Radio and Optical Wave Propagation – Prof. L. Luini,
June 29th, 2018

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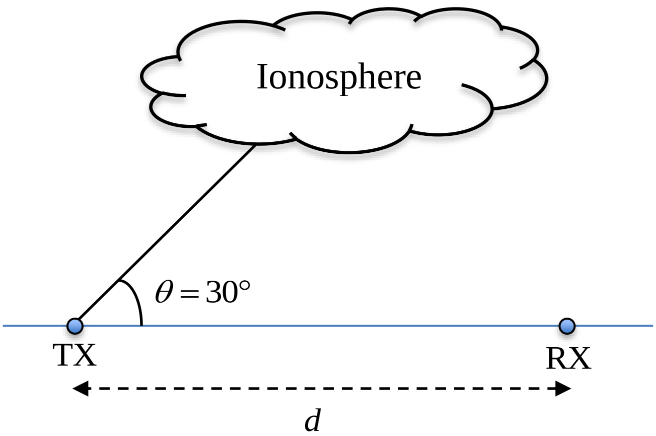
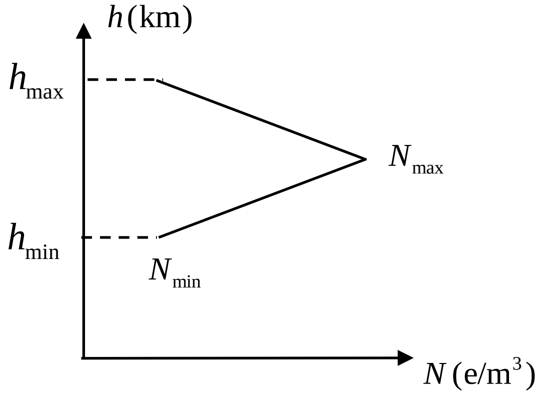
Exercise 1

Making reference to the figure below, the transmitter TX wants to reach the users on the ground by exploiting the ionosphere. TX transmits with elevation angle θ = 30°. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where $h_{\max}$ = 400 km, $h_{\min}$ = 100 km, N_{max} = 4×10^{13} e/m^3 and N_{min} = 10^{11} e/m^3.

1) Assuming that the system can use different carrier frequencies, determine the minimum d_1 and maximum distance d_2 of RX from TX such that the link between TX and RX can be established using the ionosphere (keep the same elevation angle θ).
2) Calculate the operational frequency range associated to d_1 and d_2.

Assume: the virtual reflection height h_V is 1.2 of the height at which the wave is actually reflected.





Solution

1) Considering the figure below, the distance between the TX and RX is given by:

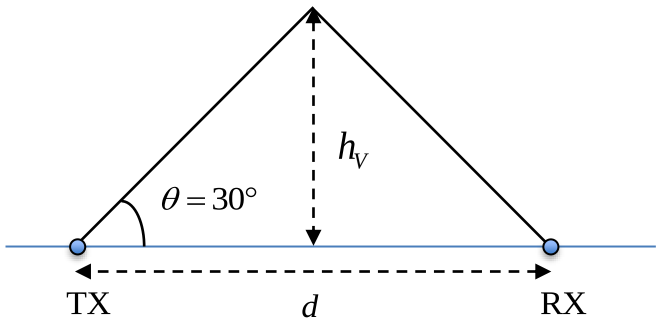
$$d = 2\frac{h_V}{\tan\theta}$$

Considering the actual reflection ($h_R = h_V/1.2$):

$$d = 2.4\frac{h_R}{\tan\theta}$$

The relationship between the operational frequency, the electron content in the ionosphere and the geometry of the link is:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N}}{f}\right)^2}$$



The first shadow zone goes from the transmitter to the distance d_1 where the first user can be reached, which occurs at the following frequency:

$$f_1 = \sqrt{\frac{81N_{\min}}{1-(\cos\theta)^2}} = 5.7 \text{ MHz}$$

The second shadow zone begins at distance d_2, beyond which no users can be reached. This occurs at the following frequency:

$$f_2 = \sqrt{\frac{81N_{\max}}{1-(\cos\theta)^2}} = 113.9 \text{ MHz}$$

The distances d_1 and d_2 are:

$$d_1 = 2.4\frac{h_{\min}}{\tan\theta} = 415.7 \text{ km}$$

$h_{mid} = h_{\min} + (h_{\max} - h_{\min})/2 = 250$ km

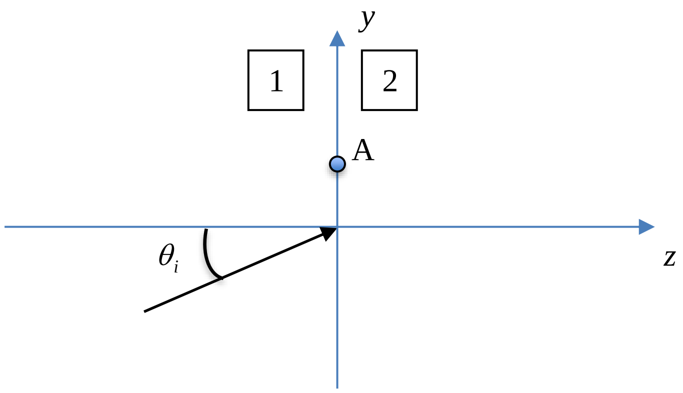
$$d_2 = 2.4\frac{h_{mid}}{\tan\theta} = 1039.2 \text{ km}$$

Exercise 2

A plane sinusoidal EM wave (incidence angle θ_i = 30°) propagates from free space into a medium with ε_{r2} = 2, μ_{r2} = 1. The expression for the incident electric field is:

$$\vec{E}_i = (8.66\vec{\mu}_y - 5\vec{\mu}_z)e^{-j90.69z}e^{-j52.36y} \text{ V/m}$$

1) Determine the polarization of the incident EM wave.
2) Determine the frequency of the EM wave.
3) Determine the magnetic field vector associated to $\vec{E}_i$.
4) Calculate the transmitted electric field at point A(x = 1 m, y = 1 m, z = 0 m).



Solution

1) The incident wave is a TM wave with linear polarization.

2) The frequency of the incident EM wave can be derived, for example, from the phase constant along z, β_z = 90.69 rad/m:

$$\beta_z = \beta \cos(\theta_i) \quad \text{and} \quad \beta = \frac{2\pi f}{c} \text{ GHz}$$

By combining and inverting such expressions → f = 5 GHz.

3) The magnetic field is perpendicular to the yz plane and to the wave propagation direction, specifically, it is oriented as $-\vec{\mu}_x$. Its absolute value is:

$$\left|\vec{H}_i\right| = \frac{\left|\vec{E}_i\right|}{\eta_0} = 26.5 \text{ mA/m}$$

Therefore:

$$\vec{H}_i = -26.5\vec{\mu}_x e^{-j90.69z}e^{-j52.36y} \text{ mA/m}$$

3) The transmission angle is given by:

$$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t \rightarrow \theta_t \approx 20.7°$$

Let us first calculate the intrinsic impedances for the TM waves:

$$\eta_1^{TM} = \eta_0 \cos\theta_i = 326.5 \ \Omega$$

$$\eta_2^{TM} = \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\eta_0 \cos\theta_t = 249.4\ \Omega$$

$$\Gamma^{TM} = \frac{\eta_2^{TM} - \eta_1^{TM}}{\eta_2^{TM} + \eta_1^{TM}} = -0.134$$

The full expression for the transmitted electric field is given by:

$$\vec{E}_t = \vec{E}_i\,(0,0,0)\frac{\cos\theta_i}{\cos\theta_t}(1+\Gamma^{TM})(\cos\theta_t\vec{\mu}_y - \sin\theta_t\vec{\mu}_z)e^{-j\beta_2(y\sin\theta_t + z\cos\theta_t)}\ \text{V/m}$$

For point A, we obtain:

$$\vec{E}_t\,(A) = \left(7.5\vec{\mu}_y - 2.8\vec{\mu}_z\right)e^{-j32.37}\ \text{V/m}$$

being β_2 = 148.1 rad/m.

Exercise 3

Given a transmitter for TV broadcasting operating at frequency f = 500 MHz installed on a tower with height h:

1) Calculate h to achieve a coverage area A_1 around the transmitter of 1000 km^2, considering standard propagation conditions.
2) Keeping the same value of h found at point 1), calculate the new coverage area A_2 if the refractivity gradient changes to dN/dh = -157 units/km.

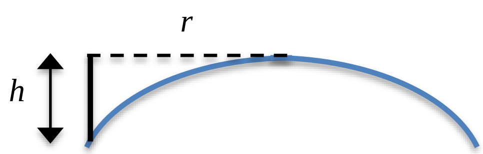
Additional data: transmit power of the broadcast repeater P_T = 100 W, repeater and user antenna gain G = 10 dB, receiver sensitivity P_{min} = 1 nW, antennas optimally pointed, no atmospheric attenuation.

Solution

1) Under standard propagation conditions, the equivalent Earth radius is R_E = 4/3 R_{earth} = 8495 km. The antenna height required to cover the area A_1 is obtained by combining the following expressions:

$$h = \frac{1}{2}\frac{r^2}{R_E} \rightarrow r = \sqrt{2hR_E} \text{ km}$$

$$A_1 = r^2\pi = 2hR_E\pi \rightarrow h = \frac{A_1}{2R_E\pi} = 18.7 \text{ m} \rightarrow r = 17.82 \text{ km}$$



Considering the link budget equation, we can calculate the power received by the users located at the edge of the coverage area:

$$P_R = P_T G_T f_T \left(\lambda/4\pi r\right)^2 G_R f_R \approx 7.2\cdot 10^{-8} \text{ W}$$

Being $P_R > P_{min}$ = 1 nW, the users at the edge of the coverage area will be properly covered.

2) For dN/dh = -157 units/km, the EM wave is curved as much as the Earth surface, and therefore, in the equivalent Earth representation, the surface appears to be flat. In this case, the coverage area is determined through the link budget equation:

$$P_R = P_T G_T f_T \left(\lambda/4\pi r\right)^2 G_R f_R > P_{\min}$$

Inverting the equation, solving for R:

$$r < \sqrt{P_T G_T f_T \frac{\lambda^2}{P_{\min}\left(4\pi\right)^2} G_R f_R} \approx 151 \text{ km}$$

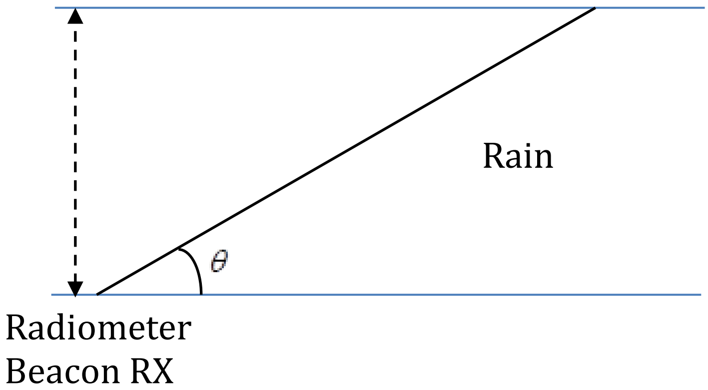
The new coverage are is:

$$A_1 = r^2\pi = 71631 \text{ km}^2$$

Exercise 4

A ground-based radiometer and collocated satellite signal receiver, both operating at 30 GHz, point to a rainy sky (assume the rain rate to be constant horizontally and vertically) with elevation angle $\theta = 30°$. The average physical temperature for the rain layer is 10° C and the radiometer measures brightness temperature $T_B = 265$ K.

1) Assuming to disregard the receiver internal noise temperature, as well as the contribution to T_B coming from gases, clouds and from the cosmic background, calculate the signal-to-noise ratio (*SNR*), using the following additional information: satellite and ground station antennas optimally pointed; satellite transmit power $P_T = 1$ kW; satellite and ground station antenna gain $G = 40$ dB; satellite signal bandwidth $B = 100$ MHz; satellite-ground station distance $R = 40000$ km.
2) Considering that the minimum *SNR* tolerable by the satellite signal receiver is $SNR = 5$ dB, repeat the calculation for $\theta = 20°$ and state whether the receiver operates properly or not (assume the same distance R despite the change in the link geometry).



Solution

1) From the radiometer, we can infer the rain attenuation by inverting

$$T_B = T_{mr}(1 - A_l)$$

where A_l is the rain attenuation but in linear scale and $T_{mr} = 10+273.15 = 283.15$ K

$$A_l = 1 - \frac{T_B}{T_{mr}} = 0.0641 \rightarrow A = 11.93 \text{ dB}$$

The *SNR* is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T (\lambda/4\pi R)^2 G_R f_R A_l}{k T_B B} = 6.94 = 8.4 \text{ dB}$$

2) The attenuation, in dB, for the cloud scenario mentioned here, can be scaled using the cosecant law, i.e.:

$$A^{20°}(\theta) = 10\log 10\left(\frac{1}{A_l}\right)\sin 30° \frac{1}{\sin 20°} = 17.4 \text{ dB} \rightarrow A_l^{20°} = 10^{-(A^{20°}/10)} = 0.018$$

Accordingly, the new brightness temperature is:

$$T_B^{20°} = T_{mr}(1 - A_l^{20}) = 278 \text{ K}$$

The new *SNR* is given by:

$$SNR^{20^\circ} = \frac{P_T G_T f_T \left(\lambda/4\pi R\right)^2 G_R f_R A_l^{20^\circ}}{kT_B^{20^\circ} B} = 1.86 = 2.7 \text{ dB}$$

Under this conditions, the system cannot operate properly.