Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini,
September 5th, 2025

1	2	3	4

do not write above

SURNAME AND NAME ______________________________

ID NUMBER ______________________________

SIGNATURE ______________________________

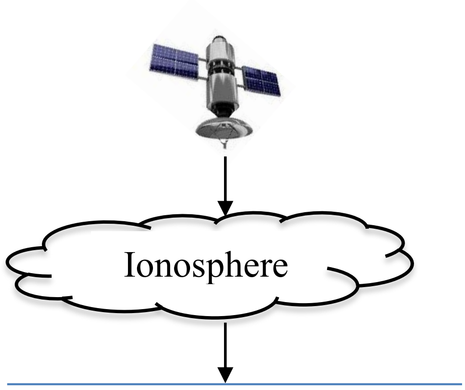
Problem 1

An Earth observation satellite, at altitude H = 800 km, downloads data to a ground station using two channels centered around the carrier frequencies f_1 = 25 MHz and f_2 = 35 MHz (zenithal link).

1) Determine T_2, the data travel time at frequency f_2, knowing T_1 = 2.774 ms (data travel time at frequency f_1).

2) If the ionospheric profile changes due to a sudden ionospheric anomaly, determine the maximum peak electron content value, N_{max}, for the system to operate correctly.

Assumption: no effects induced by the troposphere (neither delay, nor attenuation).



Solution

1) The total travel time T is due to the free space and to the ionosphere, the latter being frequency dependent. T is defined as:

$$T = T_{FS} + T_{IONO} = \frac{H}{c} + \frac{1}{2c}\frac{81}{f^2}TEC$$

where TEC is the total electron content. As a result, inverting that equation with T_1, f_1 and H → TEC = 49.7 TECU

Therefore, using the same equation, T_2 = 2.721 ms.

2) For the system to operate correctly, both channels need to download data to the ground station. When N_{max} increases, so does the critical frequency f_C. Specifically, both carrier frequencies need to be higher than the critical frequency for the waves to cross the ionosphere (zenithal pointing). Therefore:

$f_c = 9\sqrt{N_{max}} < f_1$ → $N_{max} < \frac{f_1^2}{81} = 7.72 \times 10^{12}$ e/m^3.

Problem 2

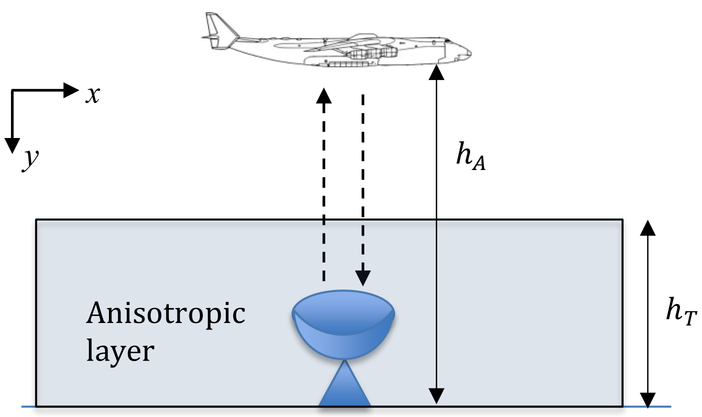
A pulsed radar with zenithal pointing (transmit power $P_T = 1$ kW), working with carrier frequency $f = 80$ GHz and with antenna gain $G = 40$ dB, illuminates an aircraft flying at high altitude. As depicted in the figure below, the wave crosses a layer consisting of anisotropic particles ($h_T = 3.5$ km), whose propagation constants are:

$\gamma_z = \alpha_z + j\,\beta_z = 6\times10^{-4} + j1675.5$ 1/m
$\gamma_x = \alpha_x + j\,\beta_x = 4\times10^{-4} + j1675.5$ 1/m

For this scenario:

1) Which polarization among the following ones allows improving the radar sensitivity to small aircrafts? Linear along *x*, linear along *z*, circular.
2) Working with the polarization chosen at point 1, calculate the minimum backscatter value σ that allows the target identification by the radar, knowing that the radar receiver sensitivity is $P_m = 1.279\times10^{-15}$ W. Consider all targets flying at altitude $h_A = 8$ km.

Assumption: no further tropospheric attenuation other than that due to the anisotropic particles.



Solution

1) From the propagation constants, it emerges that the two linear polarizations are subject to the same phase delay, but to a different attenuation: the *z*-component will be more attenuated than the *x*-component. If the radar pulse is less attenuated, it can identify smaller targets (i.e. associated to t a lower backscatter σ) → it is better to use the linear polarization along *x*.

2) Considering this scenario, the power density reaching the aircraft is:

$$S_A = \frac{P_T}{4\pi h_A^2} A_R G$$

The power received back by the radar is:

$$P_R = \frac{S_A \sigma}{4\pi h_A^2} A_R G \frac{\lambda^2}{4\pi}$$

The tropospheric attenuation A_R is given by:

$$A_R^{dB} \alpha_x \cdot 8.686 \cdot 1000 \cdot h_T = 12.16\text{ dB} \;\rightarrow\; A_R = 0.0608$$

Inverting the equation by setting $P_R = P_m$, and solving for the minimum backscatter → $\sigma = 2$ m^2.

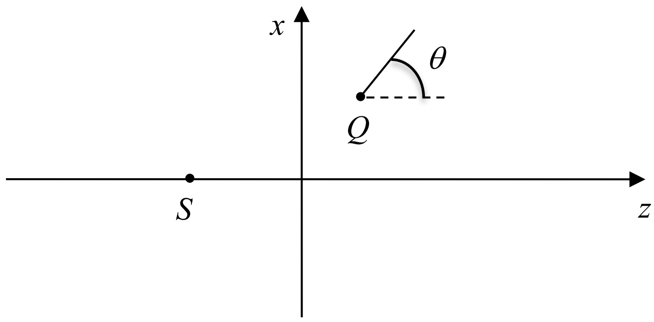
Problem 3

A uniform plane wave (frequency f = 100 MHz) propagates along the z axis in a perfect dielectric with $\varepsilon_{r1} = 9$ and $\mu_{r1} = 4$, and hits the surface of a medium characterized by conductivity $\sigma = 5\times10^{-2}$ S/m, $\varepsilon_{r2} = 9$ and $\mu_{r2} = 1$. The incident electric field is polarized along x and its value in (0,0,0) is E_0 = 1 V/m (see figure below).

Calculate:

a) The electric field at point S(0,0,-2 m).
b) The power absorbed by the linear antenna located in Q(1 m,1 m,1 m), lying on the xz plane ($\theta = 45°$).

Assume that both linear antennas have an effective area $A_E = 2$ m^2.



Solution

a) For the second medium, the loss tangent is (no approximations possible):

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_{r2}\varepsilon_0} = 1$$

The reflected and transmitted waves can be calculates through the reflection coefficient:

$$\eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 251.3\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma + j\omega\varepsilon_2}} = 97.6 + j40.4\ \ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.4214\ +\ j0.1646$$

The electric field in S is the combination of the incident and the reflected waves:

$\vec{E}(S) = E_0\vec{\mu}_x e^{-j\beta_1 z_S} + \Gamma E_0\vec{\mu}_x e^{j\beta_1 z_S} = 0.581 + j0.185$ V/m

with $\beta_1 = \omega\sqrt{\varepsilon_1\mu_1} = 12.57$ rad/m

b) As the electric field has a linear polarization along x and the antenna has a 45° tilt, only part of the power density carried by the transmitted wave will be received. Specifically:

$$\gamma_2 = \sqrt{j\omega\mu_2(\sigma + j\omega\varepsilon_2)} = \alpha_2 + j\beta_2 = 2.86 + j6.91\ 1/\text{m}$$

$$S(Q) = \frac{1}{2}\frac{\left|E_0(1+\Gamma)\cos(\theta)\right|^2}{|\eta_2|}\cos(\mathrm{R}\,\eta_2)\,\mathrm{e}^{-2\alpha_2 z_Q} \ \text{W/m}^2$$

Therefore:

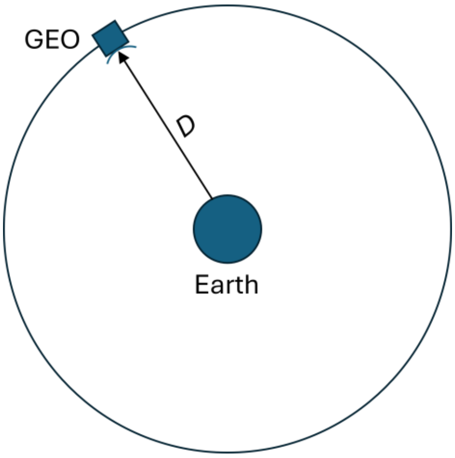
$$S(Q) = 2.6\times10^{-6} \ \text{W/m}^2$$

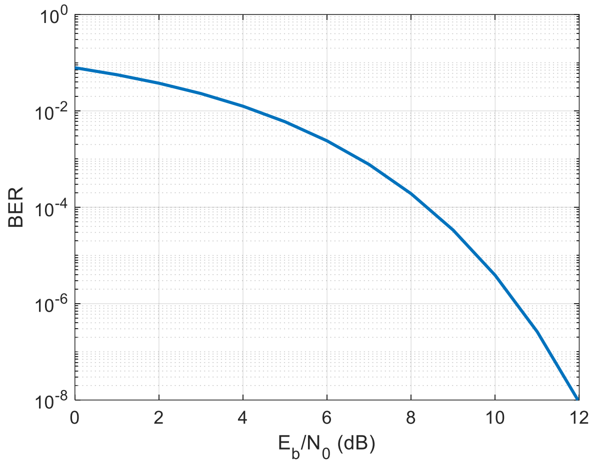
Finally, the power received by the antenna is:

$$P(Q) = S(Q)A_E = 5.2\times10^{-6} \ \text{W}$$

Problem 4

Referring to the figure below, consider the uplink to a GEO satellite, that is seen at 45° elevation angle from the gateway on the Earth. The path length is D = 40000 km and the two antennas are perfectly pointed. The gain of the satellite antenna and of the gateway antenna are G_S = 20 dB and G_G = 34 dB, respectively. The main beam of the satellite antenna covers an area with brightness temperature T_B = 200 K.





1. Determine the BER of the link, considering the BER graph for the 2-PSK modulation reported above.
2. What is the maximum data rate that could be achieved for the conditions at point 1? How can this data rate be achieved?

Additional data:

- carrier frequency f = 18.1 GHz
- power transmitted by the gateway: P_T = 1260 W
- bandwidth of the receiver: B = 1 MHz
- data rate: R = 1 Mb/s
- internal noise temperature of the receiver: T_R = 100 K
- CCDF of the zenithal tropospheric attenuation A: $P(A) = 100e^{-1.15A}$ (A in dB and P in %)
- target service level: P_{AV} = 99.9%
- mean radiating temperature of the troposphere: T_{mr} = 260 K.

Solution

1) The signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_G f_G (\lambda/4\pi D)^2 f_S G_S A}{k T_{sys} B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T_{sys} is the total noise temperature (summation of T_R and the antenna noise temperature T_A), $f_G = f_S = 1$ (perfect pointing) and A is the tropospheric attenuation exceeded for $P_{OUT} = 100 - P_{AV}$ = 0.01% of the time. Inverting the CCDF expression and scaling the attenuation from zenith to the elevation angle → A = 8.5 dB = 0.1414. Regarding T_A, given the geometry, it is calculated as:

$$T_A = T_B A + T_{mr}(1 - A) = 251.5 \text{ K}$$

The resulting SNR is approximately 10 dB. Recalling that $SNR = E_b R / N_0 B$, we obtain $E_b/N_0 = 10$ dB → BER ≈ 4×10^{-6}.

2) The maximum theoretical data rate is given by Shannon's formula:

$$C = B\log_2(1 + SNR) = 3.46 \text{ Mbit/s}$$

This data rate can be achieved by using complex information coding strategies (e.g. Turbo codes).