Radio and Optical Wave Propagation – Prof. L. Luini,
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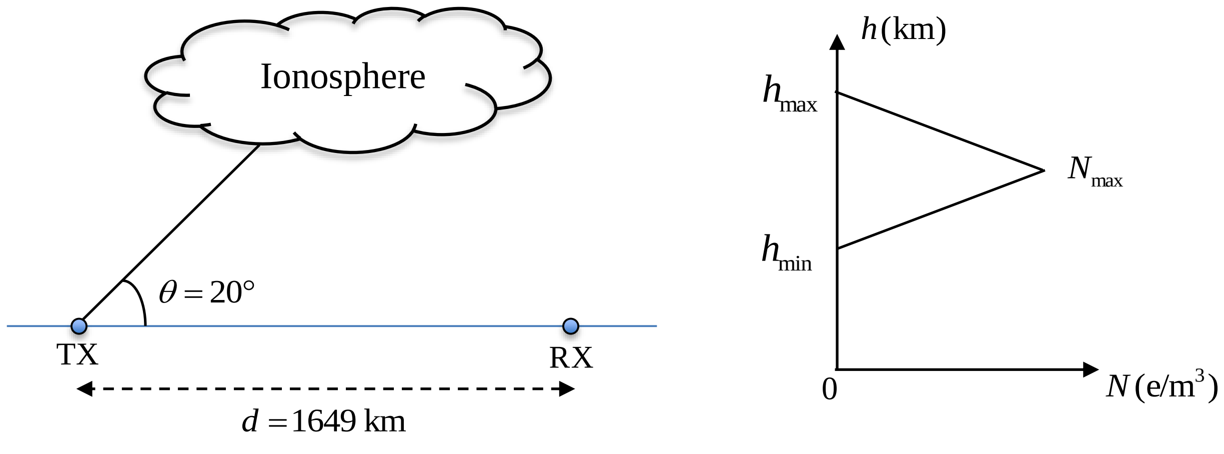
Exercise 1

Making reference to the figure below, the transmitter TX, working at f_1 = 52.6 MHz, reaches the user RX, at a distance d = 1649 km, by exploiting the ionophere. TX transmits with elevation angle θ = 20°. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where h_{max} = 400 km and h_{min} = 100 km.
1) Calculate the maximum electron density N_{max}.
2) Considering the same ionospheric profile and that the frequency of the transmitter increases to f_2 = 150 MHz, evaluate if TX can reach a geostationary satellite seen at elevation angle of 10°.

Assume: the virtual reflection height h_V is 1.2 of the height at which the wave is actually reflected.



Solution:
Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$$h_V = \frac{d}{2}\tan\theta = 300 \text{ km}$$

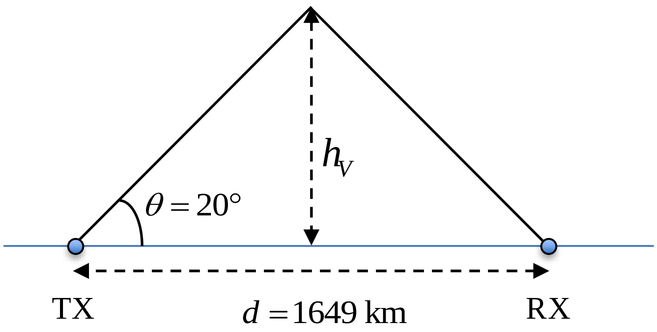
Actual reflection occurs at $h_R = h_V/1.2$ = 250 km. This is also the height at which N_{max} lies:
$h_N = h_{min} + (h_{max} - h_{min})/2$ = 250 km

The value of N_{max} can be derived from:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f_1}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_{max}}}{f_1}\right)^2}$$

The inversion of such equation yields:

$$N_{max} = \frac{\left(1-(\cos\theta)^2\right) f_1^2}{81} \approx 4\times10^{12}\ \text{e/m}^3$$



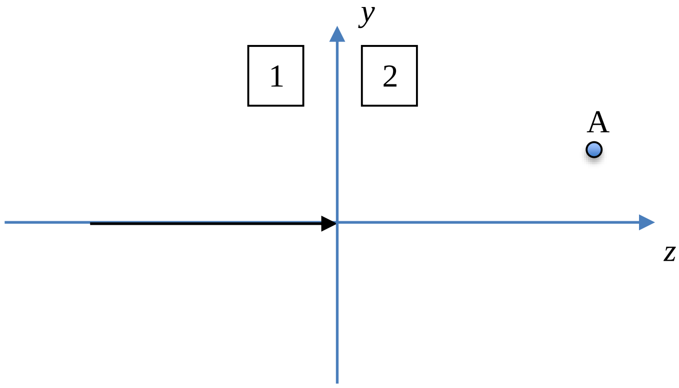
If the transmission frequency increases to f_2 = 150 MHz in the same ionospheric conditions, the maximum elevation angle to obtain total reflection decreases to:

$$\theta_{max} = \cos^{-1}\left[\sqrt{1-\left(\frac{f_P}{f_2}\right)^2}\right] = \cos^{-1}\left[\sqrt{1-\left(\frac{9\sqrt{N_{max}}}{f_2}\right)^2}\right] \approx 6.9°$$

Being the new elevation angle of the link equal to 10°, i.e. higher than θ_{max}, the wave will not be reflected by the ionosphere (only partially refracted) and the TX will be able to reach the GEO satellite.

Exercise 2

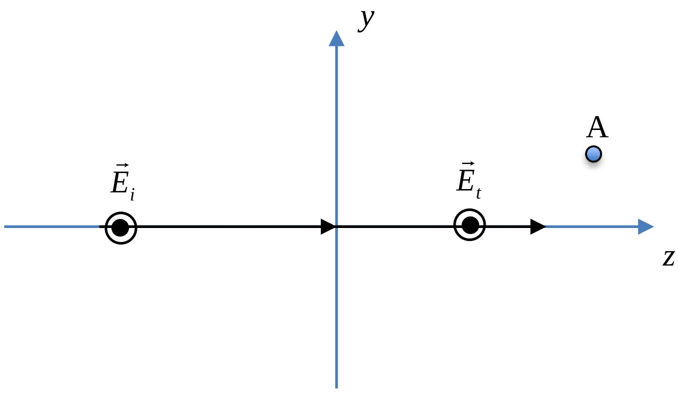
A plane sinusoidal wave at f = 3 GHz propagates in the vacuum and impinges orthogonally on a medium characterized by ε_{r2} = 4-j4, μ_{r2} = 1. The absolute value of the electric field in (z = 0 m) is E_0 = 5 V/m (assume phase equal to zero). Write the full expression of the magnetic field in the second medium and calculate the power received by the antenna in A(z = 0.2 m, y = 0.01 m) that has equivalent area A_E = 1 m^2.



Solution:

The problem concerns a TEM plane wave: the choice of the wave polarization is irrelevant. Let's assume the electric field is as the one in the figure below, i.e.:

$\vec{E}_i = -E_0 \vec{\mu}_x e^{-j\beta_1 z}$ V/m



The intrinsic impedances of the two media (no approximations are possible for the lossy medium) are:

$\eta_1 = \eta_0 = 377\ \Omega$

$\eta_2 = \sqrt{\dfrac{j\omega\mu_0\mu_{r2}}{j\omega\varepsilon_0\varepsilon_{r2}}} = 146.3 + j60.6\ \Omega$

The reflection coefficient is:

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.4271 + j0.1647$

The electric field at the boundary between the two media is given by:

$\vec{E}_t(0) = \vec{E}_i\ (1+\Gamma) = -5\vec{\mu}_x(0.5783 + j0.1647) = (\text{-}2.8916\ \text{-}\ j0.8233)\vec{\mu}_x$ V/m

The full expression of the electric field in the second medium is:

$\vec{E}_t(z) = \vec{E}_t(0)e^{-\gamma_2 z} = \vec{E}_t(0)e^{-(\alpha_2 + j\beta_2)z}$ V/m $\rightarrow \vec{E}_t(z = 0.2 \text{ m}) = \vec{E}_t(0)e^{-\gamma_2 0.2} = 1.95\times10^{-5} + j2.56\times10^{-5}$ V/m

The propagation constant in the second medium is:

$\gamma_2 = \sqrt{j\omega\mu\, j\omega\varepsilon} = 57.2 + j138.2$ m^{-1}

Thus, the full expression of the magnetic field in the second medium is:

$$\vec{H}_t(z) = \frac{(\text{-}2.8916 \text{ - } j0.8233)\vec{\mu}_y}{\eta_2} e^{-\gamma_2 z} = (-0.0189 + j0.0022)\vec{\mu}_y\, e^{-(\alpha_2 + j\beta_2)z} \text{ A/m}$$

The power received by the antenna in A is:

$$P = S_t A_E = \frac{1}{2}\left|\vec{E}_t(z = 0.2 \text{ m})\right|^2 \text{Re}\left\{\frac{1}{\eta_2}\right\} A_E = \frac{1}{2}\frac{\left|\vec{E}_t(z = 0.2 \text{ m})\right|^2}{|\eta_2|}\cos\left(\angle\eta_2\right) A_E \approx 3 \text{ pW}$$

Exercise 3

Given a transmitter for TV broadcasting operating at frequency f = 18 GHz installed on a tower with height h = 20 m, calculate:

1) The area A_1 covered by the transmitter in standard propagation conditions, i.e. assuming the refractivity gradient dN/dh = -37 units/km.

2) The refractivity gradient for which the area covered by the antenna is $A_2 = 1.5\,A_1$.

3) The power margin (in dB) required at the transmitter to guarantee the full coverage of A_2, assuming that the whole area is affected by a constant rain rate R = 5 mm/h.

Assume: the specific attenuation due to rain (dB/km) at 18 GHz, for vertical polarization and 0° link elevation angle is given by $\gamma = kR^{\alpha}$ where k = 0.0771 and α = 1.0025.

Solution:

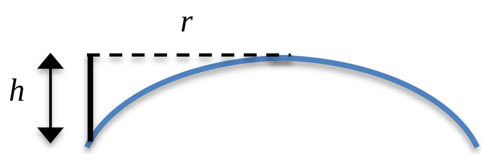
1) Under standard propagation conditions, the equivalent Earth radius is $R_E = 4/3\ R_{earth}$ = 8495 km.

The radius of the area A_1 covered by the antenna is given by the inversion of:

$$h = \frac{1}{2}\frac{r^2}{R_E} \rightarrow r = \sqrt{2hR_E} = 18.4 \text{ km}$$

Therefore the area A_1 will be:

$$A_1 = r^2\pi = 1067.5 \text{ km}^2$$



2) When the propagation conditions change, the area covered by the antenna becomes:

$A_2 = 1.5\,A_1 = 1601.3 \text{ km}^2$

which corresponds to the new radius:

$$r_2 = \sqrt{A_2/\pi} = 22.6 \text{ km}$$

The new equivalent Earth radius is given by:

$$R_E = \frac{1}{2}\frac{r_2^2}{h} = kR_{earth} = \left(\frac{1}{1 + R_{earth}\, {dn}/{dh}}\right) R_{earth} = 12743 \text{ km}$$

Inverting this equation, we obtain:

$$\frac{dn}{dh} = \left(\frac{R_{earth}}{R_E} - 1\right)\frac{1}{R_{earth}} = -78.5\times 10^{-6} \text{ 1/km} \rightarrow \frac{dN}{dh} = -78.5 \text{ units/km}$$

3) Under rainy conditions, the total path attenuation due to rain (assuming constant rain rate along the path) is:

$$A_R = \gamma_R r_2 = 0.387\, r_2 = 8.7\,\mathrm{dB}$$

This is also the power margin to be allocated to the transmitter to cover the whole area A_2 under rainy conditions.

Exercise 4

Consider a zenithal link (elevation angle θ = 90°) from a LEO satellite to a ground station, operating at f = 19 GHz, in which the signal goes through a uniform ice cloud (of thickness h = 4 km) consisting of equioriented ice needles. The specific attenuation of the ice cloud $\alpha_V = \alpha_H = \alpha$ = 0.025 dB/km (V and H are associated to the vertical and horizontal wave polarization, respectively) is constant and uniform through the whole cloud and, in addition, the ice needles cause a differential phase shift (between H and V) equal to 90°/km.
Knowing that the satellite transmits a right-end circular polarization (RHCP):

1) Calculate the signal-to-noise ratio assuming vacuum between the transmitter and the receiver.

2) Determine the polarization in front of the receiver, considering the presence of the ice cloud.

3) Calculate the signal-to-noise ratio, considering the presence of the ice cloud.

Assumptions:

- no other sources of noise apart from the atmosphere (no cosmic background noise)
- always assume vacuum for the calculation of the wavelength
- antennas optimally pointed
- the antenna on the ground receives RHCP waves

Additional data:

- cloud temperature T_{ice} = -5 °C
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 10 dB
- power transmitted by the satellite: P_T = 100 W
- altitude of the LEO satellite: H = 400 km
- bandwidth of the receiver: B = 5 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution:

1) In case no clouds are present, the signal-to-noise ratio (SNR) is simply given by the link budget equation:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R}{kTB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T is the total noise temperature (summation of T_R and the antenna noise T_A), $f_R = f_T = 1$ (antenna optimally pointed). Note that if there are no attenuating media between the transmitter and the receiver, then T_A = 0 K. Considering $\lambda = c/f$ = 0.0158 m, and using the data available → SNR = 4.77 = 6.78 dB.

2) An RHCP wave consists of two orthogonal linear components with the same amplitude and a differential phase shift of -90°. The ice cloud, overall, causes the same attenuation on both components:

$$A_{dB} = \alpha h = 0.1 \text{ dB}$$

which means that at the receiver the two components will have the same amplitude.

The cloud also causes a total differential phase shift of:

$$\Delta\phi = 90° h = 360°$$

Therefore at the receiver the two linear components will still produce an RHCP wave.

3) There are three effects on the SNR due to the presence of the ice cloud:

- Depolarization → no effects as the RHCP is preserved
- Additional atmospheric attenuation → $A = 10^{-\frac{A_{dB}}{10}} = 0.977$
- Increase of the total receiver noise due to the antenna noise $T_A = T_{ice}\,(1-A) = 6.1$ K

The link budget becomes:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R A}{k\left(T_R + T_A\right)B}$$

We obtain:

SNR = 4.56 = 6.59 dB