Radio and Optical Wave Propagation – Prof. L. Luini,
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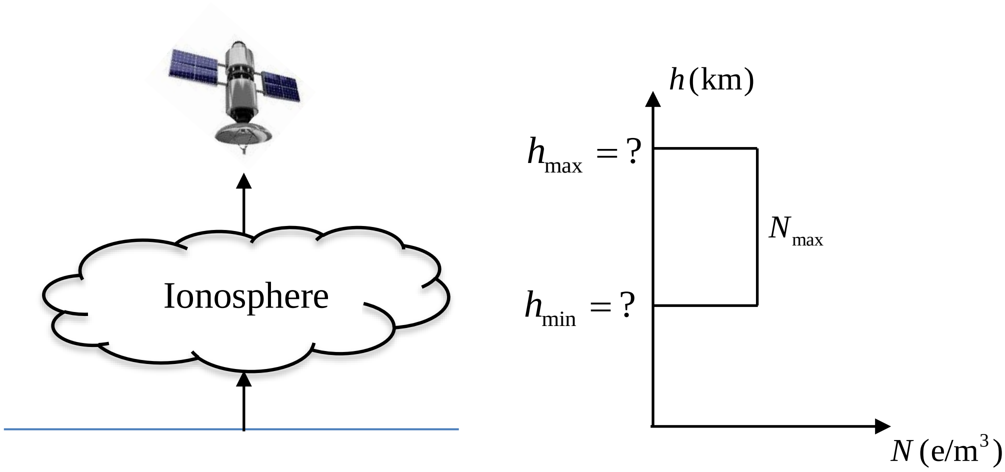
Exercise 1

Making reference to the figure below, a ground station transmits to a low Earth orbit (LEO) satellite along a zenithal link. The satellite altitude is D = 900 km. The profile of the electron content N is constant with height, and its peak value is $N_{\max} = 4\times10^{12}$ e/m^3 (see figure below).

1) Calculate the minimum frequency f_{LEO} for the ground station to be able to reach the satellite.
2) Assuming that the ground station transmits a pulse with carrier frequency $f = 1.1 f_{LEO}$ and that the time required for such signal to reach the satellite is T = 4 ms, calculate the TEC along the path to the satellite.
3) Calculate the thickness of the ionospheric layer Δ_{iono}.



Solution

The critical frequency $f_C = f_{\text{LEO}}$ is associated to N_{max} as follows:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f_{LEO}}\right)^2}$$

where θ is the elevation angle, here set to 90°, Solving for the frequency f_{LEO}, we obtain:

$$f_{LEO} = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} = 18 \text{ MHz}$$

2) The time required for the pulse to reach the LEO satellite depends both on the distance between the ground station and the satellite, and on the presence of the ionosphere. The total time is calculated as the summation of the free space travel time (i.e. as if ionosphere was not present) and of the ionospheric delay:

$$T = T_{FS} + T_{IONO} = \frac{D}{c} + \frac{1}{2c}\frac{81}{f^2}TEC$$

where f = 1.1f_{LEO} = 19.8 MHz.

Therefore, solving for TEC, we obtain:

$$TEC = \left(T - T_{FS}\right)\frac{2cf^2}{81} = 2.904\times10^{18} \text{ e/m}^2$$

3) The TEC is obtained by integrating N with height:

$$TEC = \int_{h_{\min}}^{h_{\max}} N(h)dh = N_{\max}\left(h_{\max} - h_{\min}\right) = N_{\max}\Delta_{iono}$$

Therefore:

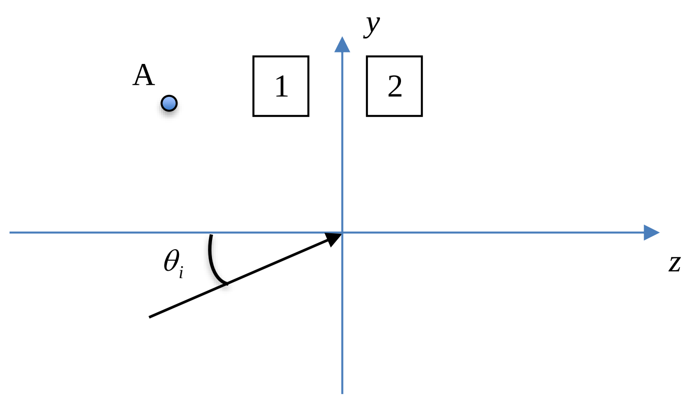
$$\Delta_{iono} = \frac{TEC}{N_{\max}} = 726 \text{ km}$$

Exercise 2

A plane sinusoidal EM wave (frequency f = 5 GHz) propagates from free space into a medium with ε_{r2} = 4, μ_{r2} = 1 (slant incidence). The expression for the electric field is:

$$\vec{E}(z,y) = \vec{\mu}_x e^{-j90.69z} e^{-j52.36y} \text{ V/m}$$

1) Determine the incidence angle of the EM wave.
2) Determine the polarization of the incident EM wave.
3) Calculate the power received by the antenna at point A(x = 2, y = 2 m, z = -2 m,) that has equivalent area A_E = 1 m^2 (consider only the contribution of the reflected wave).



Solution

1) The angle of the incident EM wave can be derived, for example, from the phase constant along z, β_z = 90.69 rad/m:

$$\beta_z = \beta \cos(\theta_i) \quad \text{and} \quad \beta = \frac{2\pi f}{c} \text{ GHz}$$

By combining and inverting such expressions → θ_i = 30°

2) The polarization of the EM wave, which is a TE wave, is linear along x.

3) The transmission angle is given by:

$\sqrt{\varepsilon_{r1}\mu_{r1}} \sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}} \sin\theta_t$ → $\theta_t \approx 14.5°$

To find the reflected wave, let us first calculate the intrinsic impedances for the TE wave:

$$\eta_1^{TE} = \frac{\eta_0}{\cos\theta_i} = 435.3 \ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t \sqrt{\varepsilon_{r2}}} = 194.7 \ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.382$$

The full expression for the reflected electric field is given by:

$$\vec{E}_r = \vec{E}_0 \, \Gamma^{TE} e^{-j\beta_1 (y \sin\theta_r - z\cos\theta_r)} \text{ V/m}$$

Considering that:

$$\theta_r = \theta_i = 30°$$

We obtain:

$$\vec{E}_r = -\vec{\mu}_x \, 0.382 \, e^{-j52.36y} e^{j90.69z} \text{ V/m}$$

The power received by the antenna in A (only reflected wave) is therefore:

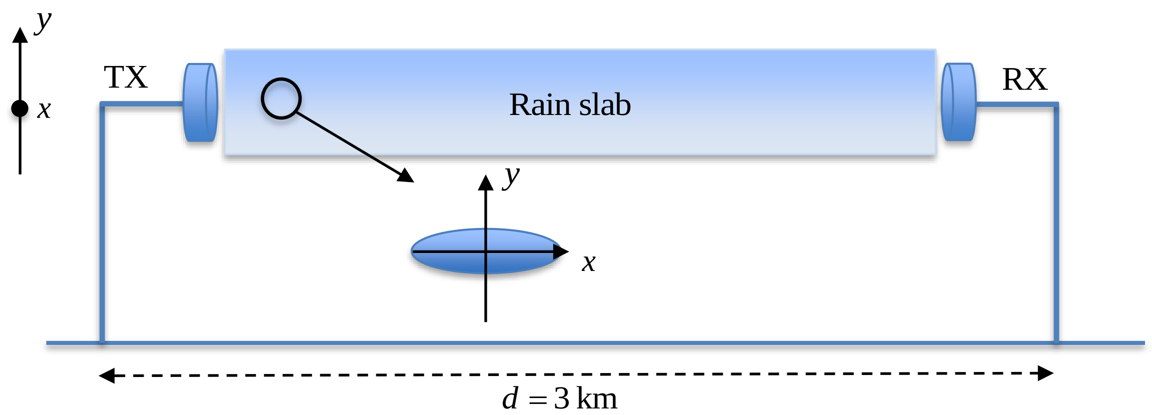
$$P_A = \frac{1}{2} \frac{\left|\vec{E}_r(A)\right|^2}{\eta_0} A_E = 0.19 \text{ mW}$$

Exercise 3

A terrestrial link, with path length d = 3 km and operating at f = 30 GHz, is subject to rain. TX emits the electric field $\vec{E}_0 = \vec{\mu}_x$ V/m and, as shown in the figure below, rain drops are all aligned horizontally (see the figure below). Both antennas at TX and RX are suited for horizontal polarized waves.

1) Calculate the depolarization ratio δ at RX.
2) Calculate the power to be transmitted by TX for the link to be operative; to this aim consider the RX sensitivity to be P_{min} = 1 pW and that the rain rate R = 30 mm/h is constant along the path.
3) Assuming to select the proper kind of antennas, is the horizontal polarization the best one to be used in this scenario to maximize the link availability at rain rates higher than 30 mm/h? If not, identify the best one and explain why.

Assumptions: consider the field emitted by TX as a plane wave, that the Earth is flat and that there are no reflections from the ground. Also assume that the specific attenuation at 30 GHz and 0° elevation angle is $A_{spec} = aR^b$, where a_V = 0.2291, b_V = 0.9129 for vertical polarization, a_H = 0.2403, b_H = 0.9485 for horizontal polarization. Also assume that the propagation constant β is the same for both V and H. Finally consider that antennas are optimally pointed and that they both have gain G = 20 dB.



Solution:

1) Being the wave polarization horizontal and the drops all aligned horizontally too, there is no depolarization, and therefore δ = 0.

2) It is necessary to consider the path attenuation induced by rain:

$A_{spec}^{H} = a_H R^{b_H} = 6.1$ dB/km → $A_R = A_{spec}^{H} d = 18.3$ dB → $A_R^{lin} = 0.0148$ dB

Link budget for the link is:

$$P_R = P_T G_T f_T \left(\lambda/4\pi d\right)^2 G_R f_R A_R^{lin}$$

By inverting this equation, we obtain:

$$P_T = \frac{P_R}{G_T f_T \left(\lambda/4\pi d\right)^2 G_R f_R A_R} = 0.096 \text{ W}$$

3) The best polarization is the linear vertical because no depolarization is induced either and the attenuation caused by rain is lower than for the horizontal polarization.

Exercise 4

A terrestrial point-to-point link operating at f = 30 GHz connects two points that are D = 8 km apart. The height for both antennas is h = 7 m.

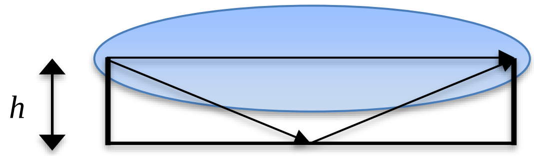
1) Does this condition properly take into account the Fresnel's ellipsoid for the link?
2) Consider the direct ray and the ray reflected by the ground: determine the value of the ground reflection coefficient that maximizes the received power.

Assumption: consider the Earth surface to be flat.

Solution

1) The semi-minor axis of the first Fresnel's ellipsoid is (λ = 0.02 m):

$a = \frac{\sqrt{\lambda D}}{2} \approx 4.47$ m → being $h > a$, the Fresnel's ellipsoid is correctly taken into account.



2) In these conditions, the two rays combine at the receiver with different phases. The total electric field is given by:

$E = E_0\left(1 + \Gamma e^{-j\beta\delta}\right)$

where E_0 is the field received from the direct wave.

Also:

$\beta = {}^{2\pi}/_{\lambda} = 628.31$ 1/m

$\delta = \frac{2hh}{D} = 1.23$ cm

In order to maximize the E, the second part of the sum in the equation has to be equal to 1:

$\Gamma e^{-j\beta\delta}$=1

This can be more easily solved as follows:

$\Gamma e^{-j\beta\delta} = |\Gamma| e^{j\sphericalangle\Gamma} e^{-j\beta\delta} = |\Gamma| e^{j(\sphericalangle\Gamma - \beta\delta)} = 1$

This translates into two conditions:

$|\Gamma| = 1$

and

$e^{j(\sphericalangle\Gamma - \beta\delta)} = 1$ → $\sphericalangle\Gamma - \beta\delta = 2\pi N$ → $\sphericalangle\Gamma = 2\pi N + \beta\delta$

with N = -1:

$\sphericalangle\Gamma = -2\pi + \beta\delta = 1.4137 = 81°$

As a result:

$E = 2E_0$