Radio and Optical Wave Propagation – Prof. L. Luini,
July 7th, 2017

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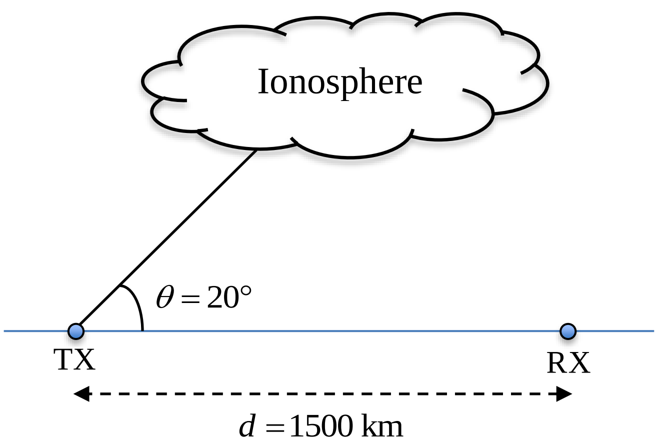
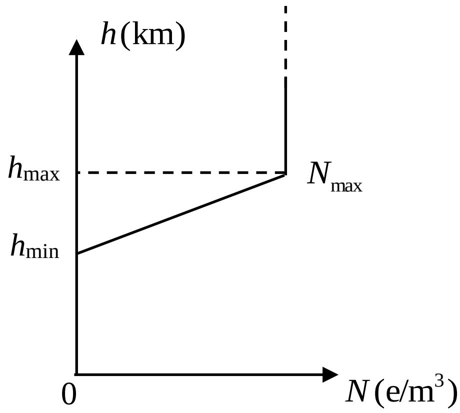
Exercise 1

Making reference to the figure below, we want the transmitter TX to reach the user RX, at a distance d = 1500 km, by exploiting the ionosphere. TX transmits with elevation angle $\theta = 20°$. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where $N_{max} = 4\times10^{12}$ e/m^3, h_{min} = 100 km and h_{max} = 400 km.

1) Calculate the link frequency f_1 to reach RX.
2) Now assume to use $f_2 = 1.5\, f_1$: keeping the same elevation angle, determine whether the TX can still reach the RX.
3) Answer the same question as in 2), but considering $f_3 = 2\, f_1$.

Assume: the virtual reflection height h_V is 1.2 of h_R, the height at which the wave is actually reflected.





Solution:

1) Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$$h_{V1} = \frac{d}{2}\tan\theta = 273 \text{ km}$$

The actual reflection occurs at $h_{R1} = h_{V1}/1.2 = 227.5$ km. The value of electron density N associated to h_R can be obtained from the following linear expression (easily derivable from the profile sketch):

$$N = \frac{N_{\max}}{h_{\max} - h_{\min}}\left(h - h_{\min}\right)$$

Thus, for $h = h_{R1}$, we obtain:

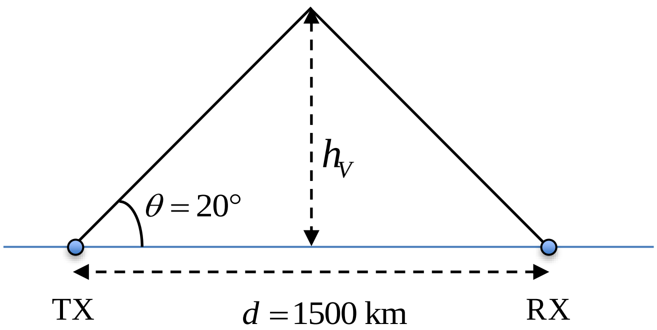
$N_1 = 1.7\times10^{12}$ e/m^3

The elevation angle and the frequency are linked by the following expression:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f_1}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_1}}{f_1}\right)^2}$$

Solving for the frequency f_1, we obtain:

$$f_1 = \sqrt{\frac{81N_1}{1-\left[\cos(\theta)\right]^2}} \approx 34.3 \text{ MHz}$$



2) If the transmission frequency increases to $f_2 = 1.5f_1 = 51.45$ MHz in the same ionospheric conditions, the reflection will occur at a higher layer compared to the situation in 1). In fact, manipulating again the equation above, we obtain the value N_2 causing the reflection at f_2.

$$N_2 = \frac{\left[1-\left(\cos\theta\right)^2\right]f_2^2}{81} \approx 3.82\times10^{12} \text{ e/m}^3$$

This value of electron density corresponds to h_{R2}:

$$h_{R2} = \frac{N_2\left(h_{\max} - h_{\min}\right) + N_{\max}h_{\min}}{N_{\max}} \approx 387 \text{ km}$$

Thus:

$$h_{V2} = 1.2h_{R2} \approx 464 \text{ km}$$

Finally, the new distance reached by TX will be:

$$d_2 = \frac{2h_{V2}}{\tan\theta} \approx \ 2550 \text{ km}$$

RX cannot be reached anymore.

3) For $f_3 = 2f_2 = 68.6$ MHz, we obtain:

$$N_3 = \frac{\left[1-(\cos\theta)^2\right] f_3^2}{81} \approx 6.8\times10^{12} \text{ e/m}^3$$

Being $N_3 > N_{max}$, the wave crosses the ionosphere and RX cannot be reached.

Exercise 2

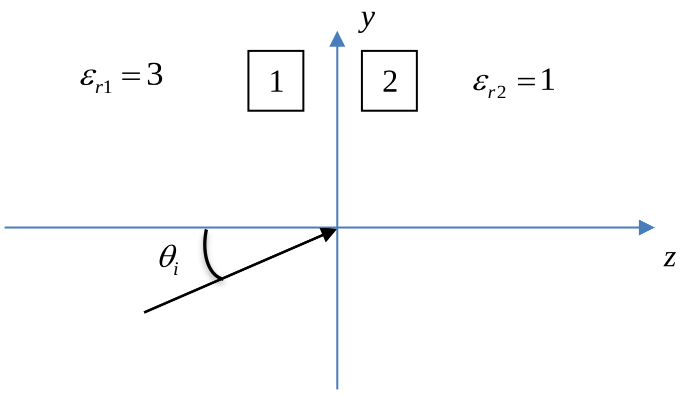
A plane sinusoidal EM wave propagates from a medium with electric permittivity ε_{r1} = 3 into vacuum with incidence angle θ_i = 30° (assume μ_r = 1 for both media). The expression for the electric field is:

$$\vec{E}(z,y) = \vec{E}_{TE} + \vec{E}_{TM} = \left[j10\vec{\mu}_x + \left(10\cos\theta\,\vec{\mu}_y - 10\sin\theta\,\vec{\mu}_z\right)\right] e^{-j\cos\theta 181.4 z} e^{-j\sin\theta 181.4 y} \text{ V/m}$$

1) Determine the frequency of the EM wave.

2) Determine the polarization of the incident EM wave.

3) Determine the polarization of the reflected wave.



Solution:

1) The frequency of the incident EM wave can be derived from the phase constant β = 181.4 rad/m:

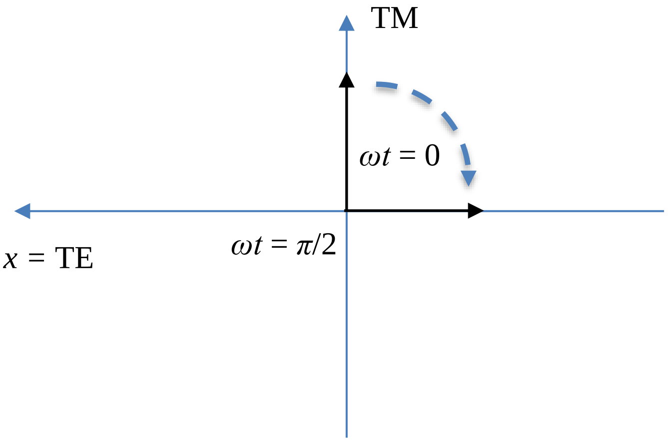
$$\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}} \quad \Rightarrow \quad f = \frac{c\beta}{2\pi\sqrt{\varepsilon_{r1}}} = 5 \text{ GHz}$$

2) The polarization of the incident wave is RHCP because the two TE and TM components have the same amplitude (10 V/m) and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can easily understand the electric field rotation direction:

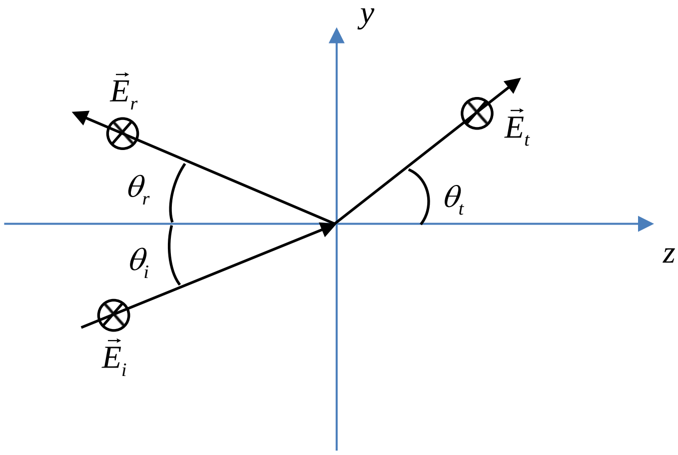
$$\vec{E}(0,0,t) = \mathrm{Re}\left\{\left[j10\vec{\mu}_x + \left(10\cos\theta\,\vec{\mu}_y - 10\sin\theta\,\vec{\mu}_z\right)\right] e^{j\omega t}\right\} = 10\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} + 10\cos(\omega t)\,\vec{\mu}_{TM} \text{ V/m}$$

Thus, for t = 0 → $\vec{E}(0,0)\big|_{\omega t=0} = 10\vec{\mu}_{TM}$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0,0)\big|_{\omega t=\pi/2} = -10\vec{\mu}_{TE}$ V/m



3) The incidence of the wave on the discontinuity will give birth to a reflected wave and a transmitted wave.



We need to consider the two components separately. For the TM, it is worth checking the Brewster angle:

$$\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As the incident angle is the Brewster angle, the TM component is completely transmitted into the second medium (no reflection).

For the TE component, let us check the transmission angle:

$$\theta_t = \sin^{-1}\left(\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 60°$$

The TE component is partially transmitted and partially reflected. As a result, the reflected wave will consist only of the TE component, and will therefore be linearly polarized along the *x* direction.

Exercise 3

A transmitter for TV broadcasting operates at frequency f = 700 MHz, and is installed on a tower with height h = 30 m. The transmit power is P_T =1 W, and the antenna can be considered to be isotropic.

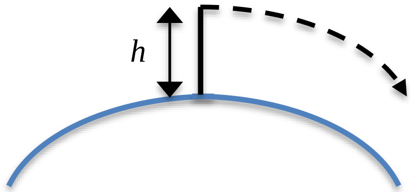
1) Calculate the area A covered by the transmitter considering that:

- The refractivity gradient is dN/dh = -157 units/km.
- The minimum power reaching the receivers at the border of the coverage area must be $P_{\min}$ = 1 pW.
- The receiver antenna is isotropic too.
- Both antennas have perfect efficiency (η = 1).

2) What is the coverage area if the propagation conditions change to dN/dh = -37 units/km?

Solution:

1) Under propagation conditions characterized by dN/dh = -157 units/km, the equivalent Earth radius tends to infinity, which means that the EM wave is bent so as to follow exactly the Earth's curvature. Thus, the limits on the coverage area A do not lie in the visibility, but can be derived from $P_{\min}$.



The received power is given by:

$$P_R = P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R$$

Assuming D is the distance between the TX and the RX at the border of the coverage area. The link budget is further simplified by the fact that both antennas are isotropic and with perfect efficiency, which means: $G_R = G_T = f_R = f_T = 1$.

In order to find D, we have to impose that $P_R > P_{\min}$ = 1 pW.

Thus:

$$P_R = P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R = P_T \left(\lambda/4\pi D\right)^2 > P_{\min}$$

Solving for D:

$$D < \frac{\lambda}{4\pi} \frac{1}{\sqrt{\frac{P_{\min}}{P_T}}} \approx 34.1 \text{ km}$$

The coverage area A is: $A = \pi D^2 = 3653.1 \text{ km}^2$.

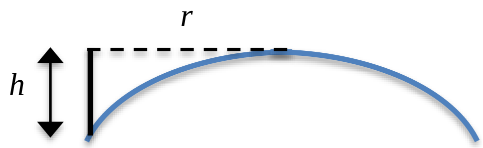
2) In this case, EM waves are bent less than the Earth's curvature, and therefore, this aspect needs to be taken into account (besides the $P_{\min}$). The equivalent Earth radius is $R_E = 4/3\ R_{earth} = 8495$ km.

The radius of the coverage area due to visibility is given by:

$$h = \frac{1}{2} \frac{r^2}{R_E} \rightarrow r = \sqrt{2hR_E} = 22.6 \text{ km}$$

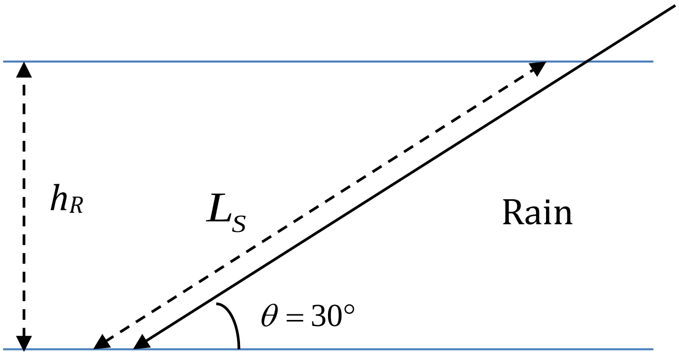
Being $r < D$, Earth's curvature and ray bending become the factors limiting the coverage area A, which is:

$$A = r^2 \pi = 1601.3 \text{ km}^2$$



Exercise 4

Consider a link from a GEO satellite to a ground station and assume that, initially, there is no atmospheric attenuation: associate this condition to the reference signal-to-noise ratio SNR_0. Evaluate the decrease in the SNR (as a function of SNR_0) when rain begins to affect the link. Consider: no cosmic background noise, link frequency f = 20 GHz, rain height h_R = 2 km, rain rate (constant vertically and horizontally) R = 10 mm/h, T_{mr} for rain = 10 °C, specific rain attenuation given by $\gamma = kR^{\alpha}$ (dB/km) with k = 0.0956 and α = 0.9933, receiver internal noise temperature T_R = 300 K.



Solution:

The SNR is given by (no attenuation):

$$SNR_0 = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R}{kT_R B}$$

where D is the distance between the satellite and the ground station and B is the RX bandwidth.

With rain attenuation, the SNR changes to:

$$SNR_1 = \frac{P_R A^{rain}}{P_N^{rain}} = \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R A^{rain}}{kT^{rain} B}$$

where A^{rain} is the rain induced attenuation and T^{rain} is the RX noise temperature in case of rain.

As a matter of fact, the SNR decreases both because of the additional attenuation induced by rain and because of the increase in the noise received by the antenna.

As for A^{rain}:

$$\left(A^{rain}\right)_{dB} = kR^{\alpha} L_S = kR^{\alpha} \frac{h_R}{\sin\theta} \approx 3.8 \text{ dB} \rightarrow A^{rain} = 10^{-(3.8/10)} = 0.42$$

As for T^{rain}:

$$T^{rain} = T_R + T_A = T_R + (1 - A^{rain}) T_{mr} \approx 464 \text{ K}$$

In rainy conditions the RX noise increases by a factor 1.55 if compared to the initial conditions; i.e.:

$$\frac{T^{rain}}{T_R} = \frac{T_R + (1 - A^{rain})T_{mr}}{T_R} \approx 1.55 \quad \rightarrow \quad T^{rain} = 1.55 T_R$$

As a result:

$$SNR_1 = \frac{P_R A^{rain}}{P_N^{rain}} = \frac{A^{rain}}{1.55} \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R}{k T_R B} = \frac{A^{rain}}{1.55} SNR_0 = 0.27 SNR_0$$