Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini
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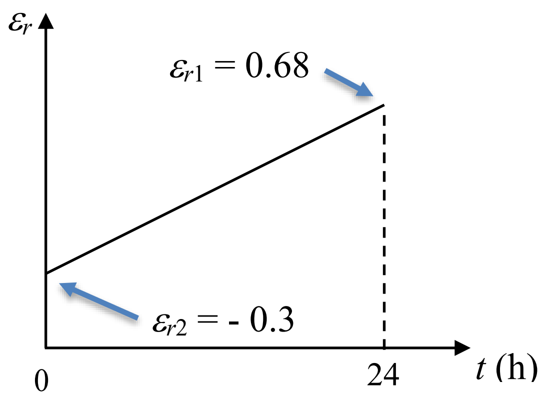
Problem 1

A pulsed space-borne radar, whose carrier frequency is f = 10 MHz, illuminates (nadir pointing) the Earth surface to measure the ground altitude. Assuming a constant electron content N throughout the ionosphere (thickness of the ionosphere equal to H = 400 km), for a specific day, the ionospheric equivalent relative electric permittivity (ε_r) evolves in time as reported in the sketch below. The radar operates correctly when the two-way path attenuation A is lower than 8 dB. The following equation can be used to estimate A from the Total Electron Content (TEC):

$$A = 0.4(TEC - 50) + 10 \quad (TEC \text{ in TECU and } A \text{ in dB})$$

1. Suggest the best polarization to be used.
2. Determine the percentage of the daily time for which the radar can operate correctly.

Assumption: neglect any tropospheric effects.



Solution

1) The ionosphere induces a depolarization effect (Faraday rotation) that affects linear polarizations. Therefore, the most suitable choice is a circularly polarized wave.

2) First and foremost, when $\varepsilon_r \leq 0$, the wave will not cross the ionosphere. The equation expressing ε_r is:

$$\varepsilon_r = 0.0408(t - 24) + 0.68$$

Therefore, the radar will provide results only for $\varepsilon_r > 0$, which occurs for $t > 7.347$ h during the specific day (by inversion of the equation above). The condition for the radar to operate correctly is to have a two-way path attenuation lower than 8 dB. Setting $A < 8$ dB → TEC < 45 TECU. Assuming a constant electron content N throughout the ionosphere (thickness of the ionosphere equal to $H = 400$ km), the value of the N can be simply derived as:

$$N = \frac{\text{TEC}}{H}$$

Therefore, the condition is satisfied when $N < 1.125 \times 10^{12}$ e/m^3. Given the operational frequency, the target N corresponds to the following ε_r:

$$\varepsilon_r = 1 - \left(\frac{9\sqrt{N}}{f}\right)^2 = 0.0887$$

This corresponds to $t = 9.52$ h, so the percentage of the daily time for which the radar can operate correctly is:

$$t_p = 100\frac{24 - 9.52}{24} \approx 60.3\ \%$$

Problem 2

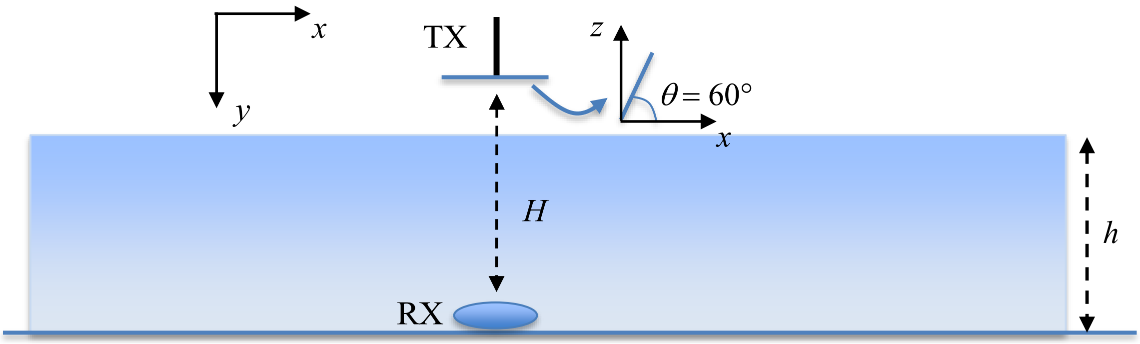
Consider the zenithal downlink from an Earth observation LEO satellite to a ground station (altitude $H = 800$ km, carrier frequency $f = 26$ GHz). The TX antenna is linear with a 60° tilt on the *xz* plane as shown in the figure below. The wave crosses an anisotropic tropospheric layer of thickness $h = 3$ km, consisting of rain drops (constant rain rate $R = 10$ mm/h). The customary power-law coefficient for the specific attenuation in dB/km ($\gamma = kR^{\alpha}$) for the two directions are: $\alpha_z = 0.9421$ and $k_z = 0.1669$, $\alpha_x = 0.9421$ and $k_x = 0.3087$.

For this scenario:

1) Determine the best antenna type at RX to maximize the received power (the effective area A_{RX} is fixed to 0.4 m^2).
2) Using the antenna determined at point 1, calculate the received power.

Assumptions:

- The EIRP is 20 dBW.
- The antennas are optimally pointed.



Solution

1) Looking at the power law coefficients, the *x* component of the electric field will induce more attenuation than the *z* one. Regarding the propagation velocity, it is sensible to assume the same β for both components. Therefore, the received wave will still have linear polarization, but the tilt angle θ is expected to increase. Let us denote as E_0 the transmitted electric field; therefore, the electric field at the RX will be:

$$E_z^{RX} = E_0^{TX} K \sin(\theta) e^{-\gamma_z h} = E_0^{RX} \sin(\theta_{RX})$$

$$E_x^{RX} = E_0^{TX} K \cos(\theta) e^{-\gamma_x h} = E_0^{RX} \cos(\theta_{RX})$$

where K is a constant common to both components, accounting for free space loss. Also, E_0^{RX} is the linear electric field at RX. The tilt angle of the linear polarization at RX is given by:

$$\theta_{RX} = \tan^{-1}\left(\frac{E_z^{RX}}{E_x^{RX}}\right) \approx 70°$$

Therefore, the best antenna at RX to maximize the received power is a linear antenna, with tilt angle θ_{RX}.

2) The received power is:

$$P_{RX} = \frac{\text{EIRP}}{4\pi H^2} A_E A_R$$

where A_R is the rain attenuation, which can be calculated in dB as follows:

$$A_R = -20\log_{10}\left(\frac{E_0^{RX}/K}{E_0^{TX}}\right) = -20\log_{10}\left(\frac{\sqrt{(E_z^{RX})^2 + (E_x^{RX})^2}/K}{E_0^{TX}}\right) =$$

$$= -20\log_{10}\left(\frac{\sqrt{(E_0^{TX}K\sin(\theta)e^{-\gamma_z h})^2 + (E_0^{TX}K\cos(\theta)\,e^{-\gamma_x h})^2}/K}{E_0^{TX}}\right) =$$

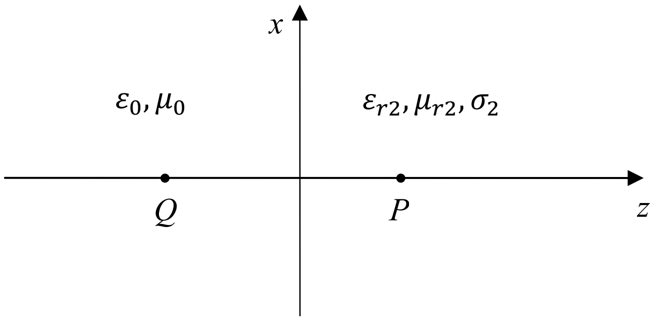
$$= -20\log_{10}\left(\frac{E_0^{TX}\sqrt{(\sin(\theta)e^{-\gamma_z h})^2 + (\cos(\theta)\,e^{-\gamma_x h})^2}}{E_0^{TX}}\right) =$$

$$-20\log_{10}\left(\sqrt{(\sin(\theta)\,e^{-\gamma_z h})^2 + (\cos(\theta)\,e^{-\gamma_x h})^2}\right) = 5.0561\text{ dB} = 0.3122$$

Note that, in the equation above, the impact of *K* has been removed to isolate only the attenuation due to rain. Indeed, the free space component is already included in the link budget equation. Therefore, converting both EIRP and A_R to the linear scale → $P_{RX} = 1.43$ pW.

Problem 3

A uniform plane wave (frequency $f = 10$ GHz) propagates in free space and impinges orthogonally on a dielectric material ($\varepsilon_{r2} = 16, \mu_{r2} = 16, \sigma_2 = 2$ S/m). The power density of the wave in P(1,1,10λ_2) is S(P) = 1 μW/m^2 (λ_2 is the wavelength in the second medium). Calculate the power density of the incident wave in Q(2,2,-10λ_1) (λ_1 is the wavelength in the first medium).



Solution

First, let us calculate the intrinsic impedance for the two media. For the first one, $\eta_1 = \eta_0 \approx 377\ \Omega$. For the second one, let us check the loss tangent, which is $\sigma_2 / \omega\varepsilon_2 \approx 0.2247$. Therefore, no approximations can be considered:

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = 92.5 + j10.3\ \ \Omega$$

The reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.6053 + j0.0351$$

Also, let us calculate the propagation constant for the second medium:

$$\gamma_2 = \alpha_2 + j\beta_2 = \sqrt{j\omega\mu_2\left(\sigma_2 + j\omega\varepsilon_2\right)} = 93.6 + j843.5\ \ \text{1/m}$$

The wavelength is obtained from β_2 as:

$$\lambda_2 = \frac{2\pi}{\beta_2} \approx 0.074\ \ \text{m}$$

Therefore, P has coordinates (1,1,0.074 m).

The power density in P is:

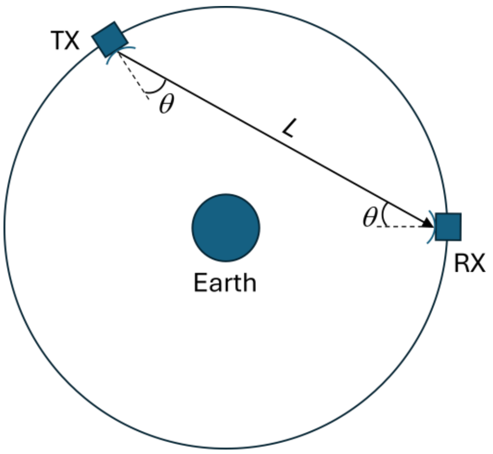
$$S(P) = S_i(1 - |\Gamma|^2)e^{-2\alpha_2 z_P}$$

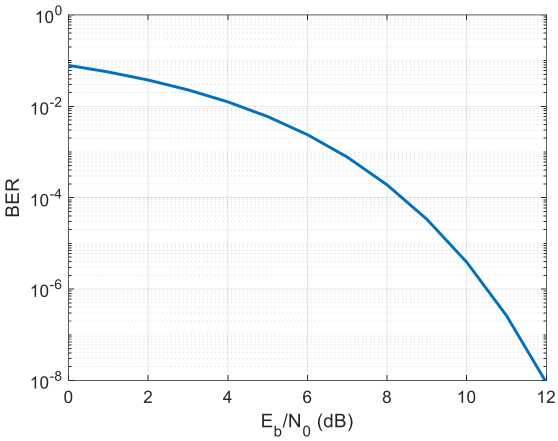
where S_i is the incident power density: as there are no losses in medium 1, S_i is constant throughout medium 1, including in Q. Inverting such equation, we obtain:

$$S(Q) = \frac{S(P)}{(1 - |\Gamma|^2)e^{-2\alpha_2 z_P}} = 1.8\ \ \text{W/m}^2$$

Problem 4

The figure below depicts two GEO satellites. The satellite RX is in charge of monitoring the signal transmitted towards the Earth by the satellite TX (digital television broadcast) to spot any possible issue in TX. The distance between the two satellites is L = 75000 km and θ = 20° is the angle from the boresight direction of the two circular parabolic reflectors with gain G_T = G_R = 26 dB and radiation pattern $f_T = f_R = \cos(\theta)^2$.





1. Based on the scenario described above, what is the most likely polarization of the link?
2. Determine the BER of the link, considering the BER graph for the 4-PSK modulation reported above.

Additional data:

- carrier frequency f = 20 GHz
- no additional losses at the TX and RX
- power transmitted by the satellite: P_T = 850 W
- bandwidth of the receiver: B = 1 MHz
- data rate: R = 2 Mb/s
- internal noise temperature of the receiver: T_R = 150 K

Solution

1) The GEO satellite signal is typically transmitted using both linear vertical and linear horizontal polarizations to double the channel capacity. This is possible because the GEO satellite appears as if it was fixed in the sky to any observer on the Earth: no temporal change in the geometry between transmitting and receiving antennas allows the use of geometry-sensitive polarizations.

2) Assuming free space conditions, the signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T (\lambda/4\pi L)^2 f_T G_T}{k T_{sys} B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T_{sys} is the total noise temperature (summation of T_R and the antenna noise T_A). As there is no attenuation from the channel (free space), $T_A = T_C$ = 2.73 K (cosmic background temperature). The resulting SNR is approximately 11 dB. Recalling that $SNR = E_b R / N_0 B$, we obtain E_b/N_0 = 8 dB → BER ≈ 2×10^{-4}.