Electromagnetics and Signal Processing for Spaceborne Applications
July 18th, 2025

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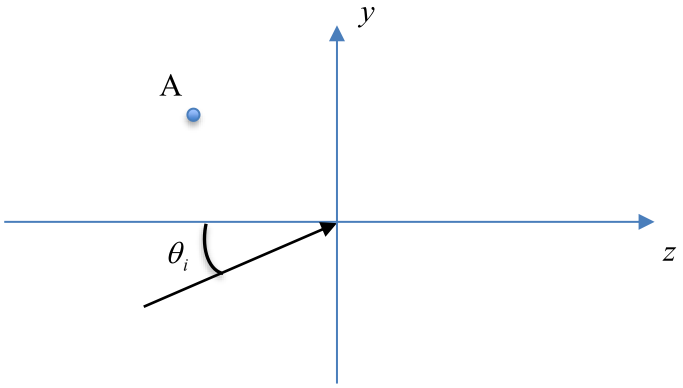
Problem 1

Consider a sinusoidal plane EM wave (f = 1 GHz) with linear polarization (only TE component) propagating into free space from a lossless dielectric material with $\varepsilon_r = 4$ and $\mu_r = 1$. Consider the propagation vector to lie on the zy plane. In particular:

1. Define the minimum incident angle θ_i to obtain total reflection.
2. Write the equation of the incident electric field using θ_i determined at point 1.
3. Calculate the total power absorbed by a dipole antenna with gain G = 3 dB, positioned in A(1,1,-1) and perpendicular to the zy plane (consider only the reflected field).
4. Repeat the calculation at point 3, considering that the dipole antenna now lies on the yz plane, and it is oriented along the propagation direction of the reflected wave.



Solution

1) For total reflection to take place, we need the incidence angle to be at least the critical angle for evanescent waves:

$$\theta_i = \theta_C = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

Any angle larger than θ_i will induce total reflection.

2) A possible TE incident wave is:

$$\vec{E}_i(y,z) = \vec{\mu}_x e^{-j\frac{\beta}{2}y} e^{-j\frac{\sqrt{3}}{2}\beta z} = \vec{\mu}_x e^{-j36.28y} e^{-j20.94z} \text{ V/m}$$

with β = 41.89 rad/m

3) The dipole will receive completely the electric field of the reflected wave, as it is parallel to the antenna direction. In case of evanescent waves, the absolute value of the reflection coefficient is 1. Therefore, the received power is:

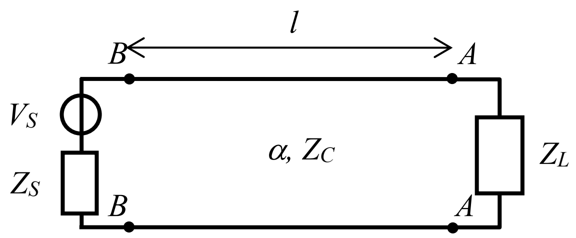
$$P_R = S_i|\Gamma|^2 \frac{\cos(\theta_i)}{\cos(\theta_r)} A_E = \frac{1}{2}\frac{|E_{TE}|^2}{\eta_1}|\Gamma|^2 \frac{\lambda^2}{4\pi} G = 9.5 \times 10^{-6} \text{ mW}$$

being $\theta_i = \theta_r$ and $A_E = 0.0036$ m^2.

4) Given the geometry of the electric field and of the antenna, no power is received.

Problem 2

A transmission line is connected to a source with voltage V_S = 10 V and internal impedance Z_S = 75 Ω. The line, with characteristics impedance Z_C = 75 Ω and attenuation coefficient α = 40 dB/km, is terminated on a load Z_L = 27 Ω. The operating frequency is f = 500 MHz and the line length is l = 6 m. Determine the power absorbed by the load and the power dissipated by the line.



Solution

The wavelength is $\lambda = c/f$ = 0.6 m. The line length is a multiple of λ, which simplifies calculations.
The reflection coefficient at the load section is:

$$\Gamma = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.4706$$

Given the specifics of the circuit, the power absorbed by the load is:

$$P_L = \frac{|V_S|^2}{8Z_S} e^{-2\alpha l}(1 - |\Gamma|^2) = 0.123 \text{ W}$$

The reflection coefficient at the input section is:

$$|\Gamma_B| = |\Gamma_G| = |\Gamma| e^{-2\alpha l} = 0.4453$$

The power crossing section BB is:

$$P_B = \frac{|V_S|^2}{8Z_S}(1 - |\Gamma_G|^2) = 0.137 \text{ W}$$

The power dissipated along the line is:

$$P_d = P_B - P_L = 0.011\ W$$

Problem 3

A signal $s(t)$ with total (two-sided) bandwidth B = 200 KHz arrives at two receivers following two distinct paths. The signals output at the two receivers are modeled as:

$$d_1(t) = s(t) + s(t - \tau_1)$$
$$d_2(t) = s(t) + s(t - \tau_2)$$

Where $\tau_1 =$ 10 microseconds and $\tau_2 = 20$ microseconds.

1. Calculate the expression of the Fourier Transforms of $d_1(t)$ and $d_2(t)$ and draw the graphs of their absolute values, having care to highlight the positions where they are null (for this point, you can approximate *S*(*f*) to a rectangular pulse in the frequency domain).
2. Propose a procedure to restore s(t) using only $d_1(t)$. Is it possible to obtain s(t) with no errors?
3. Propose a procedure to restore s(t) using both $d_1(t)$ and $d_2(t)$. Is it possible to obtain s(t) with no errors?
4. Assuming s(t) is a perfect cardinal sine, calculate the expression of the cross-correlation between $d_1(t)$ and $d_2(t)$.

Solution

The Fourier transforms of d1 and d2 are:

$$D_1(f) = rect\left(\frac{f}{B}\right)\left(1 + exp(-j2\pi f\tau_1)\right)$$
$$D_2(f) = rect\left(\frac{f}{B}\right)\left(1 + exp(-j2\pi f\tau_2)\right)$$

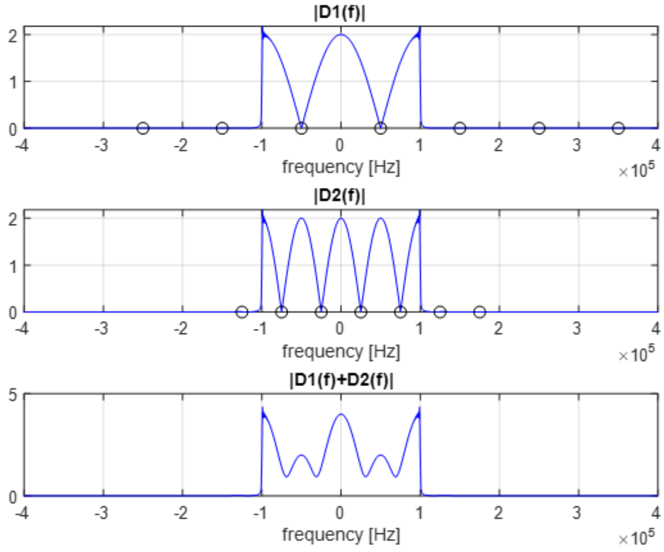
Those are null when the exponential is equal to -1, hence when $2\pi f\tau_1 = \pi + 2k\pi$ with k any integer. It follows that the frequencies when null values occur are:

$$f_1 = \frac{1}{2\tau_1} + \frac{k}{\tau_1}$$
$$f_2 = \frac{1}{2\tau_2} + \frac{k}{\tau_2}$$

In the interval (-100,100) Khz we have zeroes in D1 for f=[-50 50] Khz and in D2 for f=[-75 -25 25 75] KHz. See graphs below.

To restore s(t) from d1(t) one can apply an inverse filter, but of course it is never possible to invert zeros. If both d1(t) and d2(t) are available different solutions are possible. Noting that zeroes in dq and d2 occur in different positions, a very simple one consists in taking the sum, which does not show null values. From this, one can restore the signals by an inverse filter:

$$H_i(f) = \left(2 + exp(-j2\pi f\tau_1) + exp(-j2\pi f\tau_2)\right)^{-1}$$



The cross correlation is simply obtained by linearity:

$$R(t) = d_2(t) * d_1(-t)$$

$$R(t) = E \cdot \left(sinc(tB) + sinc\big((t-\tau_2)B\big) + sinc\big((t+\tau_1)B\big) + sinc\big((t+\tau_1-\tau_2)B\big) \right)$$

The term E represents the energy of the cardinal sine. This can be easily derived in the frequency domain as the interval of its squared FT, hence E = B.

Problem 4

An antenna array of N elements operating at the frequency $f_0 = 5$ GHz is designed to transmit a signal $s(t)$ along a single beam directed along direction ψ, with ψ the angle w.r.t. the normal to the array. The beamwidth is required to be 10° or less for any angle ψ in the interval (-45°,45°).

1. Determine the required number of elements and array length.
2. Determine the phase to be applied to each element to steer the beam along direction ψ.
3. Explain how to simultaneously transmit two distinct signals $s_1(t)$ and $s_2(t)$.along two distinct directions ψ_1 and ψ_2 in the interval (-45°,45°). Upon which conditions is it possible to receive the two signals with no relevant interference?
4. Determine the position of ambiguous beams that would arise with an array spacing of two wavelengths for a single transmission at $\psi = 0°$.

Solution

The condition of a single beam requires that $dx = \frac{\lambda}{2}$. The beamwidth is obtained as:

$$\Delta = \frac{\lambda}{Lcos(\psi)}.$$

The condition that the beamwidth is finer than 10° in the interval (-45°,45°) is fulfilled by setting $L \geq \frac{\lambda}{\Delta cos(\psi)}$, with $\Delta = \frac{10}{180}\pi$ and $\psi = \frac{45}{180}\pi$. We get L>0.487 m, which can be obtained with N = 17 elements.

The phase term to be used to steer the beam at ψ is $\frac{2\pi}{\lambda} sin(\psi) x_n$, with x_n the position of the n-th antenna elements along the array.

Simultaneous transmission of two (or more) signals is simply obtained by having the n-th element of the array transmit the signal:

$$g_n(t) = s_1(t) exp\left(j\frac{2\pi}{\lambda} sin(\psi_1) x_n\right) + s_2(t) exp\left(j\frac{2\pi}{\lambda} sin(\psi_2) x_n\right),$$

It is possible to avoid mutual interference upon the conditions that the two beams are well separated, hence $|\psi_1 - \psi_2| > \frac{10}{180}\pi$.

If $dx = 2\lambda$ we have ambiguous spatial frequencies at $f_a = \frac{k}{2\lambda}$, with k any integer. Following the relation that links angles and spatial frequencies: $sin(\psi) = \lambda f_x$, we have that ambiguous angles are the solutions of the equations:

$$\psi_a = asin(\lambda f_a) = asin\left(\frac{k}{2}\right),$$

So we have replicas at -30°, 30°,-90°, 90°.