Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini,
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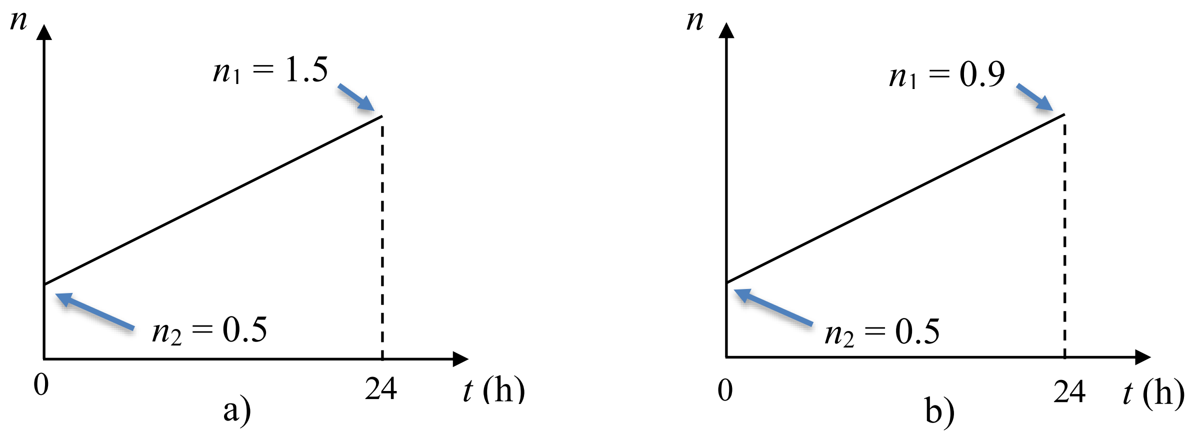
Problem 1

Referring to the sketch below, a ground station points to a geostationary satellite with link elevation angle θ and operational frequency f. Assuming a constant electron content throughout the ionosphere (thickness of the ionosphere equal to H = 300 km), for a specific day, the ionospheric refractive index evolves in time as reported below in a) or in b):

1. Determine which scenario is not realistic and justify why.
2. Considering the realistic scenario, determine the link elevation angle θ that allows establishing the link to the GEO satellite for 80% of the daily time.



Solution

1) Scenario a) is not realistic because the ionospheric refractive index is always lower than one. In fact, it is defined as:

$$n = \sqrt{1 - \left(\frac{9\sqrt{N}}{f}\right)^2}$$

2) The condition for which there is no total reflection, i.e. the ground station can reach the satellite is:

$$n > \cos(\theta)$$

Considering scenario b), the equation expressing *n* is:

$$n = \frac{0.4}{24}t + 0.5$$

80% of the daily time correspond to 0.8·24 = 19.2 hours. Considering that *n* needs to be larger than cos(θ) for 80% of the daily time, the target time is $\bar{t} = 24 - 19.2 = 4.8$ hours. The target *n* value is therefore:

$$\bar{n}(\bar{t}) = 0.58$$

Setting $\bar{n} = \cos(\theta)$ and inverting this equation, the target elevation angle is obtained:

$$\theta = \cos^{-1}(\bar{n}) \approx 54.55°$$

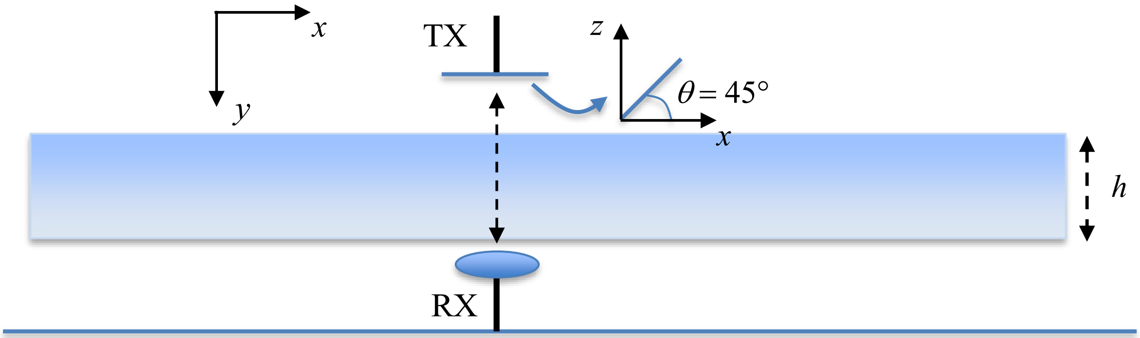
Problem 2

Consider the zenithal link in the figure below. The TX antenna is linear with a 45° tilt on the *xz* plane as shown in the figure below, while the RX antenna is designed to receive waves with any circular polarization. As depicted in the figure below, the wave crosses a very thin anisotropic tropospheric layer of thickness h (mixed ice-water particles), whose propagation constants are:

$\gamma_z = \alpha_z + j\,\beta_z = 1.6\times10^{-3} + j251.3274$ 1/m
$\gamma_x = \alpha_x + j\,\beta_x = 1.6\times10^{-3} + j251.3588$ 1/m

For this scenario:

1) Determine the minimum layer thickness h that maximizes the received power: to this aim, neglect the tropospheric attenuation affecting the wave.
2) Using the h value determined at point 1, calculate the tropospheric attenuation affecting wave.



Solution

1) The linear antenna obviously emits a linear polarization with the same tilt as the antenna itself. Thus, the electric field can be split into the two orthogonal *x* and *z* components, having the same amplitude and phase. Neglecting the attenuation induced by the particles, the received power is maximized when the wave polarization changes from linear to circular. For this to occur, given the same amplitude for the two linear components, the total differential phase shift $\Delta\phi$ after the anisotropic layer needs to be equal at least to $\pm\,\pi/2$. In mathematical terms:

$$\Delta\phi = (\beta_x - \beta_z)h = \pi/2$$

Inverting this equation → $h \approx 50$ m.

2) The attenuation induced by the anisotropic layer is given by:

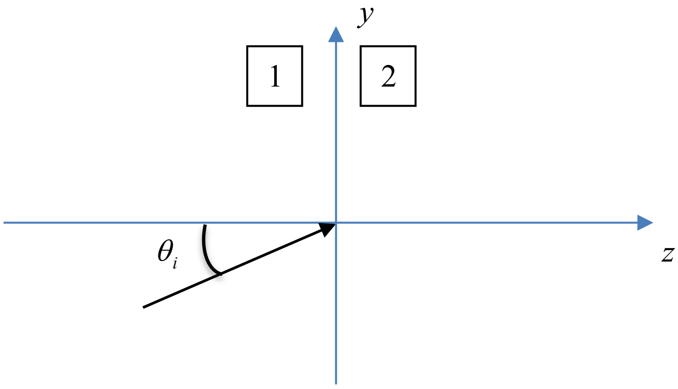
$$A = e^{-2\alpha_z h} = 0.8534 = 0.6887 \text{ dB}$$

Problem 3

A plane sinusoidal EM wave propagates from free space into medium with electric permittivity $\varepsilon_{r2} = 9$ (assume $\mu_{r2} = 1$) with incident angle θ_i. The expression for the incident electric field is:

$$\vec{E}(z,y) = \left[-\vec{\mu}_x + 2j\left(0.5\vec{\mu}_y - 0.866\vec{\mu}_z\right)\right] e^{-j104.72z} e^{-j181.38y}$$

1) Determine the incident angle θ_i.
2) Determine the frequency of the wave.
3) Determine the polarization of the incident EM wave.
4) Determine the power density, propagating along z, associated to the TE component of the transmitted wave.



Solution

1) The incident angle θ_i can be derived from the TM component of the wave. For example:

$$0.5\vec{\mu}_y = \cos(\theta_i)\vec{\mu}_y \rightarrow \theta_i = 60°$$

2) The frequency of the wave can be derived, for example, from β_z:

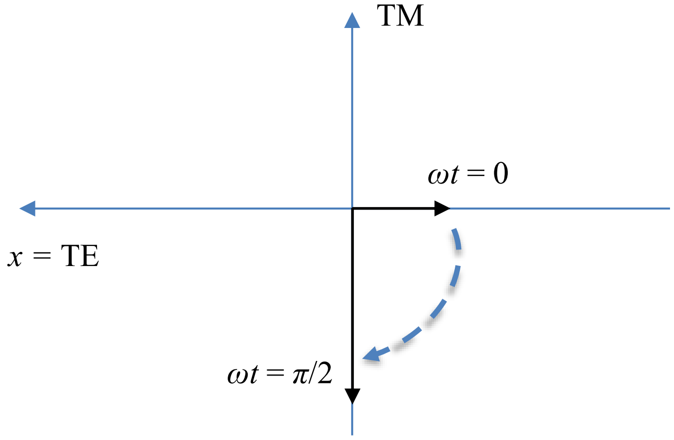
$$\beta_z = \frac{2\pi f}{c}\cos(\theta_i) \rightarrow f = 10 \text{ GHz}$$

3) The polarization of the incident wave can be derived from the two TE and TM components. In fact, setting y and z to 0, and expressing the dependence on time:

$$\vec{E}(z,y,t) = Re\left\{\left[-\vec{\mu}_x + 2j\left(0.5\vec{\mu}_y - 0.866\vec{\mu}_z\right)\right]e^{j\omega t}\right\}$$

Thus, for $t = 0 \rightarrow \vec{E}(0,0,0) = -\vec{\mu}_x$ V/m
Thus, for $\omega t = \pi/2 \rightarrow \vec{E}(0,0,\omega t) = -2\vec{\mu}_{TM}$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



It is a RHEP wave.

4) From Snell's law:

$$\sin(\theta_i)\sqrt{\varepsilon_{r1}} = \sin(\theta_t)\sqrt{\varepsilon_{r2}} \;\rightarrow\; \theta_t = 16.8°$$

Therefore:

$$\eta_1^{TE} = \frac{\eta_1}{\cos(\theta_i)} = 754\ \Omega$$

$$\eta_2^{TE} = \frac{\eta_2}{\cos(\theta_t)} = 131.3\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.7$$

The TE wave power density of the transmitted wave along z is:

$$S_{t,TE}^{z} = S_{i,TE}^{z}\left(1-\left|\Gamma^{TE}\right|^2\right) = S_{i,TE}\cos\theta_i\left(1-\left|\Gamma^{TE}\right|^2\right) = \frac{1}{2}\frac{\left|\vec{E}^{TE}\right|^2}{\eta_1}\cos\theta_i\left(1-\left|\Gamma^{TE}\right|^2\right)$$

Therefore:

$$S_{t,TE}^{z} = 3.35 \times 10^{-4}\ \mathrm{W/m^2}$$

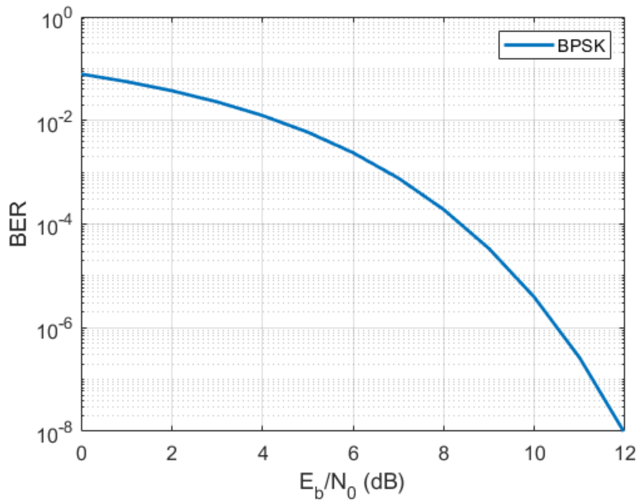
Problem 4

Consider a download link from a LEO satellite to a ground station, operating at f = 26 GHz and with elevation θ = 90°. The signal crosses a uniform ice cloud (of thickness h = 2 km) consisting of equioriented ice needles. The specific attenuation of the ice cloud is negligible and the ice needles cause a differential phase shift (between H and V) equal to 90°/km. The satellite transmits a right-end circular polarization (RHCP):

1) Calculate the signal-to-noise ratio assuming free space conditions (i.e. no clouds, nor any other atmospheric effect).
2) Calculate the expected BER, considering the presence of the ice cloud and BPSK modulation.

Assumptions:

- antennas optimally pointed
- no additional losses at the transmitter and at the receiver
- the antenna on the ground receives LHCP waves
- cloud temperature T_{ice} = -2 °C
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 13 dB
- power transmitted by the satellite: P_T = 93 W
- distance to the LEO satellite: H = 500 km
- bandwidth of the receiver: B = 3 MHz
- data rate: R = 3 Mb/s
- internal noise temperature of the receiver: T_R = 300 K



Solution

1) Assuming free space conditions, the signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T (\lambda/4\pi H)^2 G_T}{k T_{sys} B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T_{sys} is the total noise temperature (summation of T_R and the antenna noise T_A). As there are no attenuating media between, $T_A = T_C$ = 2.73 K (cosmic background temperature). A point not yet considered though is that the two antennas are designed to transmit and receive orthogonal polarizations → the received power is close to 0.

2) The presence of the cloud will not induce any attenuation, but a depolarization. In fact, an RHCP wave consists of two orthogonal linear components with the same amplitude and a differential

phase shift of -90°. The ice cloud causes no attenuation, which means that at the receiver the two components will have the same amplitude. Also, the cloud causes a total differential phase shift of:

$$\Delta\phi = 90°h = 180°$$

Therefore, at the receiver the two linear components will have a differential phase shift of +90°, so the polarization will change from right-hand circular to left-hand circular, i.e. the same one received by the antenna on the ground. Given that the presence of the cloud does not introduce any attenuation, the SNR is still calculated using the first equation above, which yields:

$$SNR \approx 10 \text{ dB}$$

In this case $SNR = E_b/N_0$, therefore, from the BPSK curve → BER ≈ 4×10^{-6}.