Radio and Optical Wave Propagation – Prof. L. Luini,
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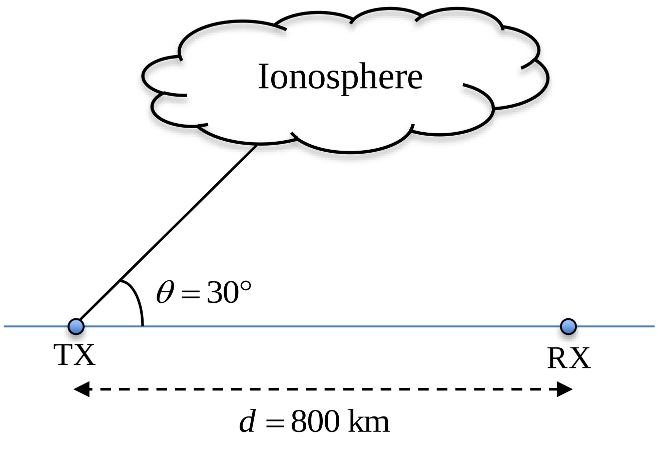
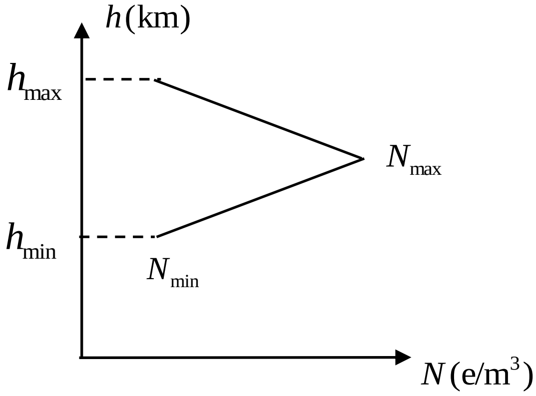
Exercise 1

Making reference to the figure below, the transmitter TX wants to reach the user RX, at a distance d = 800 km, by exploiting the ionosphere. TX transmits with elevation angle θ = 30°. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where $h_{\max}$ = 400 km, $h_{\min}$ = 100 km, N_{max} = 4×10^{12} e/m^3 and N_{min} = 10^{11} e/m^3.
1) Determine the transmission frequency f to be used to reach RX.
2) Using the same elevation angle, what happens if the operational frequency of TX becomes $f_2 = 2f$?

Assume: the virtual reflection height h_V is 1.2 of the height at which the wave is actually reflected.





Solution:
1) Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$$h_V = \frac{d}{2}\tan\theta = 230.9\ \text{km}$$

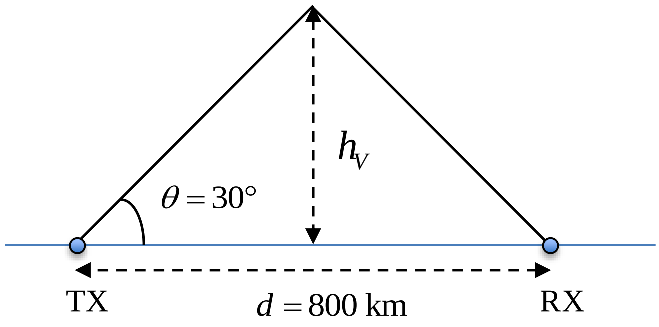
Actual reflection occurs at $h_R = h_V/1.2$ = 192.5 km.

We first need to find the value of N corresponding to h_R, i.e. the value of N causing the total reflection to reach RX. N_{max} lies at the following height:

$h_N = h_{min} + (h_{max} - h_{min})/2 = 250$ km

As a result, the function expressing N as a function of the height h (lower part) is (h is expressed in km):

$$N(h) = \frac{3.9\times 10^{12}}{150}(h-100)+10^{11}\ \text{e/m}^3$$



Therefore:

$$N(h = 192.5\,\text{km}) = N^* = 2.5\times 10^{12}\ \text{e/m}^3$$

The operational frequency f can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N^*}}{f}\right)^2}$$

which leads to:

$$f = \sqrt{\frac{81N^*}{1-(\cos\theta)^2}} = 28.46\ \text{MHz}$$

2) If the frequency becomes $f_2 = 2f = 56.92$ MHz, the critical angle is:

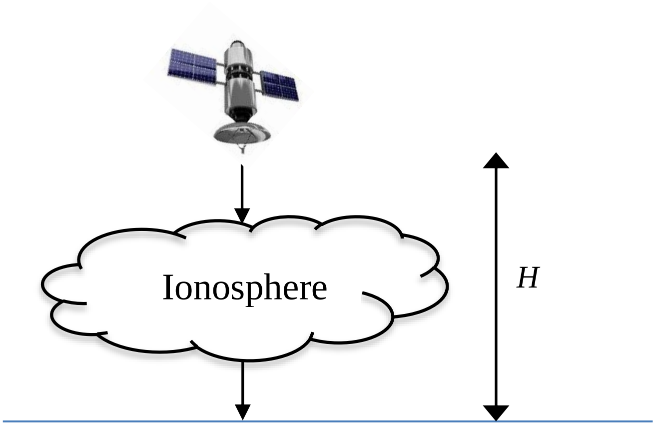
$$\theta_C = \cos^{-1}\left\{\sqrt{1-\left(\frac{9\sqrt{N_{max}}}{f_2}\right)^2}\right\} \approx 18.4°$$

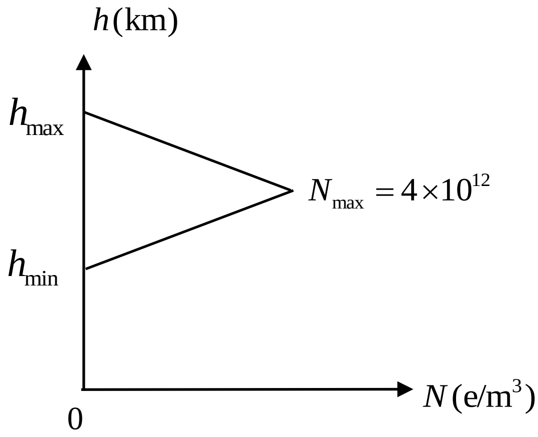
Being the link elevation angle higher than 18.4°, the signal will cross the ionosphere.

Exercise 2

Making reference to the figure below, consider a space-borne radar operating at f = 1 GHz and flying along a LEO orbit (altitude above the ground H = 800 km), which is intended to estimate the ground altitude above the mean sea level by transmitting pulses vertically. Considering the ionospheric profile reported below, where $h_{\max}$ = 600 km, $h_{\min}$ = 60 km and N_{max} = 4×10^{12} e/m^3, calculate the error in estimating the ground altitude (in meters) due to the ionosphere.

Assumption: assume no attenuation and depolarization caused by the ionosphere; assume no troposphere.





Solution:

The critical frequency is:

$$f_C = 9\sqrt{N_{\max}} = 180\ \text{MHz}$$

Being the pulse transmitted vertically and being $f > f_C$, the pulse will cross the ionosphere and will be subject to a group delay because the group velocity, function of h, is:

$$v_G(h) \approx \frac{c}{1+\frac{1}{2}\left(\frac{f_P(h)}{f}\right)^2} = \frac{c}{1+\frac{1}{2}\frac{81}{f^2}N(h)}$$

The overall delay of the pulse, τ, is given by:

$$\tau = \int_0^H d\tau(h)$$

where

$$d\tau(h) = \frac{dh}{v_G(h)}$$

Elaborating the expressions above:

$$\tau = \frac{H}{c} + \frac{1}{2c}\frac{81}{f^2}\int_{h_{\min}}^{h_{mean}} N(h)dh = \frac{H}{c} + \frac{1}{2c}\frac{81}{f^2}TEC$$

The TEC value is (similar expression as in Ex. 1):

$$TEC = 2\int_{h_{\min}}^{h_{mean}} \frac{4\times10^{14}}{270}(h-60)\,dh$$

where h_{mean} = 330 km is the height where the maximum N value lies.
Solving the expressions above, we get:
$TEC = 1.08\times10^{18}$ e/m^2
$\tau \approx 2.7$ ms

Therefore, the additional delay introduced by the ionosphere is:

$$\Delta\tau = \tau - H/c = \frac{1}{2c}\frac{81}{f^2} TEC = 1.46\times10^{-7} \text{ s}$$

Finally, the error in estimating the ground altitude is given by (the pulse crosses the ionosphere two times, i.e. satellite-to-ground and ground-to-satellite):
$\Delta h = 2\, c\, \Delta\tau = 87.6$ m

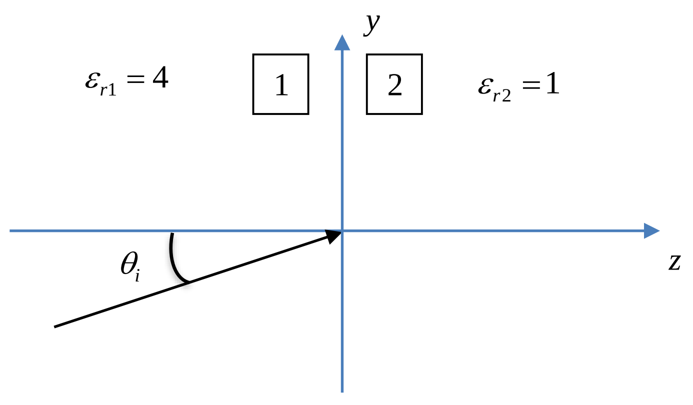
Exercise 3

A plane sinusoidal wave at f = 5 GHz propagates from a medium with electric permittivity ε_{r1} = 4 into vacuum with incidence angle θ_i = 20° (assume μ_r = 1 for both media). The absolute value of the electric field in (0,0) is $\vec{E}_0 = 10e^{j\frac{\pi}{4}}\vec{\mu}_x$ V/m:

1) Write the expression of the reflected electric field in the first medium.

2) Calculate the power density transmitted into vacuum (along direction z).

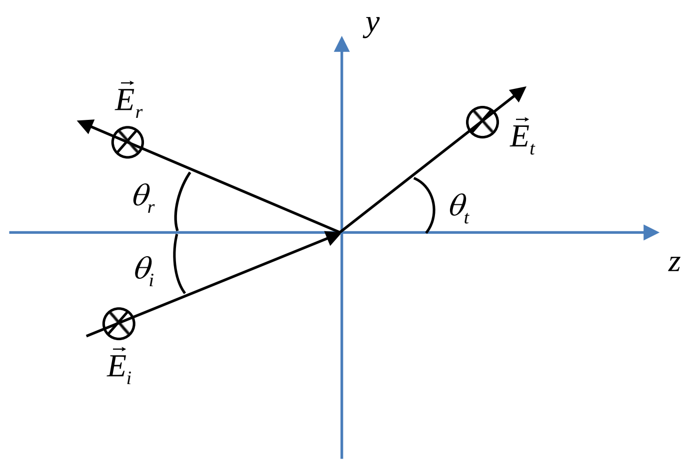
3) Calculate the power density transmitted into vacuum (along direction z) when the incidence angle changes to θ_i = 40°.



Solution:

1) The wave is a TE wave. The refraction angle θ_t in the second medium can be easily calculated from the Snell's law:

$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t \rightarrow \theta_t \approx 43.2°$



Let's calculate the reflection coefficient:

$$\eta_1^{TE} = \frac{\eta_0}{\cos\theta_i\sqrt{\varepsilon_{r1}}} = 200.6\ \ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t\sqrt{\varepsilon_{r2}}} = 516.8\ \ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.44$$

The full expression for the reflected electric field is given by:

$$\vec{E}_r = \vec{E}_0\ \Gamma^{TE} e^{-j\beta_1(y\sin\theta_r - z\cos\theta_r)}\ \text{ V/m}$$

Considering that:

$$\theta_r = \theta_i = 20°$$

and that:

$$\beta_1 = \omega\sqrt{\varepsilon_1\mu_1} = 209.6\ \ \text{rad/m}$$

we obtain:

$$\vec{E}_r = 10e^{j\frac{\pi}{4}}\vec{\mu}_x\ 0.44\ e^{-j\beta_1(y\sin\theta_r - z\cos\theta_r)} = (3.1 + j3.1)e^{-j(71.7y - 196.9z)}\ \text{ V/m}$$

2) The power density along direction *z* in the second medium can be easily calculated as:

$$S_z^2 = S_z^1\left(1 - \left|\Gamma_{TE}\right|^2\right) = \frac{1}{2}\frac{\left|\vec{E}_0\right|^2}{\eta_1}\cos\theta_i\left(1 - \left|\Gamma_{TE}\right|^2\right) = 0.2\ \ \text{ W/m}^2$$

3) If the incidence angle changes to 40°, we obtain:

$$\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\sin\theta_i = 1.28 = \sin\theta_t$$

This is the hint of an evanescent wave in the second medium: as a result, the power density along direction *z* in the second medium is zero because the incidence wave is completely reflected.

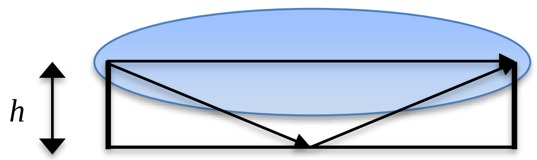
Exercise 4

A terrestrial link operating at f = 26 GHz connects two points that are D = 10 km apart. Calculate the height h above the ground for the transmitter TX and for the receiver RX (assume same height h) such that h is 1.5 times the semi-minor axis of the Fresnel's ellipsoid. In those conditions, determine if the direct and reflected rays combine in a constructive or destructive way (assume that the reflection coefficient of the ground if $\Gamma = -0.5$). Keeping the same h value, determine the minimum shift in frequency needed to maximize the power received by the two rays.

Solution

The semi-minor axis of the first Fresnel's ellipsoid is (λ = 0.0115 m):

$$a = \frac{\sqrt{\lambda D}}{2} = 5.371 \text{ m}$$



Therefore:

$h = 1.5a = 8.06$ m

In these conditions, the two rays combine at the receiver with different phases. The total electric field is given by:

$E = E_0\left(1 + \Gamma e^{-j\beta\delta}\right)$

where E_0 is the field received from the direct wave.

Also:

$\beta = {2\pi}/{\lambda} = 544.54$ 1/m

$\delta = \frac{2hh}{D} = 1.3$ cm

Therefore

$E = E_0(0.647 + j0.354)$ and $|E| = |E_0|0.737$

Thus the rays combine destructively.

In order to maximize the power received, the two rays must combine constructively. Considering that Γ is negative, this happens when:

$e^{-j\beta\delta} = -1$ → $\beta\delta = \pi$ → $\lambda_2 = 2\delta = 2.6$ cm → f_2 = 11.55 GHz

As a result the desired minimum shift in the frequency:

$\Delta f = f - f_2 = 14.45$ GHz