Electromagnetics and Signal Processing for Spaceborne Applications
June 27th, 2025

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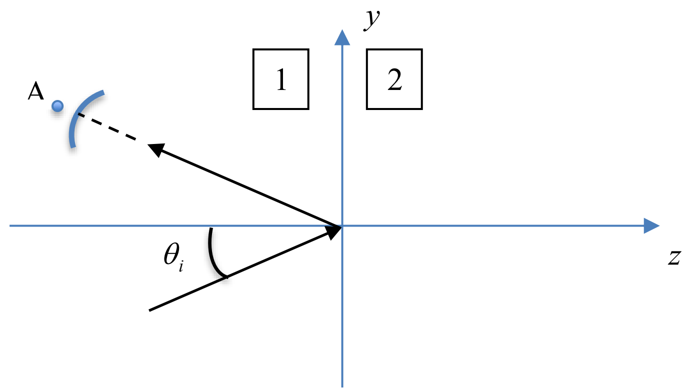
Problem 1

A plane sinusoidal EM wave propagates from free space into medium with electric permittivity $\varepsilon_{r2} = 9$ (assume $\mu_{r2} = 1$) with incident angle θ_i. The expression for the incident electric field is:

$$\vec{E}(z, y) = \left[-\vec{\mu}_x + 2j\left(0.5\vec{\mu}_y - 0.866\vec{\mu}_z\right)\right] e^{-j104.72z} e^{-j181.38y}$$

1) Determine the incident angle θ_i.
2) Determine the frequency of the wave.
3) Determine the polarization of the incident EM wave.
4) Determine the power received by a parabolic antenna in A(x = 10 cm, y = 1 m, z = -2 m), with gain G = 30 dB and oriented as in figure below (the dashed line indicates the boresight axis, that is aligned with the wave propagation direction after reflection); to this aim, consider only the TE component of the reflected wave (the antenna can receive any polarization).



Solution

1) The incident angle θ_i can be derived from the TM component of the wave. For example:

$$0.5\vec{\mu}_y = \cos(\theta_i)\vec{\mu}_y \rightarrow \theta_i = 60°$$

2) The frequency of the wave can be derived, for example, from β_z:

$$\beta_z = \frac{2\pi f}{c}\cos(\theta_i) \rightarrow f = 10 \text{ GHz}$$

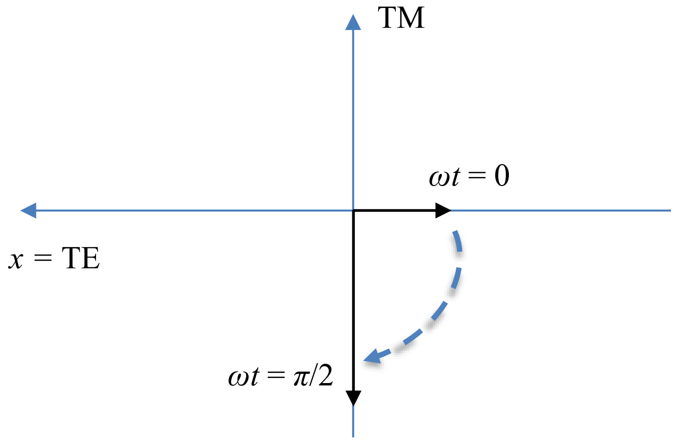
3) The polarization of the incident wave can be derived from the two TE and TM components. In fact, setting y and z to 0, and expressing the dependence on time:

$$\vec{E}(z,y,t) = Re\{[-\vec{\mu}_x + 2j(0.5\vec{\mu}_y - 0.866\vec{\mu}_z)]e^{j\omega t}\}$$

Thus, for $t = 0$ → $\vec{E}(0,0,0) = -\vec{\mu}_x$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0,0,\omega t) = -2\vec{\mu}_{TM}$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



It is a RHEP wave.

4) From Snell's law:

$$\sin(\theta_i)\sqrt{\varepsilon_{r1}} = \sin(\theta_t)\sqrt{\varepsilon_{r2}} \rightarrow \theta_t = 16.8°$$

Therefore:

$$\eta_1^{TE} = \frac{\eta_1}{\cos(\theta_i)} = 754\ \Omega \quad \text{and} \quad \eta_2^{TE} = \frac{\eta_2}{\cos(\theta_t)} = 131.3\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.7$$

The TE wave power density of the reflected wave in the propagation direction ($\theta_r = \theta_i$ is the reflection angle):

$$S_r^{TE} = S_i^{TE}\left|\Gamma^{TE}\right|^2 \frac{\cos(\theta_i)}{\cos(\theta_r)} = \frac{1}{2}\frac{\left|\vec{E}^{TE}\right|^2}{\eta_1}\left|\Gamma^{TE}\right|^2 \frac{\cos(\theta_i)}{\cos(\theta_r)} = 6.56 \times 10^{-4}\ \text{W/m}^2$$

Finally, the power received by the antenna is:

$$P_R = S_r^{TE} A_e = 4.7 \times 10^{-5}\ \text{W}$$

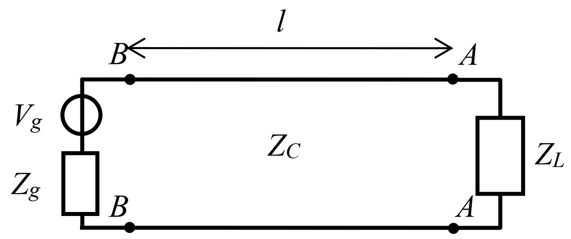
where:

$$A_e = \frac{\lambda^2}{4\pi} G = 0.0716\ \text{m}^2$$

Problem 2

A source with voltage V_g = 150 V and internal impedance Z_g = 75 Ω is connected to a lossless transmission line with characteristic impedance Z_C = 75 Ω, which terminates on a load Z_L = 50 Ω. The frequency is f = 520 MHz and the length of the line is l = 5.25 m.

1) Calculate the power absorbed by Z_L.
2) Calculate the voltage at the load section (V_{AA}) if Z_L becomes an open circuit due to a fault in the load; express V_{AA} in the time domain.



Solution

1) The generator is matched to the line, so the calculations for point 1 can be carried out in the time domain: in fact, there will be only one reflection (at the load section). The reflection coefficient at that section is:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.2$$

Therefore the power absorbed by the load is:

$$P_L = P_{AV}(1 - |\Gamma_L|^2) = \frac{|V_g|^2}{8\mathrm{Re}[Z_g]}(1 - |\Gamma_L|^2) = 36\ \mathrm{W}$$

2) In this case, we can assume $Z_L \to \infty$. Therefore:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = 1$$

The reflection coefficient at section BB is:

$$\Gamma_{BB} = \Gamma_L e^{-2j\beta l} = 0.3090\ - j0.9511$$

As a result, the input impedance is:

$$Z_{BB} = Z_C \frac{1 + \Gamma_{BB}}{1 - \Gamma_{BB}} = -j103.2\ \Omega$$

The voltage at section BB is:

$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Z_g} = 98.2\ - j71.3\ \mathrm{V}$$

The progressive wave on the line at section BB is:

$$V_{BB}^+ = \frac{V_{BB}}{1 + \Gamma_{BB}} = 75\ \mathrm{V}$$

The voltage on the load is:

$$V_{AA} = V_{BB}^+ e^{-j\beta l}(1 + \Gamma_L) = 121.3\ -\ j88.2\ \mathrm{V}$$

In the time domain:

$$V_{AA}(t) = 150\cos(2\pi f t - 0.6283)\ \mathrm{V}$$

Problem 3

A ground-based Radar transmits the signal

$$s_{Tx}(t) = g(t) \cdot cos(2\pi f_0 t),$$

where $f_0 = 10$ GHz is the carrier frequency and $g(t)$ is a cardinal sine with bandwidth $B = 100$ MHz. The same Radar is used to receive the echoes of the transmitted signal reflected by two objects placed at $r_1 = 100$ m and $r_2 = 110$ m from the Radar.

1. Write the expression of the complex envelope of the received signal (after demodulation by f_0). You can assume that the two reflections have the same intensity.
2. Write the expression of the Fourier transform of the complex envelope of the received signal and draw a qualitative graph of its magnitude.
3. Assume now that $g(t)$ is an arbitrary base-band pulse with bandwidth B (not necessarily a cardinal sine). Describe a procedure to measure the delays of the two reflections.
4. What is the minimum difference $\Delta r = |r_1 - r_2|$ for which it is possible to detect the presence of two reflections?

Solution

1) Complex envelope:

$$s(t) = g(t - \tau_1) \cdot exp(-j2\pi f_0 \tau_1) + g(t - \tau_2) \cdot exp(-j2\pi f_0 \tau_2)$$

2) Fourier Transform

$$S(t) = rect\left(\frac{f}{B}\right) \cdot \{exp(-j2\pi(f + f_0)\tau_1) + exp(-j2\pi(f + f_0)\tau_2)\}$$

With $\tau_1 = \frac{2r_1}{c}$ and $\tau_2 = \frac{2r_2}{c}$.

For a qualitative understanding of how the graph looks like, one can write:

$$S(t) = rect\left(\frac{f}{B}\right) exp(-j2\pi(f + f_0)\tau_1)\{1 + exp\big(-j2\pi(f + f_0)(\tau_2 - \tau_1)\big)\}$$

The term outside the brackets has no effect on magnitude. The term inside the brackets goes to zero whenever $2\pi(f + f_0)(\tau_2 - \tau_1) = (2k + 1)\pi$. Hence the distance between two zeroes is obtained as: $2\pi\Delta f(\tau_2 - \tau_1) = 2\pi \Rightarrow \Delta f = \frac{1}{\tau_2 - \tau_1}$

3) The best way to measure the delays is to cross-correlate the received signal with $g(t)$:

$$s_{xc}(t) = s(t) * g^*(-t)$$

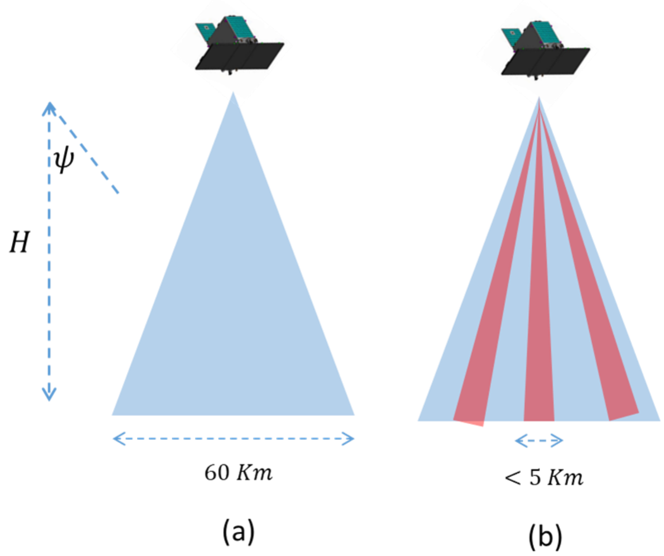
$$s_{xc}(t) = R_g(t - \tau_1) \cdot exp(-j2\pi f_0 \tau_1) + R_g(t - \tau_2) \cdot exp(-j2\pi f_0 \tau_2)$$

where R_g is a short pulse of duration 1/B.

4) The temporal resolution is $\Delta\tau = \frac{1}{B}$. Recalling that $\tau = \frac{2r}{c}$ we have $\Delta\tau = \frac{2\Delta r}{c}$, hence $\Delta r = \frac{c}{2B}$, meaning that the cross-correlation generates two detectable peaks as long as

$$|r_1 - r_2| > \frac{c}{2B}$$

Problem 4



A spaceborne transmitter is equipped with an array of radiating elements to transmit a signal to be received by ground stations. The system operates at a frequency $f_0 = 5$ GHz at an orbital height $H = 500$ Km.

Individual elements are designed to radiate over a beamwidth that determines a footprint of 60 Km on the surface, see figure 1a. The array is designed to transmit the same signal in three beams within the footprint, see figure 1b. The width of these three beams is such as to determine sub-footprints no larger than 5 km on the surface. The three beams point at $\psi = 0°$ and $\psi = \pm 3°$ (see figure 1b).

1. Determine the width of individual radiating elements.
2. Determine the separation between nearby elements and the total number of elements within the array.
3. Determine the phase shifts to be applied at each element to steer the beams by 0.5°.
4. Assume now that the array is formed by only two elements with a separation of 0.5 m and that their transmissions occur with a relative phase shift of π. Calculate beam intensity as a function of the squint angle ψ (considering only small values of ψ within the 60-km footprint).

Solution

1) 60 Km on the surface correspond to a beamwidth $\Delta\psi = \frac{60}{500} = 0.12$ rad, or 6.87°. The width of any single element is therefore $= \frac{\lambda}{\Delta\psi} = 0.5\ m$
2) For generating the 3 beams we require angular ambiguity to be $\psi_{amb} = \frac{3°}{180°}\pi = 0.052\ rad$. This corresponds to a separation $\Delta x = \frac{\lambda}{\psi_{amb}} = 1.146\ m$. By taking 6 elements, total array length is $L_{array} = 6\Delta x = 6.87$ m, which determines sub-footprints of $\frac{\lambda}{L_{array}} H = 4.36\ Km$
3) The spatial frequency is $f_x = \frac{1}{\lambda}\frac{0.5°}{180°}\pi$. The corresponding phase shift at the n-th element is given as: $\varphi_n = 2\pi f_x n \Delta x$.
4) Neglecting common amplitude and phase terms, the field is:

$$E(\psi) = exp\left(-j2\pi\frac{sin(\psi)}{\lambda}x_1\right) - exp\left(-j2\pi\frac{sin(\psi)}{\lambda}x_2\right)$$

where the sign "-" of the second element comes from the assumption of a relative phase shift of π. The two transmitters are placed at $\pm\frac{\Delta x}{2}$, hence:

$$E(\psi) = exp\left(j2\pi\frac{sin(\psi)}{\lambda}\frac{\Delta x}{2}\right) - exp\left(-j2\pi\frac{sin(\psi)}{\lambda}\frac{\Delta x}{2}\right) = 2jsin\left(\pi\frac{sin(\psi)}{\lambda}\Delta x\right)$$

For small angles one can further simplify $sin(\psi) \approx \psi$:

$$E(\psi) = 2jsin\left(\pi\frac{\psi}{\lambda}\Delta x\right)$$

Accordingly, field intensity is null at $\psi = 0°$ and peaks when $\pi\frac{\psi}{\lambda}\Delta x = \pm\frac{\pi}{2}$, that is when $\psi = \pm\frac{\lambda}{2\Delta x}$ (about 3.44°). No other peaks or zeroes are present within the 60-km footprint allowed by the size of individual radiating elements.