Radio and Optical Wave Propagation – Prof. L. Luini,
July 20th, 2017

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Exercise 1

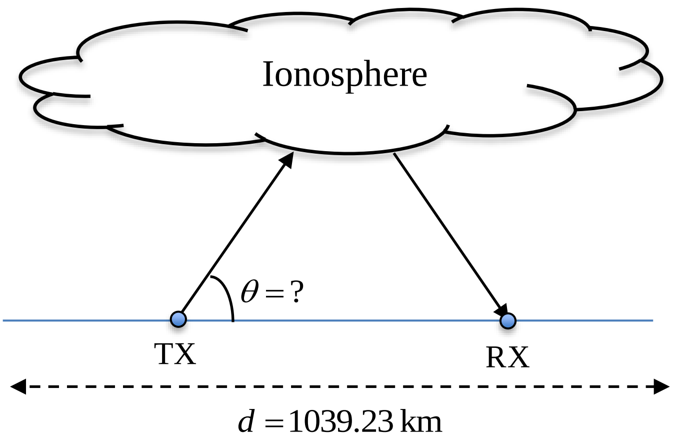
Making reference to the figure below, we want the transmitter TX to reach the user RX by exploiting the ionosphere. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where $N_{max} = 4\times10^{12}$ e/m^3, h_{min} = 300 km.

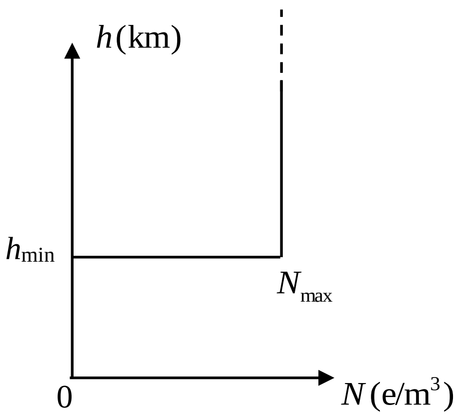
1) Assuming that the link operational frequency is lower than the critical frequency, calculate the elevation angle θ to reach RX (see left side of the figure below).
2) Determine the maximum frequency f_{max} that can be used to reach RX for the elevation angle found at point 1). Select $f = f_{max}$ as the operative frequency for the link.

The absolute value of the electric field associated to the plane wave travelling from TX towards the ionosphere is 1 V/m.

3) Calculate the power received by a parabolic antenna located in RX (perfectly pointed to the wave direction of arrival, with efficiency η_A = 0.8 and with diameter D = 6 m).

Assume: the virtual reflection height h_V is equal to h_R, the height at which the wave is actually reflected; that all EM waves are plane waves; that the attenuation in the ionosphere is negligible.





Solution:

1) Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$$h_V = d/2 \tan\theta \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{2h_V}{d}\right) = 30°$$

Indeed, given the constant profile for N, the wave is reflected as soon as the first layer of the ionosphere is hit, i.e. at height $h_V = h_{\min}$.

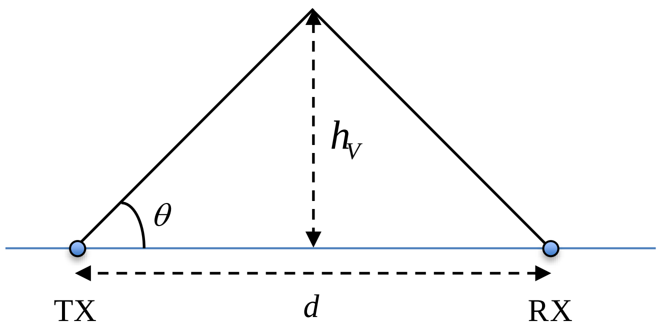
2) The elevation angle, the electron content and the frequency are linked by the following relationship:

$$\cos\theta = \sqrt{1-\left(\frac{f_C}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2}$$

Solving for the frequency f, we obtain:

$$f_{\max} = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} = 36 \text{ MHz}$$

For frequencies higher than 36 MHz, the wave will cross the ionosphere and it will be impossible to reach RX.



3) The wave is totally reflected by the ionosphere; if we assume to deal with plane waves, whatever the position, the power density carried by the wave is always:

$$S = \frac{1}{2}\frac{\left|\vec{E}_0\right|^2}{\eta} = 1.32 \text{ mW/m}^2$$

The antenna effective area is given by:

$$A_E = \eta_A\left(\frac{D}{2}\right)^2 \pi = 22.6 \text{ m}^2$$

The power received at RX is:

$P = SA_E = 29.8\ \text{mW}$

Exercise 2

A plane sinusoidal EM wave (frequency f = 10 GHz) propagates from a dielectric medium into vacuum with incidence angle θ_i = 30° (assume μ_r = 1 for both media). The expression for the electric field is:

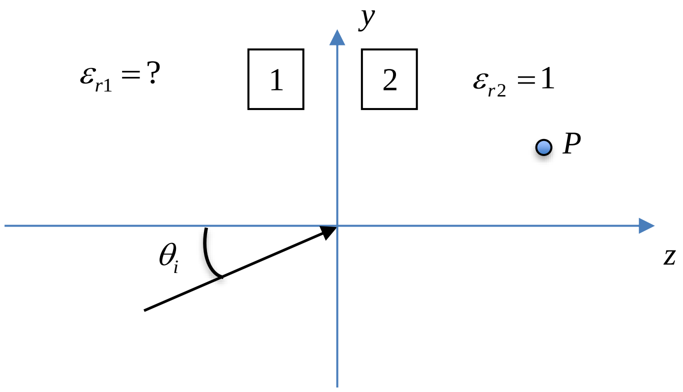
$$\vec{E}(z,y) = \left[\vec{\mu}_x + \left(0.866\,\vec{\mu}_y - 0.5\,\vec{\mu}_z\right)\right] e^{-j362.76z} e^{-j209.44y} \text{ V/m}$$

1) Determine the dielectric permittivity ε_{r1} of the first medium.

2) Determine the polarization of the incident EM wave.

3) Determine the polarization of the reflected wave.

4) Calculate the power received by an antenna located in point P(x = 1 m, y = 1 m, z = 2 m) with effective area A_E = 3 m^2.



Solution:

1) The electric permittivity of the first medium can be derived from the propagation constant β, which, in turn, can be determined from the electric field equation. For example:

$$e^{-j362.76z} = e^{-j\beta\cos\theta z} \quad \Rightarrow \quad \beta = \frac{362.76}{\cos\theta} = 418.88 \text{ rad/m}$$

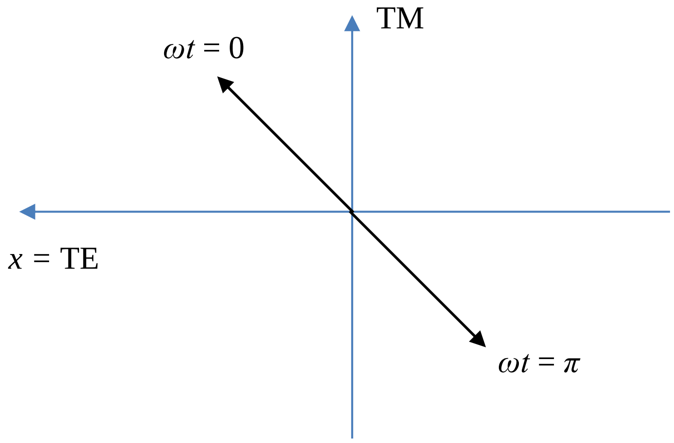
$$\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}} \quad \Rightarrow \quad \varepsilon_{r1} = \left(\frac{c\beta}{2\pi f}\right)^2 = 4$$

2) The polarization of the incident wave is linear: in fact, there is no phase shift between any of the component along x, y and z, i.e. they all oscillate in phase. More specifically, the x component defines the TE wave, while the y and z components define together the TM wave. In fact:

$$\vec{E}(0,0,t) = \text{Re}\left\{\left[\vec{\mu}_x + \left(0.866\,\vec{\mu}_y - 0.5\,\vec{\mu}_z\right)\right] e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_{TE} + \cos(\omega t)\,\vec{\mu}_{TM} \text{ V/m}$$

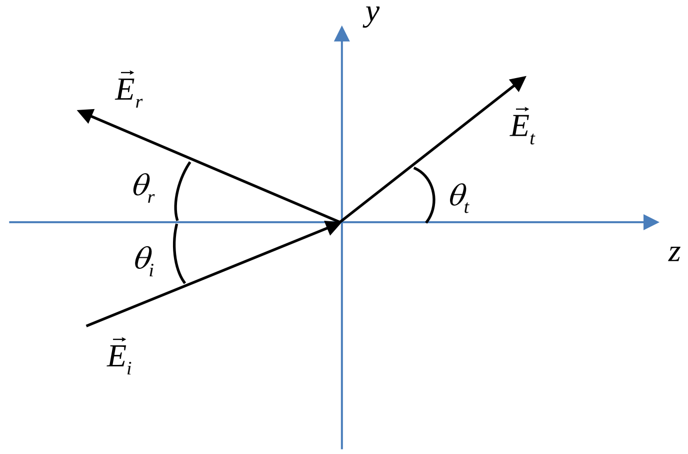
Thus, for t = 0 → $\vec{E}(0,0)\big|_{\omega t=0} = \vec{\mu}_{TM} + \vec{\mu}_{TE}$ V/m

Thus, for $\omega t = \pi \rightarrow \vec{E}(0,0)\Big|_{\omega t=\pi} = -\vec{\mu}_{TM} - \vec{\mu}_{TE}$ V/m



Being the amplitude of the TE and TM waves equal, the tilt angle of the linear polarization is 45°, as indicated in the figure above.

3) The incidence of the wave on the discontinuity will give birth to a reflected wave and a transmitted wave.



We need to consider the two components separately, i.e. calculate the reflection coefficients for the TE and TM waves. In turn, to that aim, we need to calculate the refraction angle:

$$\theta_t = \sin^{-1}\left(\sin\theta_i \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 90°$$

As it turns out, both waves are evanescent in the second medium, i.e. both waves are completely reflected. As a result, the polarization of the reflected wave is still linear, with the same tilt angle.

4) As both waves are completely reflected back by the interface between the two media, no power is transferred into the vacuum and therefore no power will reach the antenna located in *P*.

Exercise 3

Consider a terrestrial link operating at f = 20 GHz and connecting two antennas at distance D = 15 km apart. Calculate the height h above the ground for the transmitter TX and for the receiver RX (assume same height h) to maximize the power transmitted from TX to RX. Assume that the reflection coefficient of the ground if $\Gamma = 0.8$. Assuming to keep the same height h, what happens if the frequency shifts to $f_2 = 1.5\, f_1$?

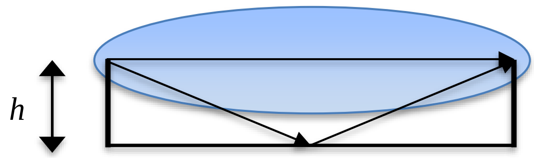
Assume: flat Earth and no ray bending.

Solution:

In order to maximize the power transmitted from TX to RX, two conditions should be met: the first Fresnel's ellipsoid has to be free and the interference between the direct and the reflected rays must be constructive.

As for the first condition, we need to impose that h is higher than the semi-minor axis of the first Fresnel's ellipsoid is (λ = 0.015 m):

$$h > a = \frac{\sqrt{\lambda D}}{2} = 7.5 \text{ m}$$



In general the direct and reflected rays combine at the receiver with different phases. The total electric field is given by:

$$E = E_0\left(1 + 0.8e^{-j\beta\delta}\right)$$

where E_0 is the field received from the direct wave. Also:

$$\beta = {}^{2\pi}/_{\lambda} = 418.88 \text{ rad/m}$$

$$\delta = \frac{2hh}{D}$$

In order to maximize E, we impose:

$$e^{-j\beta\delta} = 1 \;\rightarrow\; \beta\delta = 2N\pi \;\rightarrow\; \beta\frac{2h^2}{D} = 2N\pi \;\rightarrow\; h = \sqrt{\frac{D}{\beta}N\pi}$$

where N is an integer number. For $N = 0 \rightarrow h = 0$ m, which is obviously not acceptable.

For $N = 1 \rightarrow h = 10.6$ m. As $h > a = 7.5$ m, this solution is acceptable, and it yields:

$$E = 1.8\, E_0$$

If the frequency changes to f_2 = 30 GHz, the Fresnel's ellipsoid height reduces and b changes to:

$\beta = {}^{2\pi}/_{\lambda} = 628.32$ rad/m

Therefore:

$|E| = |E_0|\left|1 + 0.8e^{-j\beta\delta}\right| \approx \ 0.2\ |E_0|$

The combination of the two rays becomes destructive.

Exercise 4

Consider a zenithal link (elevation angle θ = 90°) from a MEO satellite to a ground station, operating at f = 30 GHz. The link is impaired by rain, with constant rain rate R = 2 mm/h (both horizontally and vertically) and height h_R = 2 km.

1) What is the best linear polarization to be used to maximize the power received on the ground from the satellite? (assume that all rain drops oriented horizontally)

The specific rain attenuation for the selected polarization is given by $A_{spec} = aR^b$, with a = 0.2291, b = 0.9129.

2) Calculate the diameter of the ground antenna (parabolic type) to guarantee that, under the rainy conditions given above, the power receiving the ground station is P_R = 1 pW. To this aim, assume that:

- antennas are optimally pointed
- the gain of the antennas on board the satellite is G_T = 10 dB
- the ground antenna has efficiency η = 0.8
- the power transmitted by the satellite is P_T = 100 W
- the altitude of the MEO satellite is H = 8000 km

3) Calculate the maximum bandwidth that can be used to guarantee a minimum SNR of 5 dB, given the P_R value above. To this aim, assume that:

- the internal received noise temperature is T_R = 300 K
- the rain drop temperature is T_{rain} = 10 °C

Solution:

1) As rain drops are oriented horizontally, given their typical oblate shape, the best polarization to be used is the vertical one.

2) Considering rain attenuation, the link budget is given by:

$$P_R = P_T G_T f_T \left(\lambda / 4\pi H\right)^2 G_R f_R A_{rain} = P_T G_T \left(\lambda / 4\pi H\right)^2 G_R A_{rain}$$

where f_T and f_R have been set to 1, as the two antennas are optimally pointed, and A_{rain} is the total rain attenuation along the path (linear scale). The latter is calculated as:

$$\left(A_{rain}\right)_{dB} = aR^b h_R = 0.8627 \quad \rightarrow \quad A_{rain} = 10^{-\frac{\left(A_{rain}\right)_{dB}}{10}} = 0.82$$

Converting also G_T to linear scale (10) and solving for G_R:

$$G_R = \frac{P_R}{P_T G_T \left(\lambda / 4\pi H\right)^2 A_{rain}} = 123249.6 \qquad \rightarrow \qquad \left(G_R\right)_{dB} = 50.9 \text{ dB}$$

The effective area of the antenna is given by:

$$A_E = \frac{G_R}{4\pi}\lambda^2 = 0.98 \text{ m}^2$$

The area of the antenna is given by:

$$A_R = \frac{A_E}{\eta} = \pi\left(\frac{D}{2}\right)^2 = 1.23 \text{ m}^2$$

Hence, the antenna diameter is:

$$D = 2\sqrt{\frac{A_R}{\pi}} = 1.25 \text{ m}$$

3) The total noise power in the RX is affected by the antenna noise temperature, that can be calculated as:

$T_A = T_{rain}\,(1-A_{rain}) + A_{rain}\,T_C = 53.2$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{k\left(T_R + T_A\right)B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). By imposing that the minimum SNR is 5 dB, we obtain:

$$\frac{P_R}{k\left(T_R + T_A\right)B} > 3.16 \qquad \rightarrow \qquad B < \frac{P_R}{k\left(T_R + T_A\right)3.16} \approx 65 \text{ MHz}$$