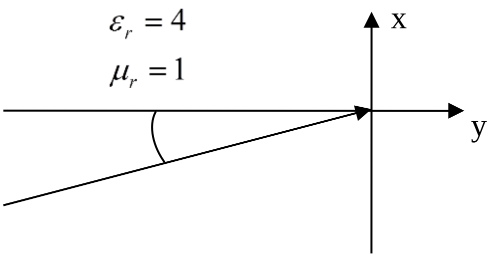
$$\eta_2^{TE} = \frac{\eta_0}{\sqrt{3}\cos(\theta_T)} = 251.3\ \Omega$$

$$\Gamma_{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.5$$

c) The reflected wave is only TE → linear polarization.
Polarization of the refracted wave → elliptical (different absolute value of the TE e TM components).

Exercise 3

Consider the plane sinusoidal wave in the figure below.



The electric field is:

$$\vec{E}(x,y,z) = \left(2e^{j\frac{\pi}{4}}\vec{\mu}_z + \sqrt{3}\vec{\mu}_x - \vec{\mu}_y\right)e^{-j8\pi\left(\sqrt{3}y+x\right)} \qquad \text{[V/m]}$$

Determine:

a) Propagation direction.
b) Frequency.
c) The magnetic field.
d) The polarization (if applicable, left- or right-ended).
e) The power density associated to the wave.

Solution:

a) and b)

$$e^{-j8\pi\left(\sqrt{3}y+x\right)} = e^{-j\beta\left(\cos(\theta)y+\sin(\theta)x\right)}$$

$$\begin{cases}\beta\cos(\theta) = 8\sqrt{3}\pi \\ \beta\sin(\theta) = 8\pi\end{cases} \Rightarrow \begin{cases}\tan(\theta) = 1/\sqrt{3} \\ \dfrac{2\pi f}{c}\sqrt{\mu_r\varepsilon_r}\sin(\theta) = 8\pi\end{cases} \Rightarrow \begin{cases}\theta = 30° \\ f = \dfrac{8c}{\sqrt{\mu_r\varepsilon_r}} = 1.2\ GHz\end{cases}$$

c) Two components treated separately:

TE → $(2e^{j\frac{\pi}{4}}\vec{\mu}_z)$

$$\vec{H}_{TE} = \frac{2e^{j\frac{\pi}{4}}}{\eta}\left(\cos(\theta)\vec{\mu}_x - \sin(\theta)\vec{\mu}_y\right) = \frac{e^{j\frac{\pi}{4}}}{\eta}\left(\sqrt{3}\vec{\mu}_x - \vec{\mu}_y\right) \quad \text{[A/m]}$$

with $\eta = \eta_0\sqrt{\mu_r}\big/\sqrt{\varepsilon_r} = 188.5\ \Omega$

TM → $(\sqrt{3}\vec{\mu}_x - \vec{\mu}_y)$

$$\vec{H}_{TM} = -\frac{2}{\eta}\vec{\mu}_z \quad \text{[A/m]}$$

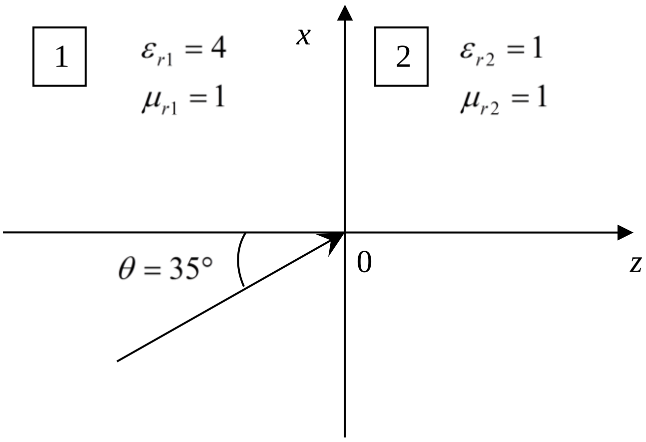
Total field → $\vec{H} = \frac{1}{\eta}\left(e^{j\frac{\pi}{4}}\left(\sqrt{3}\vec{\mu}_x - \vec{\mu}_y\right) - 2\vec{\mu}_z \right) e^{-j8\pi\left(\sqrt{3}y+x\right)}$ [A/m]

d) Right-ended elliptical.

e) $S = \frac{1}{2}\frac{\left|\vec{E}_{TE}\right|^2}{\eta} + \frac{1}{2}\frac{\left|\vec{E}_{TM}\right|^2}{\eta} = 2\frac{1}{2}\frac{4}{\eta} = \frac{4}{\eta} = 21.2$ mW/m^2

Exercize 4

Consider the plane sinusoidal wave below f = 1 GHz.



$$\vec{E}(0,0,0) = j\vec{\mu}_y \qquad [\text{V/m}]$$

Calculate:

a) The reflection coefficient.
b) The power density in the second medium in the z direction.
c) The total electric field in the first medium.

Solution:

a) TE wave

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \Rightarrow \quad \sin\theta_2 = 1.1472 \quad \Rightarrow$ evanescent wave

$$\eta_{TE}^1 = \frac{\eta_0}{\sqrt{4}} \frac{1}{\cos\theta_1} = 230.1\ \Omega$$

$$\eta_{TE}^2 = \frac{\eta_0}{\cos\theta_2} = \frac{\eta_0}{\sqrt{1-(\sin\theta_2)^2}} = \frac{\eta_0}{\pm j0.5622} \Rightarrow \eta_{TE}^2 = j670.6\,\Omega$$

$$\Gamma = \frac{\eta_{TE}^2 - \eta_{TE}^1}{\eta_{TE}^2 + \eta_{TE}^1} = 0.789 + j0.614$$

b) Evanescnt wave → total reflection → no power density in the second medium

c) $\vec{E}(x,y,z) = \vec{E}_i(x,y,z) + \vec{E}_r(x,y,z) = j\vec{\mu}_y e^{-j\frac{2\pi}{\lambda}(\cos(\theta)z+\sin(\theta)x)} + \Gamma\left(j\vec{\mu}_y\right)e^{-j\frac{2\pi}{\lambda}(-\cos(\theta)z+\sin(\theta)x)}$ V/m

with $\lambda = c/\left(\sqrt{4}f\right) = 0.15$ m.