Electromagnetics and Signal Processing for Spaceborne Applications
February 13th, 2026

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SURNAME AND NAME ____________________

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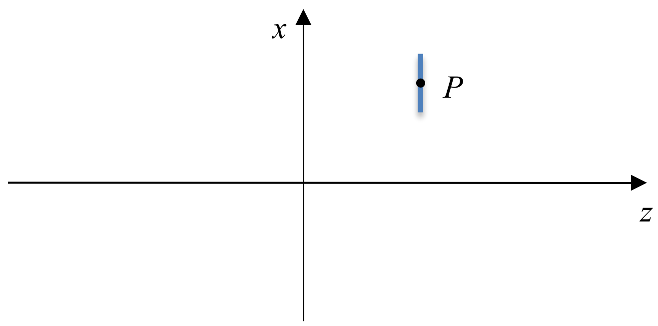
Problem 1

A uniform plane wave propagates in distilled water (ε_{r1} = 81 and μ_{r1} = 1) and impinges on the surface of a medium characterized by conductivity σ = 0.6 S/m, ε_{r2} = 9 and μ_{r2} = 1. The incident electric field is

$$\vec{E}_i = (\vec{\mu}_x + j\vec{\mu}_y)e^{-j0.0377z} \text{ V/m}$$

Determine:

1. The polarization of the incident wave.
2. The wavelength in the second medium.
3. The power absorbed by the dipole in P(1,1,1 m), lying on the xz plane. To this end, consider that following data for the dipole: directivity D = 6 dB and efficiency η = 0.3.



Solution

1) The field has two components, and they have a phase difference of $\pi/2$ → the wave polarization if circular. The rotation can be determined by shifting to the space-time domain, which yields:
$\vec{E}_i(z=0,t) = \cos(2\pi f t)\vec{\mu}_x + \cos(2\pi f t + \pi/2)\vec{\mu}_y$ V/m → LHCP

2) First, it is necessary to calculate the frequency. Based on $\beta_1 = 0.0377$ rad/m:

$$f = \frac{\beta_1 c}{2\pi\sqrt{\varepsilon_{r1}}} = 200 \text{ kHz}$$

The wavelength in the second medium depends on the propagation constant. The loss tangent for this medium is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_2} \approx 6 \times 10^3$$

Thus, the good conductor approximations can be used:

$$\gamma_2 = \alpha_2 + j\beta_2 = \sqrt{\frac{\omega\mu_2\sigma}{2}} + j\sqrt{\frac{\omega\mu_2\sigma}{2}} = 0.688(1+j)\ \ 1/\text{m}$$

The wavelength in the second medium is:

$$\lambda_2 = \frac{2\pi}{\beta_2} = 9.13 \text{ m}$$

3) The electric field reaching point P depends on the reflection coefficient, which, in turn, depends on the intrinsic impedance of the medium. Using the same kind of approximation:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = 41.9\ \Omega$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma}} = (1+j)1.15\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.945 + j0.052$$

The gain of the dipole is:

$$G = D\eta = 1.19 = 0.77 \text{ dB}$$

The effective area of the dipole is:

$$A_e = G\frac{\lambda^2}{4\pi} = 7.92 \text{ m}^2$$

The dipole can receive only the x-component of the wave. The power absorbed is:

$$P = SA_e = 0.0025 \text{ W}$$

where:

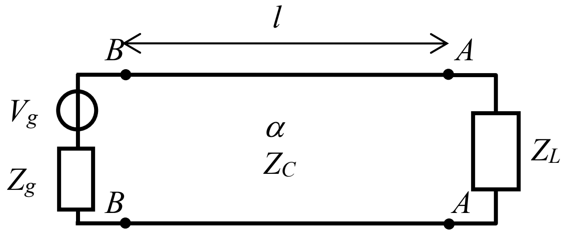
$$S = \frac{1}{2}\frac{|E_2^x(z=0)|^2}{|\eta_2|}e^{-2\alpha_2 z_P}\cos(\measuredangle\eta_2) = 3.125 \times 10^{-4}\ \frac{\text{W}}{\text{m}^2}$$

and

$$|E_2^x(z=0)| = |E_i^x(z=0)||1+\Gamma| = 0.0754 \text{ V/m}$$

Problem 2

Consider the circuit below. The receiving antenna is modeled as a voltage generator V_g = 20 V (with internal impedance Z_g = 75 Ω), which is connected to the receiver RX through a lossy transmission line (TL) with characteristic impedance Z_C and attenuation constant α = 35 dB/km. The input impedance of the receiver is Z_L = 100 Ω, the line length is l = 7 m and the frequency is f = 1 GHz. Determine which of the following values for Z_C maximizes the power absorbed by Z_L: Z_C = 75 Ω or Z_C = 100 Ω.



Solution

a) $Z_C = Z_g = 75\ \Omega$

In this case there is no discontinuity at section BB, but a mismatch at section AA; therefore, the power absorbed by the load is:

$$P_L = P_{AV} e^{-2\alpha l}\left[1-\left|\Gamma_{AA}\right|^2\right]$$

where:

$$P_{AV} = \frac{\left|V_g\right|^2}{8\,\mathrm{Re}\left[Z_g\right]} = 0.667\ \mathrm{W}$$

$\alpha = 4.6\times10^{-3}$ Np/m

$$\Gamma_{AA} = \frac{Z_L - Z_C}{Z_L + Z_C} = 0.143$$

Therefore → P_L = 0.617 W

b) $Z_C = Z_L = 100\ \Omega$

In this case there is no discontinuity at section AA, but a mismatch at section BB; therefore, the power absorbed by the load is:

$$P_L = P_{AV}\left[1-\left|\Gamma_g\right|^2\right]e^{-2\alpha l}$$

where:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_L - Z_g}{Z_L + Z_g} = 0.143$$

Therefore → P_L = 0.617 W

The power transferred to the load is the same in both cases.

Problem 3

Consider the transmitted signal $s(t) = g(t)cos(2\pi f_0 t)$, with g(t) a cardinal sine with a bandwidth of 100 Hz and unitary energy, and $f_0 = 900$ Hz.

1. Write the expression of the Fourier Transform of g(t) and draw its graph.
2. Write the expression of the Fourier Transform of s(t) and draw its graph.
3. Compute the energy of s(t).
4. The signal s(t) is sampled with sampling frequency $f_s = 400$ Hz to generate the numerical sequence s_n. Write the expression of the Fourier Transform of s_n and draw its graph.
5. Would it be possible to extract the complex envelope of s(t) based on the sequence s_n? How?

Solution <u>outline</u>

1) Since g(t) is a sinc with bandwidth B, G(f) is a rectangular pulse with width B. Let A be the amplitude of G(f), the energy of g is obtained as $\int |G(f)|^2 df = A^2 B = 1$, so $A = \frac{1}{\sqrt{B}} = 0.1$

2) $S(f) = \frac{1}{2} G(f - f_0) + \frac{1}{2} G(f + f_0)$

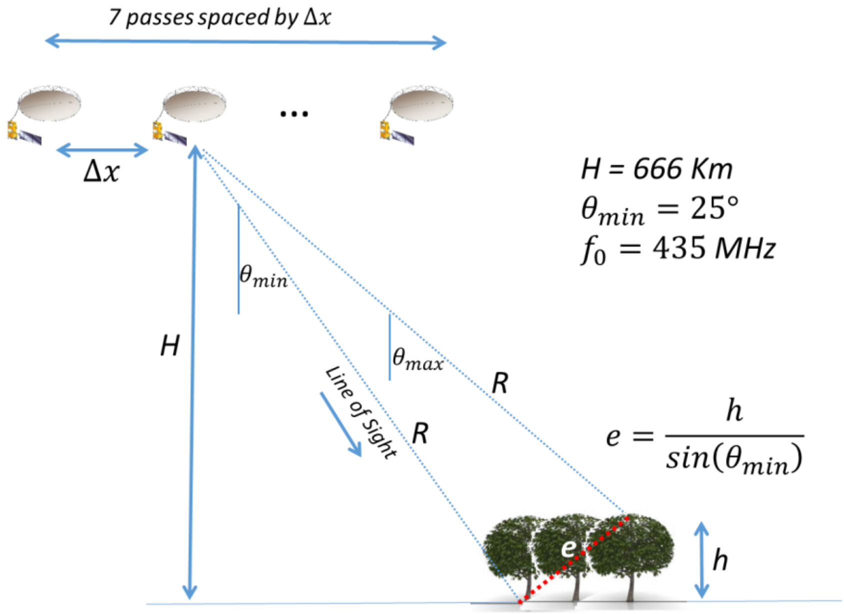
3) $E_s = \int |S(f)|^2 df = \frac{1}{4} E_g + \frac{1}{4} E_g = 0.5$

4) The spectrum of s(t) is folded with a period of 400 Hz. Two replicas fall in the interval between -200 and +200 Hz at the central frequencies of – 100 Hz and +100 Hz

5) Yes, since the replicas do not overlap. In this case it suffices to shift in frequency and filter.

Problem 4

The ESA Mission BIOMASS operates a Radar at 435 MHz to generate a 3D representation of the vegetation. System geometry is represented in the figure below (note that the orbital spacing Δx and forest height are largely exaggerated for visualization purposes):



Data from a single orbit allow for resolving the vegetation in distance. Coherent processing of data from 7 consecutive orbits is then applied to resolve different elements of the vegetation (terrain, understorey, low-, mid-, top-canopy) at the same distance from the Radar but at different angular positions.

1. The tree bottom is sensed at an incidence angle $\theta_{min} = 25°$. Make a **safe** assumption about tree top height $\boldsymbol{h}$ to derive the length of the segment $\boldsymbol{e}$ (orthogonal to the Line of Sight) and consequently the maximum angle θ_{max}.
2. On the basis of your response to point 1, discuss how to set the spacing between any two consecutive orbits $\boldsymbol{\Delta x}$.
3. On the basis of your response to point 2, calculate angular resolution and discuss how this translates to vertical resolution.

Note: remember that for a Radar the relation between angle and spatial frequencies takes on a factor 2: $f_x = \frac{2}{\lambda} sin(\theta)$.

Solution outline

This is a standard problem with antenna array. The spacing between two consecutive elements determines angular ambiguity, array length determines angular resolution.

The angular sector from which the radiation from the forest originates is by definition in between θ_{min} and θ_{max}, so that the spatial frequencies at the satellite array are within $f_{min} = \frac{2}{\lambda} sin(\theta_{min})$ and $f_{max} = \frac{2}{\lambda} sin(\theta_{max})$. The spatial bandwidth is therefore $B_x = f_{max} - f_{min}$, and the spacing between any two orbits has to be $\Delta x \leq \frac{1}{B_x}$.

Assuming a safe requirements for forest height is instrumental to ensuring that ambiguities are sufficiently far. In a tall forest the vegetation can reach 40 m (even more in tropical forests), so a safe requirements can be on the order of 80 m. This results in a spacing between orbits around 1.5 Km.

Angular resolution is obtained as always from the total array length (7 times Δx). Vertical resolution is obtained from angular resolution as (just look at the picture):

$\Delta z = \Delta\theta \cdot \mathrm{R} \cdot \sin(\theta)$.
Not surprisingly, this is equal to the assumed forest height divided by 7.