Satellite Communication and Positioning Systems – Prof. L. Luini,
June 27th, 2023

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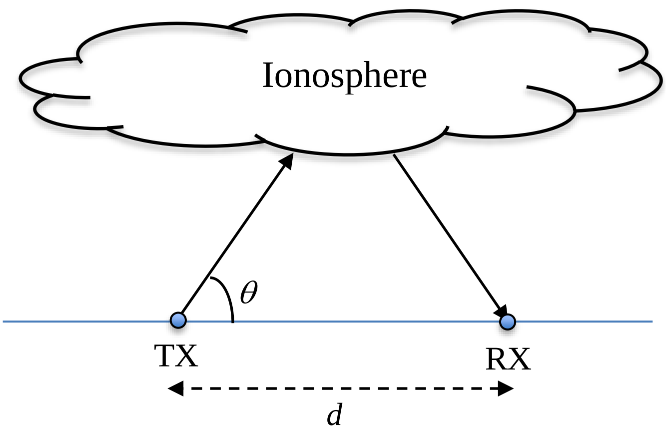
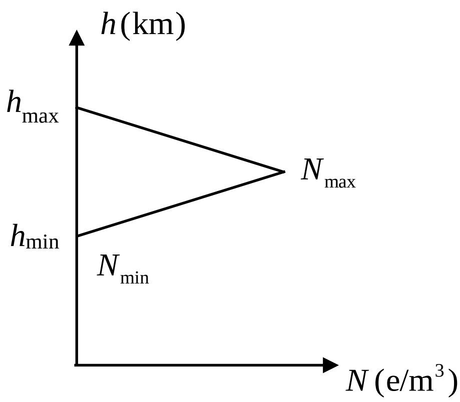
Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance d by exploiting the ionosphere (elevation angle θ = 60°). The ionosphere is modelled with the symmetric electron density profile (daytime) sketched in the figure (right side), where $N_{\max}$ = 6×10^{12} e/m^3, $N_{\min}$ = 4×10^{10} e/m^3, $h_{\min}$ = 100 km and $h_{\max}$ = 400 km.

1) Determine the maximum distance d achievable for the TX → RX link.
2) Determine the operational frequency f to achieve the conditions at point 1).
3) Indicate a reasonable margin on f found at point 2) to guarantee the TX → RX link notwithstanding ionospheric variations.
4) Indicate the best polarization to be used for the TX → RX link.

Assume: the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected; the Earth is flat; no tropospheric effects to be considered.

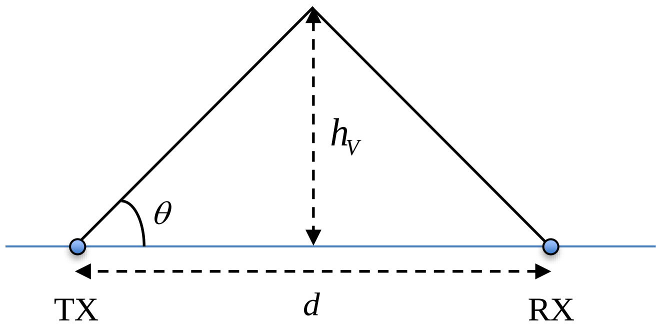




Solution

1) The distance d is maximized if the reflection occurs as high as possible in the ionosphere, i.e. at the height h_p = 250 km, correspondent to N_{max}. Considering the figure below, the distance can be found by inverting the following expression:

$h_V = 1.2\, h_p = {}^{d}/_{2}\, \tan\theta \rightarrow\; d = \frac{2.4\, h_p}{\tan\theta} = 346.4$ km



2) The link operational frequency *f* can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1 - \left(\frac{f_C}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f_m}\right)^2}$$

Solving for the frequency f_m, we obtain:

$$f_m = \sqrt{\frac{81 N_{\max}}{1 - [\cos(\theta)]^2}} = 25.5 \text{ MHz}$$

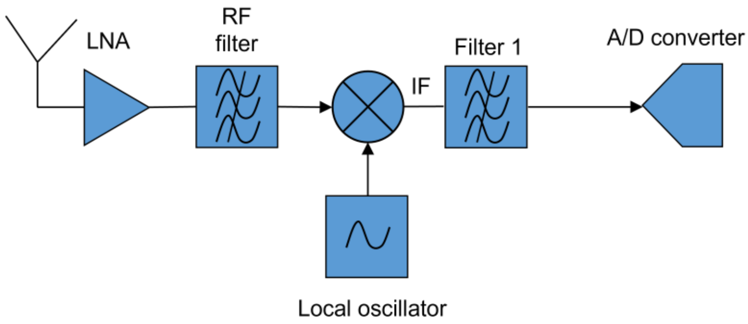
3) During the night, the peak values of the electron content will decrease: it is a good rule of thumb to use reduce by 10% the peak frequency to avoid that the wave crosses the ionosphere at nighttime. Therefore → $f' = 0.9f = 22.95$ MHz.

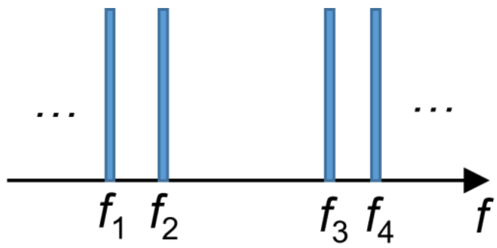
4) Depolarization in the ionosphere affects linearly polarized waves, but not circularly polarized ones. Therefore, the best polarization is LHCP or RHCP.

Problem 2

Consider the heterodyne receiver depicted below (left side), which aims at receiving the RF signal with carrier frequency f_2 = 20 GHz, to be digitalized at the end of the receiver chain by using an A/D converter with a maximum sampling frequency f_S = 4.2 GS/s. As indicated in the picture below (right side), the RF spectrum is occupied by multiple signals, all having the same bandwidth B = 200 MHz, with the closest carriers being f_1 = 19 GHz and f_3 = 25 GHz (more signals are present as indicated in the figure).

1) Design the RF filter and local oscillator considering the A/D converter features and the aim of reducing as much as possible the RF filter complexity (i.e. selectivity): specifically, provide a suitable f_{LO}, indicate the type of filter to be used for the RF filter and its cutoff(s) frequency(ies).
2) Indicate the ideal bandwidth of Filter 1.





Solution

1) When down-converting signals from RF to intermediate frequency (IF), image signals represent a problem. The same IF can be obtained using a local oscillator f_{LO} higher or lower than the target carrier frequency f_2. If $f_{LO} < f_2$, the image signals to be rejected using the RF filter are those lower than f_2; if $f_{LO} > f_2$, the image signals to be rejected using the RF filter are those higher than f_2. As a consequence, the RF filter for the case $f_{LO} < f_2$ will need to be much more selective than the one for the $f_{LO} > f_2$ case. Therefore, to meet the requirements expressed at point 1, the $f_{LO} > f_2$ case is to be selected. To reduce the RF filter complexity (low-pass filter) as much as possible, the ideal cutoff frequency is, for example, 22.5 GHz, exactly in the middle of the f_3- f_2 interval (other choices are possible). Regarding the value for the local osc illator frequency, if f_{LO} = 2.5 GHz, then $f_{IF} = f_{LO} - f_2$ = 2.5 GHz, with a maximum frequency of the IF signal equal to f_{max} = 2.6 GHz. However, according to the Nyquist theorem, in this case, the minimum sampling frequency of the A/D converter should be $f_S = 2f_{max}$ = 5.2 GS/s. This exceeds the available specifications for the A/D converter, i.e. f_S = 4.2 GS/s. Using this value as an additional constraint, a possible final optimum design is: f_{LO} = 22 GHz, $f_{IF} = f_{LO} - f_2$ = 2 GHz, f_{max} = 2.1 GHz, $f_S = 2f_{max}$ = 4.2 GS/s.

2) Given the design at point 1, the optimum Filter 1 (bandpass filter) will have a lower cut-off frequency of $f_{min} = f_{IF}-B/2$ = 1.9 GHz and $f_{max} = f_{IF}+B/2$ = 2.1 GHz.

Problem 3

We need to design a link to a deep-space probe orbiting Mars and operating at Ka-band, specifically at f = 26 GHz. The ground station is equipped with a steerable antenna to track the probe.

1) Calculate the reflector antenna diameter of the ground station (Gregorian configuration with efficiency η = 0.5) necessary to guarantee that the probe can be correctly tracked down to an elevation angle θ = 30° for 99.9% of the time in a year, i.e. that the minimum SNR is 5 dB. To this aim, assume:
   - that the atmosphere is stratified;
   - that the ground station LNA noise temperature is T_R = 50 K;
   - to neglect the cosmic background temperature;
   - that the mean radiating temperature of the atmosphere is T_{mr} = 10 °C;
   - that the probe makes use of a parabolic antenna with gain G_T = 45 dB;
   - the transmit power is P_T = 110 W;
   - the probe antenna always points at the ground station;
   - the distance between the probe satellite and the ground station is L = 225000000 km;
   - the receiver bandwidth is B = 1 kHz;
   - that the CCDF of the zenithal atmospheric attenuation A_T is modelled by:

$$P\left(A_Z^{dB}\right) = 100e^{-0.69A_Z^{dB}} \quad (A_Z \text{ in dB and } P \text{ in \%})$$

2) Calculate the maximum data rate achievable for this channel with the conditions at point 1), considering a negligible bit error rate.

Solution

1) The zenithal attenuation A_Z^{dB} is determined using the CCDF model. 99.9% availability corresponds to P = 0.1% exceedance. Inverting the CCDF formula:

$$A_Z^{dB} = -\frac{1}{0.69}\ln\left(\frac{0.1}{100}\right) \approx 10 \text{ dB}$$

Scaling the zenithal attenuation to the target elevation angle:

$$A_S^{dB} = \frac{A_Z^{dB}}{\sin(\theta)} \approx 20 \text{ dB}$$

which, in linear scale, corresponds to:

$$A_L = 10^{\frac{A_S^{dB}}{10}} \approx 0.01$$

The system noise temperature is (for the Gregorian configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}(1 - A_L) = 330.3 \text{ K}$$

The SNR is given by:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_L}{kT_{sys}B}$$

where f_R = 1 and f_T = 1.

Inverting the expression above to solve for G_R (by setting SNR = SNR_{min} = 5 dB):

$G_R \approx$ 74 dB

Recalling that:

$$\frac{\eta A_g}{G_R} = \frac{\lambda^2}{4\pi}$$

where A_g is the geometrical area of the antenna:

$$A_g = \left(\frac{D_R}{2}\right)^2 \pi$$

the antenna diameter D_R is obtained:

$D_R \approx 26$ m

This is indeed the dimension of Ka-band deep-space antennas installed at NASA Deep Space Network (DSN) sites (Goldstone, Madrid, Canberra).

2) The reply is given by the Shannon limit:
$C = B\log_2(1 + SNR_{min}) \approx 2$ kb/s

Problem 4

A two-frequency GNSS receiver, installed at sea level, is correctly providing the PVT solution. The equivalent noise temperature of the receiver is $T_R = 1000$ K. The error due to the code correlator is associated to the SNR after dispreading as follows (consider the L1 C/A code):

$$d^{C/A} = 60SNR^{-0.2} \quad \text{(SNR in dB, } d^{C/A} \text{ in m)}$$

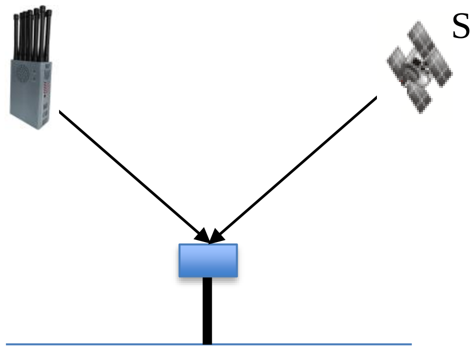
The refractivity profile is modeled as sketched in the figure below, where N_0 = 900 ppm and h_0 = 9.5 km, while the zenithal TEC value is 40 TECU.

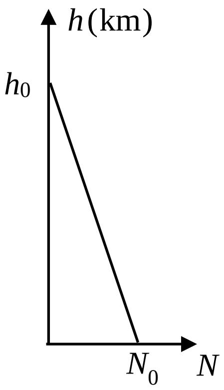
1) Determine the range error for a specific GPS satellite, visible at elevation angle θ = 45°, at L = 20000 km from the receiver.

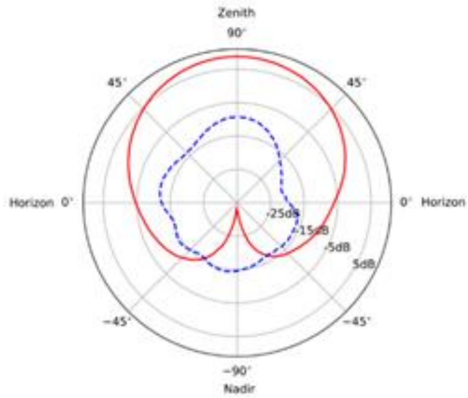
OPTIONAL: at some point, a jammer, also at 45° elevation angle, begins transmitting a wideband signal evenly covering the full L band and reaching the GPS receiver antenna with power spectral density S_J in W/(m^2·Hz).

2) For the same satellite, determine the maximum value of S_J for the range error to be limited below 42 m.

Assume that: the zenithal atmospheric attenuation is A_Z = 0.4 dB, the mean radiating temperature is T_{mr} = 260 K, the distance between the receivers and the satellites is L = 20000 km.







Solution

As the GNSS receiver is correctly providing the PVT solution, there is no clock bias. The pseudorange is therefore affected by the following error sources:

$\rho = r + d^{C/A} + d^I + d^T$

where:

$d^T = \frac{h_0 N_0 10^{-6}}{2\sin(\theta)} = 6.05$ m is the slant tropospheric error (given the profile in the figure).

$d^I \approx 0$ m, as a dual frequency receiver is considered.

$d^{C/A}$ depends on the SNR.

The L1 frequency is f = 1575.42 MHz, so the wavelength is $\lambda = c/f$ = 0.1904 m. The received power can be calculated as:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi L}\right)^2 G_R f_R A_{ATM}$$

Making reference to the specifications of GPS satellites, we can assume P_T = 21.9 W and $G_T f_T$ = 13.4 dB = 21.9 (worst case). The slant path atmospheric attenuation is:

$A = A_Z/\sin(45°)$ = 0.57 dB → A_{lin} = 0.88

The gain of the receiver antenna can be derived from the radiation pattern figure: at 45°, the receiver gain is G_R = 5 dB = 3.16. This value also includes f_R.

The received power is calculated directly from the equation above, yielding:

$P_R = 7.63 \times 10^{-16}$ W

The system noise power depends on the antenna noise temperature T_A and on the receiver noise temperature T_R:

$T^{sys} \approx T_C A_{lin} + T_{mr}(1 - A_{lin}) + T_R = 1034.1$ K

After despreading, the bandwidth for the calculation of the noise power is B = 50 Hz. The noise power is calculated as:

$P_N = kT_{sys}B = 7.14 \times 10^{-19}$ W

As a result:

SNR = 30.29 dB → $d^{C/A} = 30.3$ m.

The total range error is: $d = d^{C/A} + d^{T} = 36.4$ m.

2) The jammer will increase the noise power as follows:

$$P_N' = P_N + S_J B A_E = P_N + S_J B G_T f_T \frac{\lambda^2}{4\pi}$$

Considering that d^T is fixed, and setting d = 42 m → $d^{C/A} = 35.95$ m → SNR_m = 12.94 dB = 19.7.
As a result:

$P_N' = \frac{P_R}{SNR_m} = 3.88 \times 10^{-17}$ W

Therefore:

$S_J B G_T f_T \frac{\lambda^2}{4\pi} = P_N' - P_N = 3.8 \times 10^{-17}$ W → $S_J = 1.2 \times 10^{-17} \frac{\mathrm{W}}{\mathrm{m^2 Hz}}$.