Satellite Communication and Positioning Systems – Prof. L. Luini,
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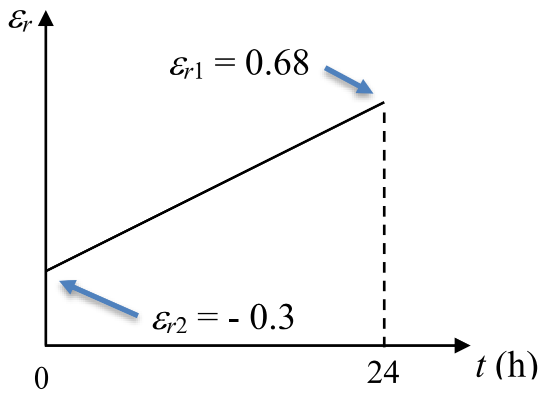
Problem 1

A pulsed space-borne radar, whose carrier frequency is f = 10 MHz, illuminates (nadir pointing) the Earth surface to measure the ground altitude. Assuming a constant electron content N throughout the ionosphere (thickness of the ionosphere equal to H = 400 km), for a specific day, the ionospheric equivalent relative electric permittivity (ε_r) evolves in time as reported in the sketch below. The radar operates correctly when the two-way path attenuation A is lower than 8 dB. The following equation can be used to estimate A from the Total Electron Content (TEC):

$$A = 0.4(TEC - 50) + 10 \quad (TEC \text{ in TECU and } A \text{ in dB})$$

1. Suggest the best polarization to be used.
2. Determine the percentage of the daily time for which the radar can operate correctly.

Assumption: neglect any tropospheric effects.



Solution

1) The ionosphere induces a depolarization effect (Faraday rotation) that affects linear polarizations. Therefore, the most suitable choice is a circularly polarized wave.

2) First and foremost, when $\varepsilon_r \le 0$, the wave will not cross the ionosphere. The equation expressing ε_r is:

$$\varepsilon_r = 0.0408(t - 24) + 0.68$$

Therefore, the radar will provide results only for $\varepsilon_r > 0$, which occurs for $t > 7.347$ h during the specific day (by inversion of the equation above). The condition for the radar to operate correctly is to have a two-way path attenuation lower than 8 dB. Setting $A < 8$ dB → TEC < 45 TECU. Assuming a constant electron content N throughout the ionosphere (thickness of the ionosphere equal to $H = 400$ km), the value of the N can be simply derived as:

$$N = \frac{\text{TEC}}{H}$$

Therefore, the condition is satisfied when $N < 1.125\times10^{12}$ e/m^3. Given the operational frequency, the target N corresponds to the following ε_r:

$$\varepsilon_r = 1 - \left(\frac{9\sqrt{N}}{f}\right)^2 = 0.0887$$

This corresponds to $t = 9.52$ h, so the percentage of the daily time for which the radar can operate correctly is:

$$t_p = 100\frac{24 - 9.52}{24} \approx 60.3\ \%$$

Problem 2

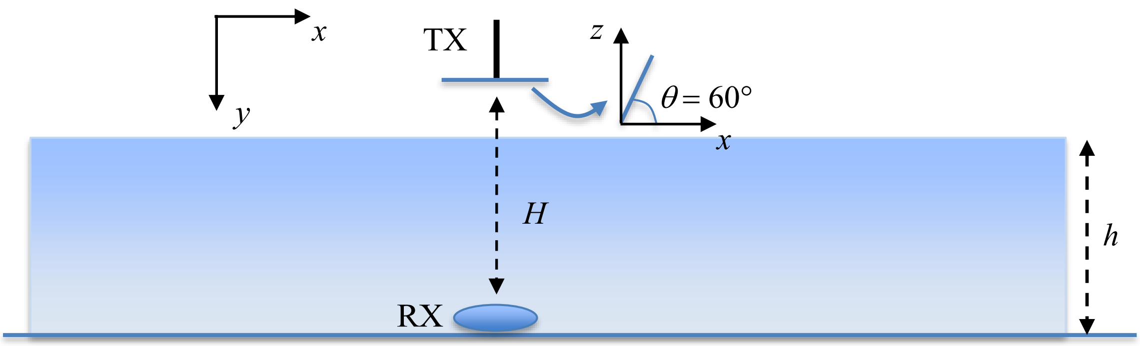
Consider the zenithal downlink from an Earth observation LEO satellite to a ground station (altitude H = 800 km, carrier frequency f = 26 GHz). The TX antenna is linear with a 60° tilt on the xz plane as shown in the figure below. The wave crosses an anisotropic tropospheric layer of thickness h = 3 km, consisting of rain drops (constant rain rate R = 10 mm/h). The customary power-law coefficient for the specific attenuation in dB/km ($\gamma = kR^{\alpha}$) for the two directions are: $\alpha_z = 0.9421$ and $k_z = 0.1669$, $\alpha_x = 0.9421$ and $k_x = 0.3087$.

For this scenario:

1) Determine the best antenna type at RX to maximize the received power (the effective area A_{RX} is fixed to 0.4 m^2).
2) Using the antenna determined at point 1, calculate the received power.

Assumptions:

- The EIRP is 20 dBW.
- The antennas are optimally pointed.



Solution

1) Looking at the power law coefficients, the x component of the electric field will induce more attenuation than the z one. Regarding the propagation velocity, it is sensible to assume for rain drops the same β for both components. Therefore, the received wave will still have linear polarization, but the tilt angle θ is expected to increase. Let us denote as E_0^{TX} the transmitted electric field; therefore, the electric field at the RX will be:

$$E_z^{RX} = E_0^{TX} K \sin(\theta) e^{-\gamma_z h} = E_0^{RX} \sin(\theta_{RX})$$
$$E_x^{RX} = E_0^{TX} K \cos(\theta) e^{-\gamma_x h} = E_0^{RX} \cos(\theta_{RX})$$

where K is a constant common to both components, accounting for free space loss. Also, E_0^{RX} is the linear electric field at RX. The tilt angle of the linear polarization at RX is given by:

$$\theta_{RX} = \tan^{-1}\left(\frac{E_z^{RX}}{E_x^{RX}}\right) \approx 69.4°$$

Therefore, the best antenna at RX to maximize the received power is a linear antenna, with tilt angle θ_{RX}.

2) The received power is:

$$P_{RX} = \frac{\text{EIRP}}{4\pi H^2} A_E A_R$$

where A_R is the rain attenuation, which can be calculated in dB as follows:

$$A_R = -20\log_{10}\left(\frac{E_0^{RX}/K}{E_0^{TX}}\right) = -20\log_{10}\left(\frac{\sqrt{(E_z^{RX})^2 + (E_x^{RX})^2}/K}{E_0^{TX}}\right) =$$

$$= -20\log_{10}\left(\frac{\sqrt{(E_0^{TX}K\sin(\theta)e^{-\gamma_z h})^2 + (E_0^{TX}K\cos(\theta)\,e^{-\gamma_x h})^2}/K}{E_0^{TX}}\right) =$$

$$= -20\log_{10}\left(\frac{E_0^{TX}\sqrt{(\sin(\theta)e^{-\gamma_z h})^2 + (\cos(\theta)\,e^{-\gamma_x h})^2}}{E_0^{TX}}\right) =$$

$$-20\log_{10}\left(\sqrt{(\sin(\theta)\,e^{-\gamma_z h})^2 + (\cos(\theta)\,e^{-\gamma_x h})^2}\right) = 5.0561\ \text{dB} = 0.3122$$

Note that, in the equation above, K has been removed to isolate only the attenuation due to rain. Indeed, the free space component is already included in the link budget equation. Therefore, converting both EIRP and A_R to the linear scale → $P_{RX} = 1.6$ pW.

Problem 3

Discuss concisely the reason(s) why Pseudo-Random Noise (PRN) codes are used in several modern GNSSs and what feature(s) allow(s) improving the Position Velocity Time (PVT) solution.

Solution

PRN codes are employed in modern GNSSs mainly for two reasons:

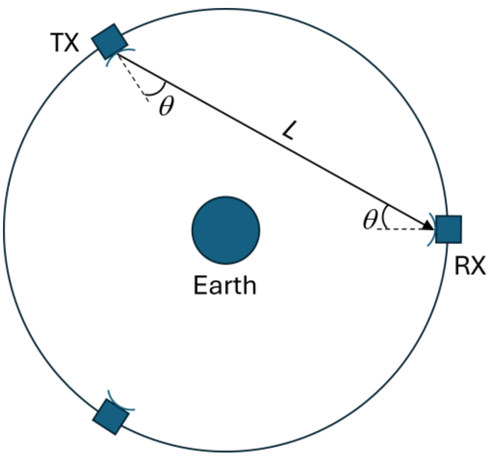
1. To allow the implementation of CDMA (code division multiple access), i.e. the possibility for different satellites to transmit simultaneously, using the same allocated bandwidth. In this sense, a specific satellite is uniquely identified by a given PRN code.
2. To allow ranging, i.e. determining the distance between the receiver and each satellite.

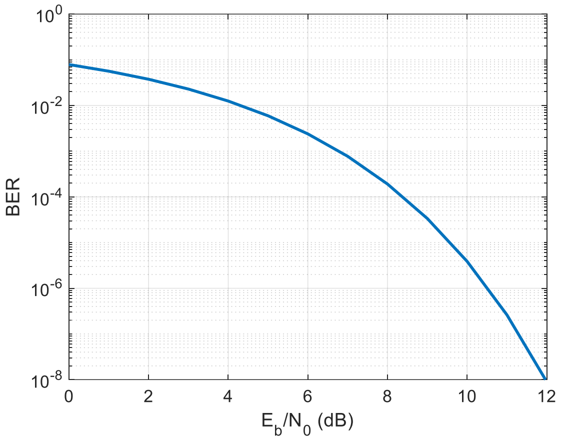
The following features of PRN codes have an impact on the PVT solution, which improves if the accuracy in determining the distance to the satellites increases. This is achieved as follows:

1. Longer PRN code, which provides a more robust identification of the correlation peak.
2. Shorter chip duration, i.e. faster chip rate, which corresponds to a lower ranging error.

Problem 4

Referring to the figure below, consider the intersatellite link between two GEO satellites that are part of the European Data Relay System (EDRS). The distance between the two satellites is L = 75000 km and θ = 20° is the angle from boresight direction of the two circular parabolic reflectors with gain $G_T = G_R$ = 26 dB and radiation pattern $f_T = f_R = \cos(\theta)^2$.





1. Suggest the best polarization to be used.
2. Determine the BER of the link, considering the BER graph for the 4-PSK modulation reported above.

Additional data:

- carrier frequency f = 20 GHz
- no additional losses at the TX and RX
- power transmitted by the satellite: P_T = 850 W
- bandwidth of the receiver: B = 1 MHz
- data rate: R = 2 Mb/s
- internal noise temperature of the receiver: T_R = 150 K

Solution

1) As the wave travels in free space conditions, there is no depolarization: any polarization is OK.

2) Assuming free space conditions, the signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T (\lambda/4\pi L)^2 f_T G_T}{k T_{sys} B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T_{sys} is the total noise temperature (summation of T_R and the antenna noise T_A). As there is no attenuation from the channel (free space), $T_A = T_C$ = 2.73 K (cosmic background temperature). The resulting SNR is approximately 11 dB. Recalling that $SNR = E_b R / N_0 B$, we obtain E_b/N_0 = 8 dB → BER ≈ 2×10^{-4}.