Radio and Optical Wave Propagation – Prof. L. Luini,
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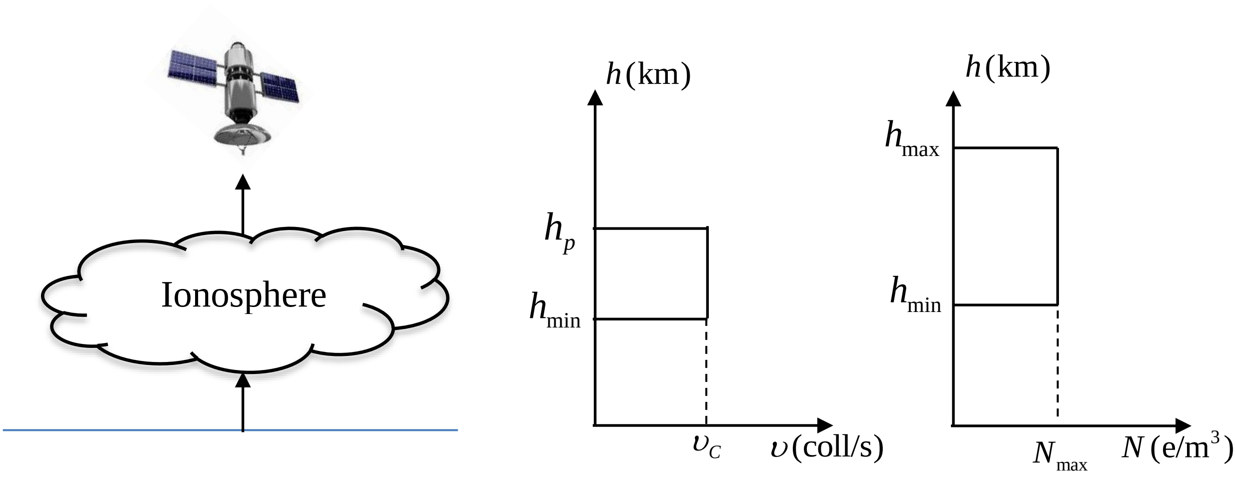
Exercise 1

Making reference to the figure below, a ground station transmits to a satellite along a zenithal link at f = 30 MHz. The satellite altitude is $d = 800$ km. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where h_{max} = 400 km and h_{min} = 100 km. The collision frequency is constant from h_{min} up to h_p = 250 km and its value is $\upsilon_C = 10^4$ collisions/s.

1) Calculate the peak electron content N_{max}, knowing that the critical frequency for the considered N profile is f_C = 18 MHz.

2) Calculate the path attenuation introduced by the ionosphere (in dB).



Solution:

1) N_{max} value is simply obtained from the critical frequency:

$$f_C \approx 9\sqrt{N_{max}} \Rightarrow N_{max} = \frac{f_C^2}{81} \approx 4 \cdot 10^{12} \text{ e/m}^3$$

2) The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{Ne^2 \nu_C}{m(\nu_C{}^2 + \omega^2)} = 3.2 \cdot 10^{-8} \text{ S/m}$$

where m = 9·10^{-31} kg is the mass of the electron and e = -1.6·10^{-19} C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{Ne^2}{m\varepsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C{}^2 + \omega^2} = 0.64$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 7.54 \cdot 10^{-6} + j0.503 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between $h_{\min}$ and h_p. Therefore

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.0655 \text{ dB/km}$$

$$A_{dB} = \alpha_{dB}(h_p - h_{\min}) = \alpha_{dB}(250 - 100) = 9.8 \text{ dB}$$

Exercise 2

Consider a transmitting antenna at height *h* from the ground. For this setup:

1) Calculate the minimum value of *h* to guarantee that the antenna covers an area with radius $R = 30$ km (assume standard atmospheric conditions).
2) Assume now that there is a receiver a distance *R* mounted on a mast with same height *h*. Calculate the minimum operational frequency to guarantee that the first Fresnel's ellipsoid is free.
3) In the conditions of point 2), calculate the value of the ground reflection coefficient to maximize the received power.

Solution:

1) The maximum distance covered by the antenna is given by (standard atmosphere → equivalent Earth radius factor $k = 4/3$):

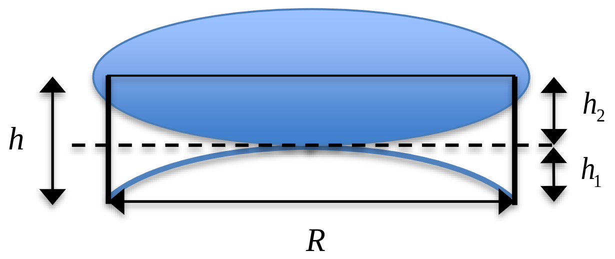
$$R = \sqrt{2hR_{eq}} = \sqrt{2hkR_E} \text{ km}$$

Inverting the above expression, *h* is determined:

$$h = \frac{1}{2}\frac{R^2}{kR_E} = 52.97 \text{ m}$$

2) Making reference to the sketch below, *h* is fixed to the value found at point 1), while h_1 is:

$$h_1 = \frac{1}{2}\frac{(R/2)^2}{kR_E} = 13.24 \text{ m}$$



Therefore $h_2 = h - h_1 = 39.73$ m.

h_2 is also equal to the semi-minor axis of the first Fresnel's ellipsoid, which allows to derive the desired minimum operational frequency:

$$h_2 = \sqrt{\lambda R}/2 \rightarrow \lambda = \frac{(2h_2)^2}{R} = 0.2105 \text{ m} \rightarrow f = c/\lambda = 1.43 \text{ GHz.}$$

For higher frequencies, λ will decrease, and so will the semi-minor axis of the first Fresnel's ellipsoid.

3) Making reference to the sketch below, defining E_0 as the electric field at the receiver associated to the direct ray, the combination of the direct and reflected rays can be assessed as:

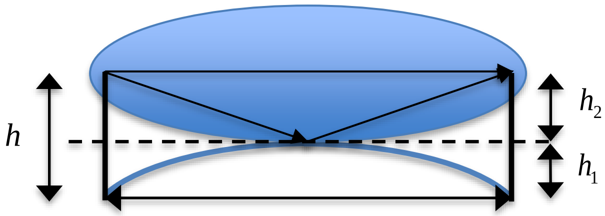
$$E = E_0(1 + \Gamma e^{-j\beta\delta}) \text{ V/m}$$

where Γ is the ground reflection coefficient and δ is the differential path, i.e. the difference between the path travelled along the reflected ray and the one travelled along the direct ray. By definition of the Fresnel's ellipsoid, $\delta = \lambda/2$.

Therefore:

$$E = E_0(1+\Gamma e^{-j\beta\delta}) = E_0(1+\Gamma e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{2}}) = E_0(1-\Gamma)\ \text{V/m}$$

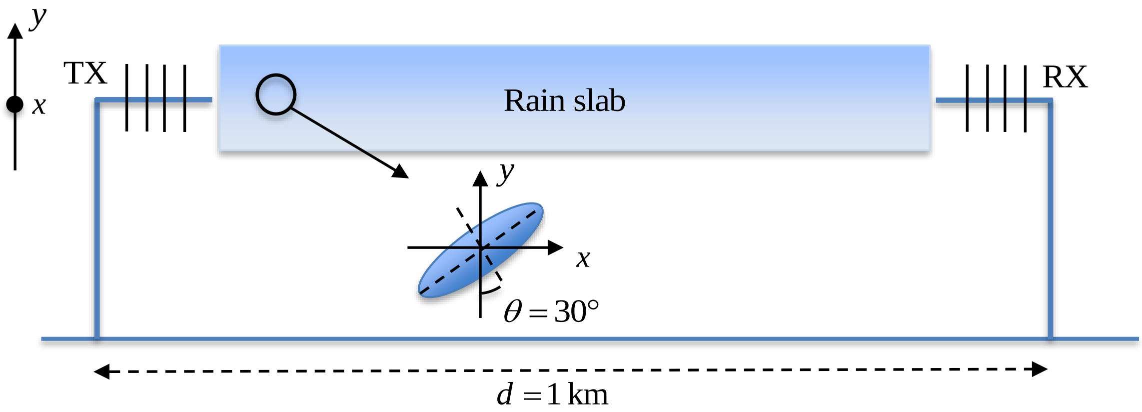
Therefore, in order to maximize the combination of the two rays, i.e. obtain $E = 2E_0$, Γ must be equal to -1.



Exercise 3

A terrestrial link, with path length d = 1 km and operating at f = 30 GHz, is subject to rain. TX emits the electric field $\vec{E}_0 = 10\,\vec{\mu}_y$ V/m and, as shown in the figure below, rain drops are tilted with angle θ = 30°. Both antennas at TX and RX are suited for vertical polarized waves. Calculate the depolarization ratio, i.e. $\delta = \dfrac{|\vec{E}_X|}{|\vec{E}_Y|}$ at RX.

Assumptions: consider the field as a plane wave, that the Earth is flat and that there are no reflections from the ground. Also assume that the specific attenuation at 30 GHz and 0° elevation angle is $A_{spec} = aR^b$,where a_V = 0.2291, b_V = 0.9129 for vertical polarization, a_H = 0.2403, b_H = 0.9485 for horizontal polarization. Also assume that the rain rate R = 30 mm/h is constant along the path and that the propagation constant β is the same for both V and H.



Solution:

In the problem, V and H corresponds to the directions of the minor and major axes of the drop, respectively. To calculate the electric field at RX, the emitted field must be first projected to V and H, and the two components will reach the RX with different attenuation constants. First, let's calculate the specific rain attenuation for V and H:

$$A_{spec}^{H} = a_H R^{b_H} = 6.1 \text{ dB/km} \rightarrow \alpha_H = \frac{A_{spec}^{H}}{8.686 \cdot 1000} = 7 \cdot 10^{-4} \text{ Np/m}$$

$$A_{spec}^{V} = a_V R^{b_V} = 5.1 \text{ dB/km} \rightarrow \alpha_V = \frac{A_{spec}^{V}}{8.686 \cdot 1000} = 5.9 \cdot 10^{-4} \text{ Np/m}$$

α_H and α_V are the attenuation constants for the two directions.

At the receiver, the two components of the electric field along direction V and H will be:

$$E_V = |\vec{E}_0| \cos\theta\, e^{-\alpha_V d} = 4.8 \text{ V/m}$$

$$E_H = |\vec{E}_0| \sin\theta\, e^{-\alpha_H d} = 2.5 \text{ V/m}$$

The two components will still be in phase at RX because their propagation constants are assumed to be the same.

Therefore, the total field at RX will be:

$E_Y = E_V \cos\theta + E_H \sin\theta = 6.3$ V/m

$E_X = -E_V \sin\theta + E_H \cos\theta = -0.2$ V/m

$\vec{E}_{RX} = 6.3\vec{\mu}_y - 0.2\vec{\mu}_x$ mV/m

The depolarization ration is therefore:

$$\delta = \frac{|\vec{E}_X|}{|\vec{E}_Y|} = 0.0317$$

Exercise 4

A terrestrial link operates at optical frequency (wavelength λ = 1.55 μm). The path length is d = 1 km, the optical beam divergence is θ = 1 mrad, the transmitted power is P_T = 1 W and the photodiode area at the receiver is A = 10 cm^2. Calculate the received power P_R, knowing that the whole link is affected by a rain slab of constant rain rate. Assume that drops are spherical, all with the same radius r = 1 mm and their concentration is N = 1000 drops/m^3.

Notes: disregard multiple scattering effects.

Solution

First, let us calculate the attenuation constant, which, if the drops all have the same dimension, can be easily expressed as:

$$\alpha = \frac{1}{2} N C_{EXT}$$

Given that, at optical frequencies, the wavelength is much smaller than the drop dimension, the extinction cross section C_{EXT} can be calculated simply as:

$$C_{EXT} = 2A_{drop} = 2r^2\pi = 6.28 \cdot 10^{-6} \text{ m}^2$$

Therefore:

$$\alpha = 0.0031 \text{ Np/m}$$

and the path attenuation due to rain is:

$$A_R = e^{-2\alpha d} = 0.002 = -27 \text{ dB}$$

The link budget provides the received power:

$$P_R = P_T \frac{A}{d^2\theta^2\pi/4} A_R = 2.6 \text{ μW}$$