

**Radio and Optical Wave Propagation – Prof. L. Luini,
September 5th, 2016**

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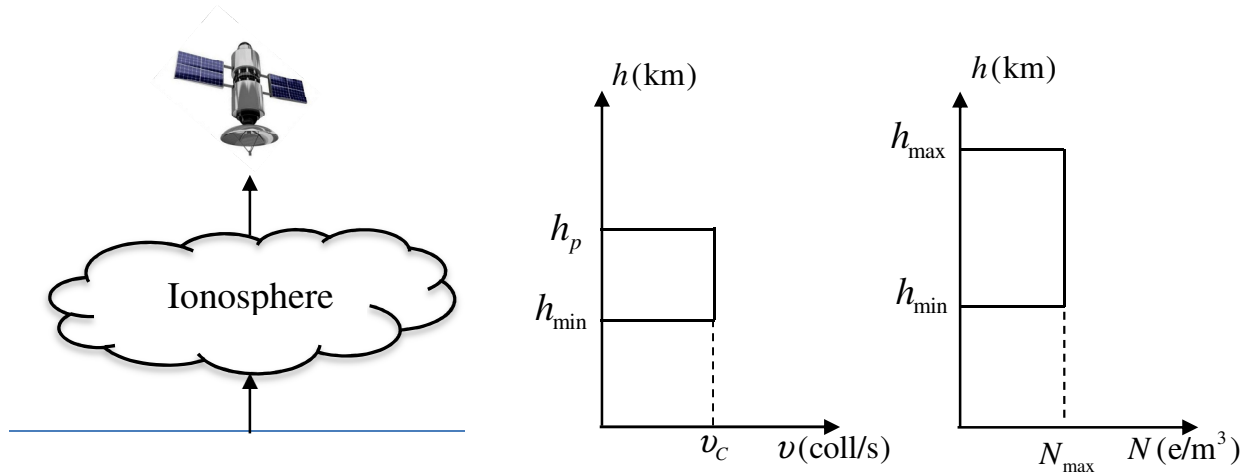
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Exercise 1

Making reference to the figure below, a ground station transmits to a satellite along a zenithal link at $f = 30$ MHz. The satellite altitude is $d = 800$ km. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where $h_{\max} = 400$ km and $h_{\min} = 100$ km. The collision frequency is constant from h_{\min} up to $h_p = 250$ km and its value is $\nu_c = 10^4$ collisions/s.

- 1) Calculate the peak electron content N_{\max} , knowing that the critical frequency for the considered N profile is $f_c = 18$ MHz.
- 2) Calculate the path attenuation introduced by the ionosphere (in dB).



Solution:

1) N_{\max} value is simply obtained from the critical frequency:

$$f_c \approx 9\sqrt{N_{\max}} \Rightarrow N_{\max} = \frac{f_c^2}{81} \approx 4 \cdot 10^{12} \text{ e/m}^3$$

2) The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{Ne^2\nu_c}{m(\nu_c^2 + \omega^2)} = 3.2 \cdot 10^{-8} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_p^2 = \frac{Ne^2}{m\epsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\epsilon_r = 1 - \frac{\omega_p^2}{\nu_c^2 + \omega^2} = 0.64$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = 4.26 \cdot 10^{-6} + j0.28 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between h_{\min} and h_p . Therefore

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.037 \text{ dB/km}$$

$$A_{dB} = \alpha_{dB}(h_{\min} - h_p) = \alpha_{dB}(250 - 100) = 5.55 \text{ dB}$$

Exercise 2

Consider a transmitting antenna at height h from the ground. For this setup:

- 1) Calculate the minimum value of h to guarantee that the antenna covers an area with radius $R = 30$ km (assume standard atmospheric conditions).
- 2) Assume now that there is a receiver a distance R mounted on a mast with same height h . Calculate the minimum operational frequency to guarantee that the first Fresnel's ellipsoid is free.
- 3) In the conditions of point 2), calculate the value of the ground reflection coefficient to maximize the received power.

Solution:

1) The maximum distance covered by the antenna is given by (standard atmosphere \rightarrow equivalent Earth radius factor $k = 4/3$):

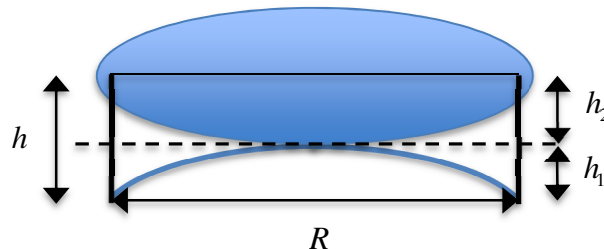
$$R = \sqrt{2hR_{eq}} = \sqrt{2hkR_E} \text{ km}$$

Inverting the above expression, h is determined:

$$h = \frac{1}{2} \frac{R^2}{kR_E} = 52.97 \text{ m}$$

2) Making reference to the sketch below, h is fixed to the value found at point 1), while h_1 is:

$$h_1 = \frac{1}{2} \frac{(R/2)^2}{kR_E} = 13.24 \text{ m}$$



Therefore $h_2 = h - h_1 = 39.73$ m.

h_2 is also equal to the semi-minor axis of the first Fresnel's ellipsoid, which allows to derive the desired minimum operational frequency:

$$h_2 = \sqrt{\lambda R}/2 \rightarrow \lambda = \frac{(2h_2)^2}{R} = 0.2105 \text{ m} \rightarrow f = c/\lambda = 1.43 \text{ GHz.}$$

For higher frequencies, λ will decrease, and so will the semi-minor axis of the first Fresnel's ellipsoid.

3) Making reference to the sketch below, defining E_0 as the electric field at the receiver associated to the direct ray, the combination of the direct and reflected rays can be assessed as:

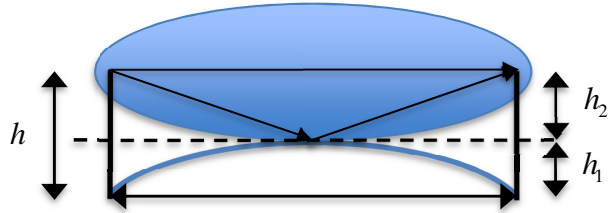
$$E = E_0(1 + \Gamma e^{-j\beta\delta}) \text{ V/m}$$

where Γ is the ground reflection coefficient and δ is the differential path, i.e. the difference between the path travelled along the reflected ray and the one travelled along the direct ray. By definition of the Fresnel's ellipsoid, $\delta = \lambda/2$.

Therefore:

$$E = E_0(1 + \Gamma e^{-j\beta\delta}) = E_0(1 + \Gamma e^{-j\frac{2\pi\lambda}{\lambda^2}}) = E_0(1 - \Gamma) \text{ V/m}$$

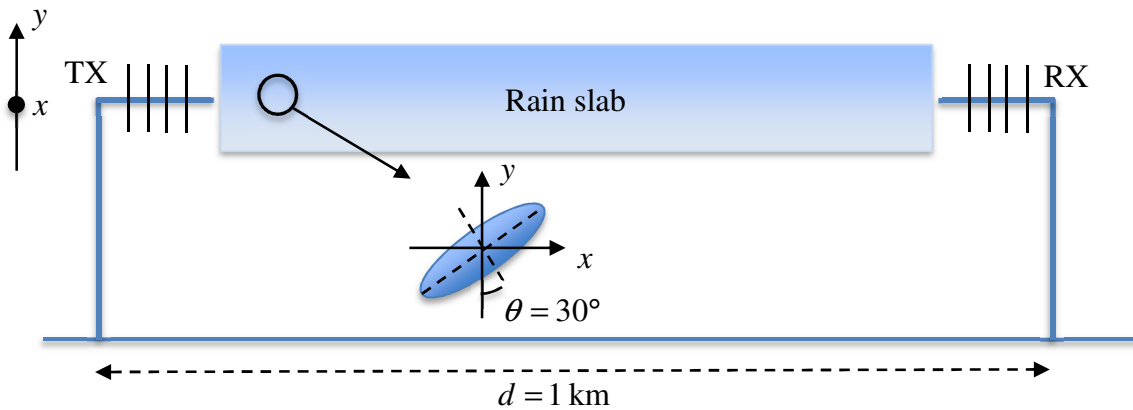
Therefore, in order to maximize the combination of the two rays, i.e. obtain $E = 2E_0$, Γ must be equal to -1.



Exercise 3

A terrestrial link, with path length $d = 1$ km and operating at $f = 30$ GHz, is subject to rain. TX emits the electric field $\vec{E}_0 = 10\vec{\mu}_y$ V/m and, as shown in the figure below, rain drops are tilted with angle $\theta = 30^\circ$. Both antennas at TX and RX are suited for vertical polarized waves. Calculate the depolarization ratio, i.e. $\delta = \frac{|\vec{E}_x|}{|\vec{E}_y|}$ at RX.

Assumptions: consider the field as a plane wave, that the Earth is flat and that there are no reflections from the ground. Also assume that the specific attenuation at 30 GHz and 0° elevation angle is $A_{spec} = aR^b$, where $a_V = 0.2291$, $b_V = 0.9129$ for vertical polarization, $a_H = 0.2403$, $b_H = 0.9485$ for horizontal polarization. Also assume that the rain rate $R = 30$ mm/h is constant along the path and that the propagation constant β is the same for both V and H.



Solution:

In the problem, V and H corresponds to the directions of the minor and major axes of the drop, respectively. To calculate the electric field at RX, the emitted field must be first projected to V and H, and the two components will reach the RX with different attenuation constants. First, let's calculate the specific rain attenuation for V and H:

$$A_{spec}^H = a_H R^{b_H} = 6.1 \text{ dB/km} \rightarrow \alpha_H = \frac{A_{spec}^H}{8.686 \cdot 1000} = 7 \cdot 10^{-4} \text{ Np/m}$$

$$A_{spec}^V = a_V R^{b_V} = 5.1 \text{ dB/km} \rightarrow \alpha_V = \frac{A_{spec}^V}{8.686 \cdot 1000} = 5.9 \cdot 10^{-4} \text{ Np/m}$$

α_H and α_V are the attenuation constants for the two directions.

At the receiver, the two components of the electric field along direction V and H will be:

$$E_V = |\vec{E}_0| \cos \theta e^{-\alpha_V d} = 4.8 \text{ V/m}$$

$$E_H = |\vec{E}_0| \sin \theta e^{-\alpha_H d} = 2.5 \text{ V/m}$$

The two components will still be in phase at RX because their propagation constants are assumed to be the same.

Therefore, the total field at RX will be:

$$E_Y = E_V \cos \theta + E_H \sin \theta = 6.3 \text{ V/m}$$

$$E_X = -E_V \sin \theta + E_H \cos \theta = -0.2 \text{ V/m}$$

$$\vec{E}_{RX} = 6.3\vec{\mu}_y - 0.2\vec{\mu}_x \text{ mV/m}$$

The depolarization ration is therefore:

$$\delta = \frac{|\vec{E}_X|}{|\vec{E}_Y|} = 0.0317$$

Exercise 4

A terrestrial link operates at optical frequency (wavelength $\lambda = 1.55 \mu\text{m}$). The path length is $d = 1 \text{ km}$, the optical beam divergence is $\theta = 1 \text{ mrad}$, the transmitted power is $P_T = 1 \text{ W}$ and the photodiode area at the receiver is $A = 10 \text{ cm}^2$. Calculate the received power P_R , knowing that the whole link is affected by a rain slab of constant rain rate. Assume that drops are spherical, all with the same radius $r = 1 \text{ mm}$ and their concentration is $N = 1000 \text{ drops/m}^3$.

Notes: disregard multiple scattering effects.

Solution

First, let us calculate the attenuation constant, which, if the drops all have the same dimension, can be easily expressed as:

$$\alpha = \frac{1}{2} N C_{EXT}$$

Given that, at optical frequencies, the wavelength is much smaller than the drop dimension, the extinction cross section C_{EXT} can be calculated simply as:

$$C_{EXT} = 2A_{drop} = 2r^2\pi = 6.28 \cdot 10^{-6} \text{ m}^2$$

Therefore:

$$\alpha = 0.0031 \text{ Np/m}$$

and the path attenuation due to rain is:

$$A_R = e^{-2\alpha d} = 0.002 = -27 \text{ dB}$$

The link budget provides the received power:

$$P_R = P_T \frac{A}{d^2 \theta^2 \pi / 4} A_R = 2.6 \mu\text{W}$$