Radio and Optical Wave Propagation – Prof. L. Luini,
September 5th, 2017

1	2	3	4

do not write above

SURNAME AND NAME ______________________________

ID NUMBER ______________________________

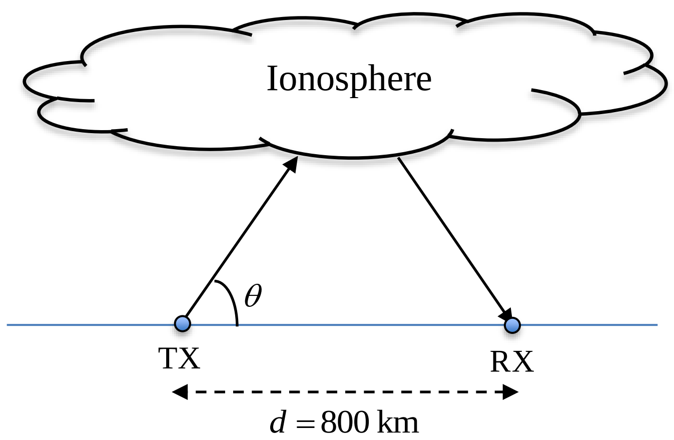
SIGNATURE ______________________________

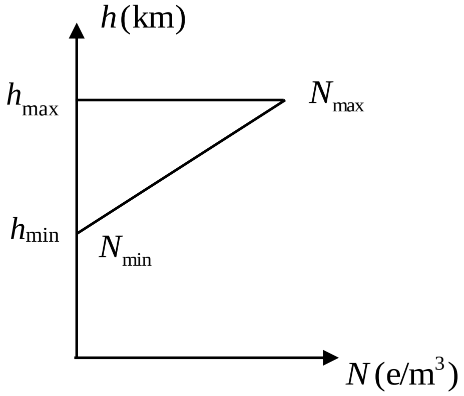
Exercise 1

Making reference to the figure below, we want the transmitter TX to reach the user RX by exploiting the ionosphere (distance d = 800 km and elevation angle θ = 50°). The ionosphere is modelled with the electron density profile sketched in the figure (right side), where $N_{max} = 4\times10^{12}$ e/m^3, $N_{min} = 1\times10^{12}$ e/m^3, h_{min} = 100 km and h_{max} = 400 km.

1) Determine the link operational frequency f to reach RX from TX.
2) For the same ionospheric profile, determine the minimum frequency f_{GEO} to be used to cross the ionosphere and reach a geostationary satellite (visible at elevation θ_{GEO} = 90°) at distance D = 36000 km from TX.
3) Assuming TX transmits a modulated signal with carrier frequency f_{GEO}, calculate the time required for such signal to travel from TX to the geostationary satellite.

Assume that the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected.





Solution:

1) Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$$h_V = \frac{d}{2}\tan\theta = 476.7\text{ km}$$

Therefore, the height at which the wave is reflected is:

$h_R = h_V/1.2 = 397.25$ km

The electron content at h_R can be derived from the linear equation $N(h)$ (see figure):

$$N(h) = \frac{N_{\max} - N_{\min}}{h_{\max} - h_{\min}}\left(h - h_{\min}\right) + N_{\min}$$

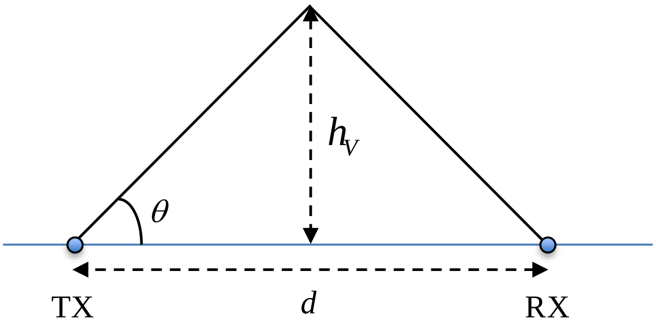
For $h = h_R \rightarrow N = 3.97\times10^{12}$ e/m^3

The elevation angle, the electron content and the frequency are linked by the following relationship:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N(h_R)}}{f}\right)^2}$$

Solving for the frequency f, we obtain:

$$f_{\max} = \sqrt{\frac{81N(h_R)}{1-\left[\cos(\theta)\right]^2}} = 23.42\text{ MHz}$$



2) The minimum frequency necessary to reach the geostationary satellite is the critical frequency f_C: that can be determined by inverting:

$$\cos\theta = \sqrt{1-\left(\frac{f_C}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2}$$

which yields (assuming in this case θ_{GEO}= 90°):

$$f_C = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} = 18 \text{ MHz}$$

3) The time required for the wave to reach the geostationary satellite depends both on the distance between TX and the satellite, and on the presence of the ionosphere. The total time can be calculated as the summation of the free space travel time (i.e. as if ionosphere was not present) and of the ionospheric delay:

$$T = T_{FS} + T_{IONO} = \frac{D}{c} + \frac{1}{2c}\frac{81}{f_{GEO}^2}TEC$$

The TEC is the total electron content and it is easily obtained by integrating *N* with height:

$$TEC = \int_{h_{\min}}^{h_{\max}} N(h)dh = \int_{h_{\min}}^{h_{\max}} \left[kh - (kh_{\min} - N_{\min})\right]dh$$

where

$$k = \frac{N_{\max} - N_{\min}}{h_{\max} - h_{\min}}$$

As a result:

$$TEC = k\left[\frac{h^2}{2}\right]_{h_{\min}}^{h_{\max}} - (kh_{\min} - N_{\min})\left[h\right]_{h_{\min}}^{h_{\max}} = 7.5\times10^{14} \text{ e/m}^2$$

Finally:

$$T = T_{FS} + T_{IONO} = 120 \text{ ms + 0.3125 } \mu\text{s}$$

Exercise 2

Consider a terrestrial link (f = 10 GHz) with path length D = 20 km and with both antennas having the same height h:

1) Determine the best minimum value for h under standard atmospheric conditions.
2) Repeat the same exercise but with the link being deployed on the Moon (radius of the Moon R_m = 1737 km).

Assumption: do not consider the ray reflected by the surface.

Solution:

1) In standard atmospheric conditions on the Earth, the equivalent Earth radius is:

$R_{eq} = 4/3 \times 6371 = 8495$ km

The half visibility is guaranteed with (see figure below):

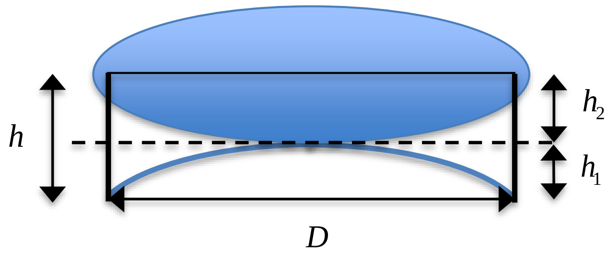
$$h_1 = \frac{1}{2}\frac{(D/2)^2}{R_{eq}} = 5.89 \text{ m}$$

For a good design of the link though, it is also necessary to account for the first Fresnel's ellipsoid, i.e. ($\lambda = c/f = 0.03$ m):

$$h_2 = \sqrt{\lambda D}/2 \approx 12.25 \text{ m}$$

Therefore the proper minimum height for the two antennas is:

$h = h_1 + h_2 = 18.14$ m



2) The same exercise can be repeated for the link on the moon: as there is no atmosphere on the moon, no ray bending occurs. Therefore, h_1 is calculated simply as:

$$h_1 = \frac{1}{2}\frac{(D/2)^2}{R_m} = 28.79 \text{ m}$$

h_2 does not depend on the propagation conditions and therefore:

$h = h_1 + h_2 = 41.04$ m

Exercise 3

Consider a ground station pointing zenithally to a LEO satellite (orbital altitude d = 500 km). The ground station transmits at f = 20 GHz and it is under rainy conditions; the rain rate is constant horizontally and it varies just as a function of the height h (km) as follows:

$$R = 50e^{-h} \text{ mm/h} \qquad 0 \le h \le h_0 = 3 \text{ km}$$
$$R = 0 \text{ mm/h} \qquad h > h_0 = 3 \text{ km}$$

Determine the power to be transmitted from the ground station to guarantee a minimum received power P_R = 0.5 pW aboard the satellite. Assume both antennas has gain G = 20 dB and that they are optimally pointed. Also assume for the calculation of the specific rain attenuation $\gamma \rightarrow k$ = 0.0939 and α = 1.0199.

Solution:

Let us calculated the total rain attenuation affecting the link:

$$A = \int_0^{h_0} \gamma(l)dl = \int_0^{h_0} kR^\alpha dl = \int_0^{h_0} k(50e^{-h})^\alpha dh = 50^\alpha k \int_0^{h_0} e^{-\alpha h} dh =$$
$$50^\alpha k\left(-\frac{1}{\alpha}\right)\left[e^{-\alpha h}\right]_0^{h_0} = 50^\alpha k\left(-\frac{1}{\alpha}\right)(e^{-\alpha h_0} - 1) = \frac{50^\alpha k}{\alpha}(1 - e^{-\alpha h_0}) = 4.74 \text{ dB}$$

$$A_{lin} = 10^{-\frac{A}{10}} = 0.3357$$

The transmit power can be derived from the link budget equation:

$$P_R = P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R A_{lin} = P_T G_T \left(\lambda/4\pi D\right)^2 G_R A_{lin}$$

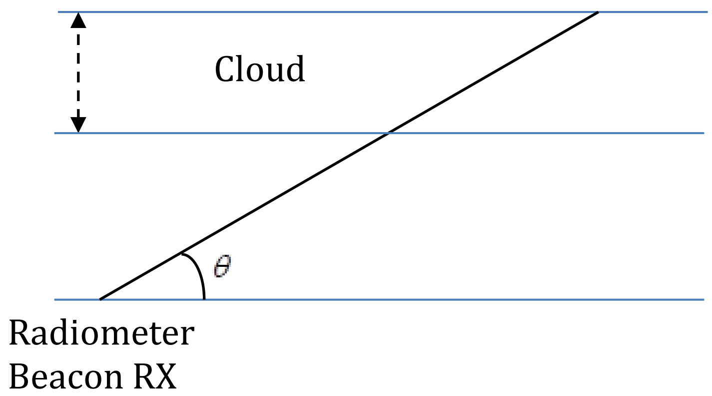
Inverting the equation above:

$P_T \approx 26.1$ W

Exercise 4

In the frame of an Earth-space EM wave propagation experiment, a ground-based radiometer points to a cloudy sky (cloud water vapour content constant horizontally and vertically) with elevation angle $\theta = 30°$ and measures the brightness temperature $T_B = 56$ K. In addition, a beacon receiver, pointing along the same path as the radiometer and collocated with the radiometer, measures a total path attenuation due to the cloud $A = 1$ dB.

1) Assuming to disregard the contribution to T_B coming from gases and from the cosmic background, calculate the physical temperature of the cloud.
2) Considering that the maximum antenna noise temperature tolerable by the beacon receiver is $T_A = 80$ K, determine the minimum elevation angle possible in order to guarantee the correct operation of the beacon receiver.



Solution:

1) The physical temperature of the cloud, T_{mr}, can be calculated by inverting the following equation:

$$T_B = T_{mr}(1 - A_l)$$

where A_l is the cloud attenuation but in linear scale:

$$A_l = 10^{-\frac{A}{10}} = 0.7943$$

Inverting the equation, we obtain:

$$T_{mr} = \frac{T_B}{1 - A_l} = 272.24 \text{ K} = -0.91\,°\text{C}$$

2) The attenuation, in dB, for the cloud scenario mentioned here, can be scaled using the sine of the elevation angle, i.e.:

$$A(\theta) = \frac{A_Z}{\sin\theta}$$

Therefore, the zenithal attenuation A_Z is:

$$A_Z = A(30°)\sin(30°) = 0.5 \text{ dB}$$

The minimum elevation angle can be derived by inverting the following equation:

$$T_A = T_{mr}\left(1 - 10^{-\frac{1}{10}\frac{A_Z}{\sin\theta}}\right)$$

Solving for θ:

$$\theta = \sin^{-1}\left(-\frac{A_Z}{10\log_{10}\left(1 - \frac{T_A}{T_{mr}}\right)}\right) \approx 19.3°$$