Electromagnetics and Signal Processing for Spaceborne Applications
September 10th, 2025

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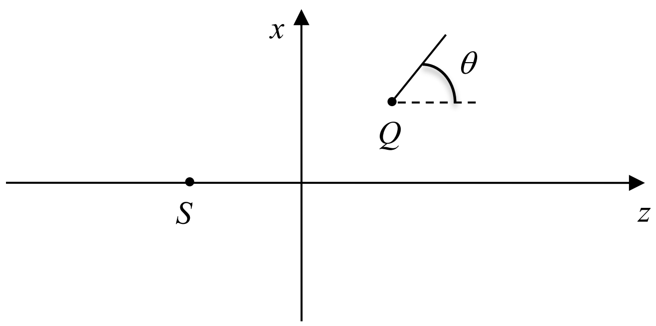
Problem 1

A uniform plane wave (frequency $f = 100$ MHz) propagates along the z axis in a perfect dielectric with $\varepsilon_{r1} = 9$ and $\mu_{r1} = 4$, and hits the surface of a medium characterized by conductivity $\sigma = 5\times10^{-2}$ S/m, $\varepsilon_{r2} = 9$ and $\mu_{r2} = 1$. The incident electric field is polarized along x and its value in (0,0,0) is $E_0 = 1$ V/m (see figure below).

Calculate:

1. The electric field at point S(0,0,-2 m).
2. The power absorbed by the linear antenna located in Q(1 m,1 m,1 m), lying on the xz plane ($\theta = 45°$).

Assume that the linear antenna has an effective area $A_E = 2$ m^2.



Solution

1) For the second medium, the loss tangent is (no approximations possible):

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_{r2}\varepsilon_0} = 1$$

The reflected and transmitted waves can be calculates through the reflection coefficient:

$$\eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 251.3\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma + j\omega\varepsilon_2}} = 97.6 + j40.4 \ \ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.4214 + j0.1646$$

The electric field in *S* is the combination of the incident and the reflected waves:
$\vec{E}(S) = E_0\vec{\mu}_x e^{-j\beta_1 z_S} + \Gamma E_0 \vec{\mu}_x e^{j\beta_1 z_S} = 0.581 + j0.185$ V/m
with $\beta_1 = \omega\sqrt{\varepsilon_1\mu_1} = 12.57$ rad/m

2) As the electric field has a linear polarization along *x* and the antenna has a 45° tilt, only part of the power density carried by the transmitted wave will be received. Specifically:
$\gamma_2 = \sqrt{j\omega\mu_2(\sigma + j\omega\varepsilon_2)} = \alpha_2 + j\beta_2 = 2.86 + j6.91$ 1/m

$$S(Q) = \frac{1}{2}\frac{|E_0(1+\Gamma)\cos(\theta)|^2}{|\eta_2|}\cos(\mathrm{R}\,\eta_2)\,\mathrm{e}^{-2\alpha_2 z_Q} \ \ \mathrm{W/m^2}$$

Therefore:
$S(Q) = 2.6 \times 10^{-6}$ W/m²

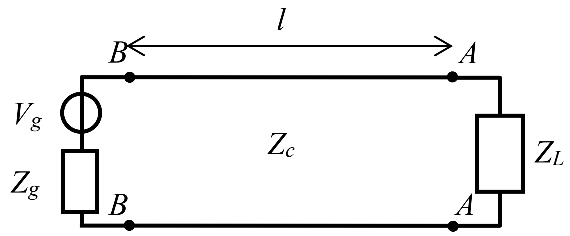
Finally, the power received by the antenna is:
$P(Q) = S(Q)A_E = 5.2 \times 10^{-6}$ W

Problem 2

A source with voltage V_g = 20 V and internal impedance Z_g = 100 Ω is connected to a load $Z_L = 15 + j15$ Ω by a transmission line with characteristic impedance Z_C = 75 Ω, the frequency is f = 300 MHz and the length of the line is l = 5.75 m.

Calculate:

1. The power absorbed by the load
2. The temporal trend of the voltage at section BB, $v_B(t)$



Solution

1) The wavelength is:

$\lambda = \lambda_0 = c/f = 1$ m

As a consequence, the length of the line normalized to the wavelength is:

$l/\lambda = 5.75 = 5.5 + 0.25$ m

In other terms, the line is a $\lambda/4$.

As a result, the load at section BB can be easily calculated as:

$$Z_{BB} = \frac{Z_C^2}{Z_L} = 187.5 - j187.5\ \Omega$$

The reflection coefficient for the source is:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = 0.52 - j0.32$$

Therefore, the power absorbed by the load is:

$$P_L = P_{AV}\left(1 - |\Gamma_g|^2\right) = \frac{|V_g|^2}{8\mathrm{Re}[Z_g]}\left(1 - |\Gamma_g|^2\right) = 0.32\ \mathrm{W}$$

2) The voltage at the beginning of the line is:

$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Z_g} = 15.12 - j3.18\ \mathrm{V} \rightarrow V_{BB} = 15.45e^{-j0.2075}\ \mathrm{V}$$

The trend of V_{BB} in time is given by:

$$v_{BB}(t) = \mathrm{Re}\left[V_{BB}\, e^{j\omega t}\right] = 15.45\cos(\omega t - 0.2075)\ \mathrm{V}$$

Problem 3

A moving audio source transmits the signal:

$$s(t) = A \cdot cos(2\pi f_0 t) \cdot rect\left(\frac{t}{T_{obs}}\right)$$

, with $f_0 = 100\ Hz$ and $T_{obs} = 1$s. The signal received by a stationary sensor is $s_{rx}(t) = s(\alpha \cdot t)$, with $\alpha = 1.1$ a temporal contraction factor caused by relative motion. The receiver knows the value of the carrier frequency f_0, but is unaware of the relative motion and does not know the values of A and T_{obs}.

1. Calculate the expression of the Fourier Transforms of $s_{rx}(t)$ and draw the graph of its absolute value. What are the effects caused by temporal contraction?
2. Determine the minimum observation time required to observe the effects of temporal contraction.
3. Write the expression of the complex envelope of the received signal, assuming to demodulate it with respect to f_0.
4. Knowing that $\alpha = \left(1 - \frac{v}{c}\right)^{-1}$, with v the transmitter velocity towards the receiver and c the propagation velocity of sound waves, propose a procedure to measure the transmitter velocity v starting from the complex envelope of the received signal.
5. How fast is the transmitter going in this exercise?

Solution

The Fourier transforms of s(t) is:

$$S(f) = A\frac{T_{obs}}{2}\{sinc(T_{obs}(f - f_0)) + sinc(T_{obs}(f + f_0))\}$$

To determine the FT at the receiver we apply the result that $s(\alpha \cdot t)$ transforms to $\frac{1}{|\alpha|} S\left(\frac{f}{\alpha}\right)$, hence:

$$S_{rx}(f) = A\frac{T_{obs}}{2\alpha}\left\{sinc\left(\frac{T_{obs}}{\alpha}(f - \alpha f_0)\right) + sinc\left(\frac{T_{obs}}{\alpha}(f + \alpha f_0)\right)\right\}$$

The signal exhibits two peaks at $f = \pm\alpha f_0$. The width of each peak is $\Delta f = \frac{\alpha}{T_{obs}}$. The effect of contraction can be summarized as follows:

- Increase in frequency
- Increase in bandwidth
- Reduction in intensity due to the shortened observation time.

The shift in frequency is $\alpha f_0 - f_0 = 10\ Hz$, which we can detect as long as frequency resolution Δf is lower than this value:$\Delta f = \frac{\alpha}{T_{obs}} < 10\ Hz$ => $T_{obs} > \frac{1.1}{10} = 0.11$ seconds.

The complex envelope is obtained as

$$s_c(t) = \mathcal{F}^{-1}(2S_{rx}(f)u(f))exp(-j2\pi f_0 t)$$

Where $u(f) = 1$ for f>0 and 0 for f<0.
We get:

$$s_c(t) = exp(j2\pi(\alpha - 1)f_0 t) \cdot rect\left(\frac{\alpha t}{T_{obs}}\right)$$

To measure transmitter velocity, we just need to determine the frequency peak in the complex envelope, which can be done as usual by computing its Fourier Transform. We know this peak is found at $f_{peak} = (\alpha - 1)f_0$, and since f0 is known this gives α. Conversion to velocity is simply obtained by inverting $\alpha = \left(1 - \frac{v}{c}\right)^{-1}$.

How fast is the transmitter going in this exercise? Just use $\alpha = 1.1 = \left(1 - \frac{v}{c}\right)^{-1}$, hence $1 - \frac{v}{c} \cong 0.9$ => $\frac{v}{c} \cong 0.1$. Assuming c = 340 m/s we get that the source is travelling at about 34 m/s towards the receiver.

Problem 4

A drone embarks a Radar sensor operating at the central frequency $f_0 = 10$ GHz to transmit electromagnetic pulses with a bandwidth of 100 MHz at the rate of 100 Hz (i.e.: it transmits a new pulse every 0.01 s). The drone is flown at the velocity of v meters per second for 1 second.

1. Determine range resolution, intended as the system capability to resolve two targets based on their distance from the Radar. *Tip: for this one point, just reason based on a single pulse.*
2. Assuming that the drone is flown at the velocity $v = 0.5$ m/s, determine angular resolution, intended as the system capability to resolve two targets based on their angular position with respect to the Radar trajectory.
3. With reference to point 2, discuss: i) in which direction you get the finest resolution; ii) whether target detection is ambiguous (i.e.: whether ghost targets can arise at angular position where no real target is actually present).
4. Repeat point 2 and 3 assuming a flight velocity $v = 2$ m/s.
5. Assuming that the Radar antenna radiates over a beamwidth of 40° ($\pm$ 20° off boresight), determine the maximum velocity that guarantees non-ambiguous detection.

Solution

Temporal resolution is 1/B. Range is related to delay via

$$\mathrm{R} = \frac{c\tau}{2}.$$

, hence range resolution is

$$\mathrm{dR} = \frac{c}{2B}.$$

At 0.5 m/s the drone flies a trajectory of length L = 0.5 m in 1 second. This is equivalent to an antenna array with 100 elements spaced by dx=5 mm. Antenna spacing is less than a quarter of a wavelength, hence no ambiguity arises. Angular resolution is

$$\mathrm{d}\psi = \frac{\lambda}{2L\cos(\psi)}.$$

The finest resolution is at $\psi = 0$, which coincides with the direction orthogonal to the trajectory (boresight).
Note that the factor 2 arises for both range and angular resolution since distances depend on the two-way delays (back and forth) in the case of a Radar.

At 2 m/s the drone flies a trajectory of length L = 2 m in 1 second. Angular resolution is now four times as fine, but we have ambiguities because the equivalent antenna spacing dx is larger than a quarter of a wavelength.

For a target at boresight, ghosts arise at

$$\psi_{amb} = asin\left(\frac{\lambda}{2dx}\right) = asin\left(\frac{\lambda}{2vT}\right) \approx \frac{\lambda}{2vT}.$$

To avoid ghosts one needs to ensure that no energy is radiated outside the antenna beamwidth, hence

$$\psi_{amb} > \psi_{max} = \frac{20}{180}\pi$$

Accordingly:

$$v > \frac{\lambda}{2T\psi_{max}} = 4.3\ m/s.$$