Electromagnetics and Signal Processing for Spaceborne Applications
July 17th, 2026

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SURNAME AND NAME ____________________

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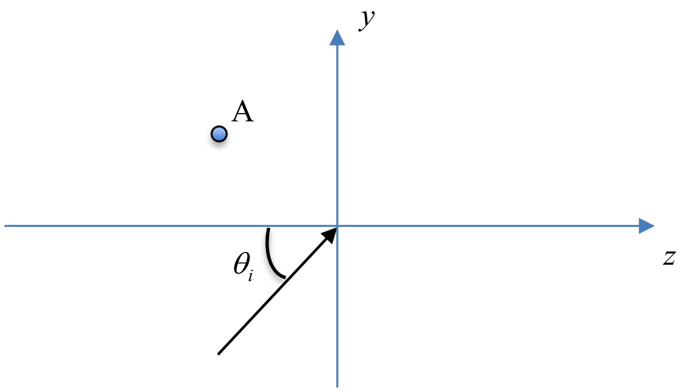
Problem 1

A plane sinusoidal EM wave (incidence angle θ_i = 60°) propagates from free space into a medium with ε_{r2} = 3, μ_{r2} = 1. The expression for the incident electric field is:

$$\vec{E}_i = (\vec{\mu}_y - 1.7321\vec{\mu}_z + 2\vec{\mu}_x)e^{-j52.36z}e^{-j90.69y} \text{ V/m}$$

1) Determine the polarization of the incident EM wave.
2) Determine the frequency of the EM wave.
3) Determine the electric field vector of the reflected wave at point A(1 m, 1 m, -1 m).
4) Draw the magnetic field of the reflected wave at point A(1 m, 1 m, -1 m) and calculate its absolute value.



Solution

1) The incident wave is composed by both TE and TM components. The TM component, lying on the *yz* plane, has absolute value equal 2 V/m. The TE component, orthogonal to the *yz* plane, has absolute value equal 2 V/m. As the two components are in phase, the polarization is linear with a tilt angle of 45° with respect to the *x*-axis in the clockwise direction.

2) The frequency of the incident EM wave can be derived, for example, from the phase constant along *z*, β_z = 52.36 rad/m:

$\beta_z = \beta\cos(\theta_i)$ and $\beta = \frac{2\pi f}{c}$ GHz

By combining and inverting such expressions → f = 5 GHz.

3) Let's check the Brester angle, which is given by:

$$sin\,\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r2}+\varepsilon_{r1}}}\right) = 60°$$

As the incident angle is equal to the Brester angle, the TM component will be completely transmitted in medium 2, so we need to consider only the reflected TE component.

The transmission angle is given by:

$$\sqrt{\varepsilon_{r1}\mu_{r1}}\,sin\,\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\,sin\,\theta_t \rightarrow \theta_t \approx 30°$$

Let us first calculate the intrinsic impedances for the TE waves:

$$\eta_1^{TE} = \eta_0\,/cos\,\theta_i = 754\ \Omega$$

$$\eta_2^{TE} = \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\eta_0/\,cos\,\theta_t = 251.3\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE}-\eta_1^{TE}}{\eta_2^{TE}+\eta_1^{TE}} = -0.5$$

The full expression for the reflected electric field is given by:

$$\vec{E}_r = \vec{E}_i^{TE}(0,0,0)\Gamma^{TE}\ e^{-j\beta_1 \sin\theta_i y} e^{j\beta_1 \cos\theta_i z}\ \text{V/m}$$

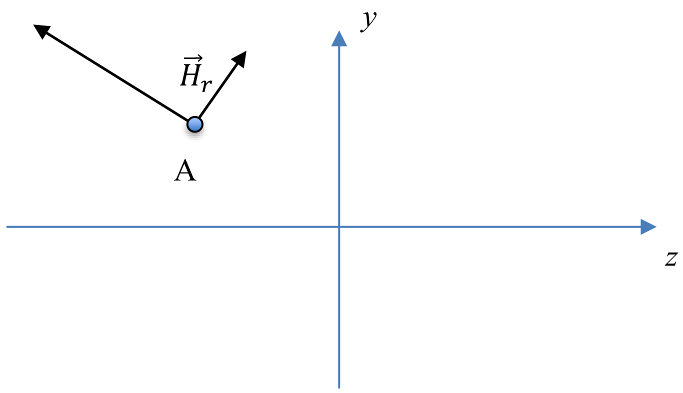
For point A, we obtain:

$$\vec{E}_r(A) = \vec{\mu}_x(-0.107 - j0.994)\ \text{V/m}$$

4) The absolute value of the magnetic field is

$$\left|\vec{H}_r(A)\right| = \frac{\left|\vec{E}_r(A)\right|}{\eta_0} = 2.65\frac{\text{mA}}{\text{m}}$$

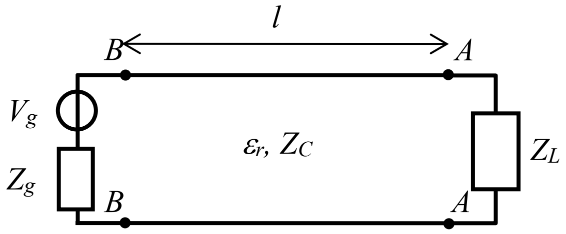
Given the direction of the reflected electric field (-$\vec{\mu}_x$), the direction of the associated magnetic field is the following



Problem 2

A source with voltage V_g = 10 V and internal impedance Z_g = 75 Ω is connected to a transmission line with characteristic impedance Z_C = 50 Ω and electric permittivity ε_r = 25. The frequency is f = 600 MHz and the length of the line is l = 5 m.

1) Determine the value of the load Z_L to maximize the absorbed power.
2) Calculate the power absorbed by the load using the value of Z_L determined at point 1)
3) Calculate the power absorbed by the internal impedance Z_g using the value of Z_L determined at point 1
4) Keeping the value of Z_L determined at point 1, and assuming that the line becomes lossy with attenuation constant α = 50 dB/km, calculate again the power absorbed by the load Z_L.



Solution

a) The wavelength is:

$\boldsymbol{\lambda = \lambda_0/\sqrt{\varepsilon_r} = c/(f\sqrt{\varepsilon_r}) = 0.1}$ m

Therefore, the length of the line normalized to the wavelength is:

$\boldsymbol{l/\lambda = 50}$

In other terms, the line is a multiple of λ. In this case:

$$Z_{BB} = Z_L$$

Therefore, to maximize the power absorbed by the load → $Z_L = Z_g$ = 75 Ω. In fact:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_L - Z_g}{Z_L + Z_g} = 0$$

b) Therefore, the power absorbed by the load is:

$$P_L = P_{AV} = \frac{|V_g|^2}{8\,Re[Z_g]} = 0.1667 \text{ W}$$

c) At section BB, the voltage on Z_g is given by:

$$V_{Zg} = V_g \frac{Z_g}{Z_g + Z_L} = \frac{V_g}{2} \text{ V}$$

Therefore, the power absorbed by Z_g is:

$$P_g = \frac{1}{2}|V_{Zg}|^2 \, Re\left[\frac{1}{Z_g}\right] = 0.1667 \text{ W}$$

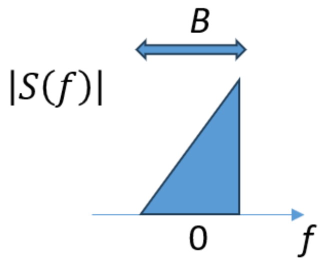
As expected in the case of perfect matching, the power absorbed by Z_g and by Z_L is the same.

d) If the line becomes lossy, we need to calculate Z_{BB}:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = 0.2$$
$$\Gamma_{BB} = \Gamma_L e^{-j2\beta l} e^{-2\alpha l} = 0.1888$$
$$Z_{BB} = Z_C \frac{1 + \Gamma_{BB}}{1 - \Gamma_{BB}} = 73.276 \text{ W}$$
$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Z_L} = 4.94 \text{ V}$$
Therefore, the progressive wave along the line is:
$$V_{BB}^{+} = \frac{V_{BB}}{1 + \Gamma_{BB}} = 4.157 \text{ V}$$
$$P_L = \frac{1}{2} \left| V_{BB}^{+} e^{-\alpha l} e^{-j\beta l} (1 + \Gamma_L) \right|^2 \mathrm{Re}\left[\frac{1}{Z_L}\right] = 0.1566 \text{ W}$$

Problem 3

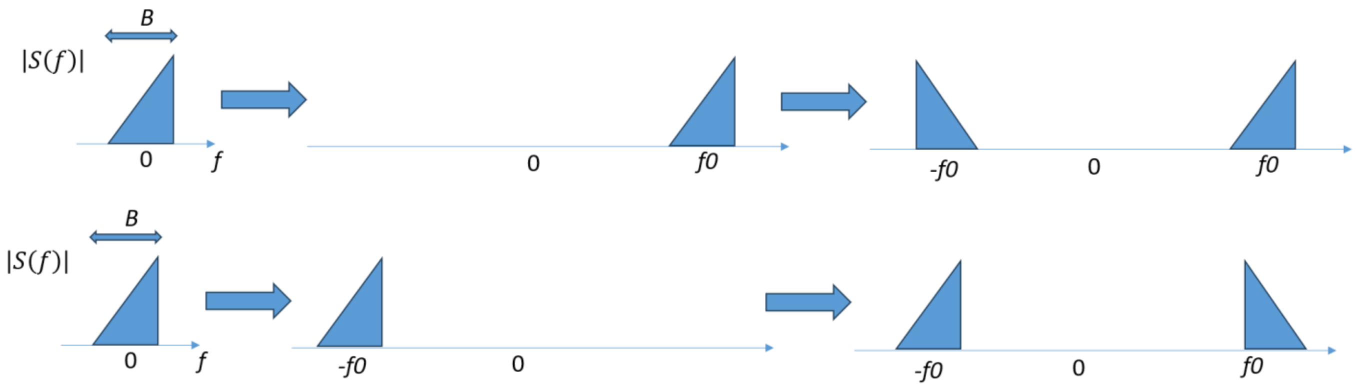
The picture below represents the magnitude of the Fourier Transform (FT) of a signal $s(t)$.



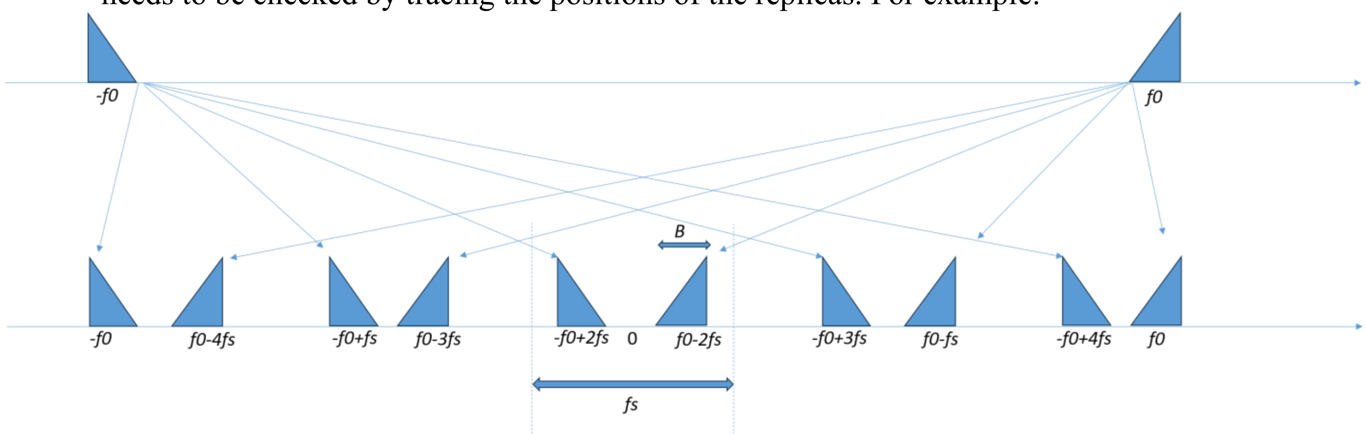
1. Is $s(t)$ real- or complex-valued? Justify your answer.
2. Draw the graphs of the FT of the signals $s_{RF}(t) = Real\big(s(t) \cdot exp(j2\pi f_0 t)\big)$ and $s'_{RF}(t) = Real\big(s(t) \cdot exp(-j2\pi f_0 t)\big)$.
3. The signal $s_{RF}(t)$ is sampled with sampling frequency f_s to generate a numerical sequence s_n. Discuss the conditions on f_s to ensure the possibility to retrieve the original signal $s(t)$ from the numerical sequence s_n. In doing this, privilege options that allow minimizing the value of f_s.
4. Invent values of f_0, B, and f_s that allow recovering $s(t)$ from s_n. Justify your choice.

Solution

1. The FT of feal-valued signals features Hermitian symmetry (magnitude even, phase odd). Since the magnitude is not an even function, the signal has to be complex.
2. Multiplication by $exp(j2\pi f_0 t)$ shifts the signal to the right, then taking the real part generates Hermitian symmetry:



3. The original signal can be recovered as long as the two replicas do not overlap. The minimum possible value is the one for which the two replicas are close to one another, with no gaps, hence $f0 \geq 2*B$. Still, this value does not guarantee absence of overlap, which needs to be checked by tracing the positions of the replicas. For example:



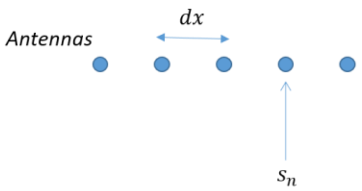
4. It's easy to find a combination of numbers that work by drawing the graph of the close-by replicas and subsequently by replicating it at any sampling frequency larger than its two-sided bandwidth. For example: B = 2, fs = 8, f0 = 18.

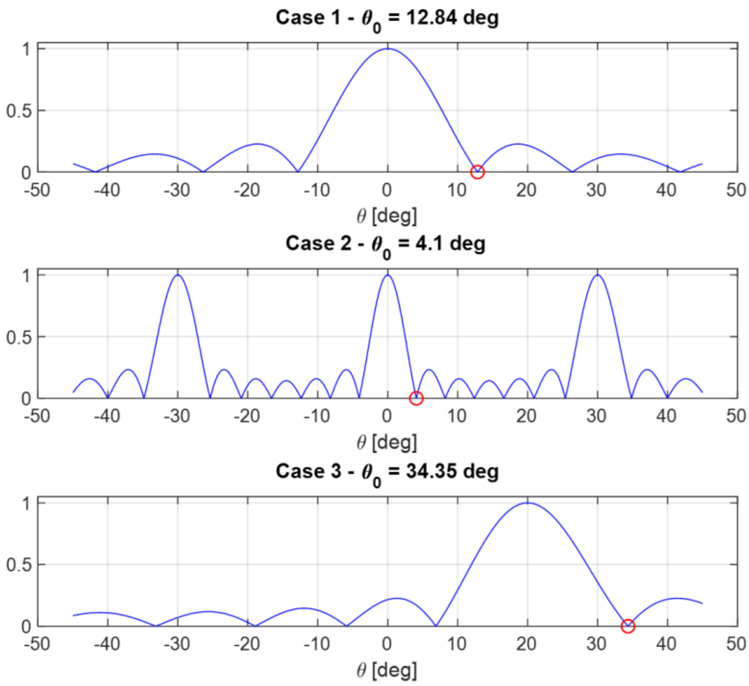
Problem 4

The three graphs below represent the beam patterns generated by three different antenna arrays operating at the same wavelength λ. The three arrays are evenly-spaced with spacing dx, and the radiation is excited by injecting at the *n-th* antenna element a signal of the form

$$s_n = exp\left(-j\frac{2\pi}{\lambda}sin(\theta_p)\cdot ndx\right).$$

The three different beams are obtained by varying the parameters θ_p, dx, and the total number of antenna elements. For each graph, θ_0 represents the position of the first zero after the main peak.





1. Write the formal expression of the beam pattern as a function of the parameters θ_p, dx, and the total number of antenna elements.
2. Determine the values of the parameters θ_p, dx, and the total number of antenna elements for each of the three graphs above. Explain clearly which features of each graph you use to infer each parameter.

Solution

1. The radiated beam can be expressed as:

$$E(\theta) = \sum_n s_n\, exp\left(j\frac{2\pi}{\lambda}sin(\theta)\cdot ndx\right)$$

Since

$$s_n = exp\left(-j\frac{2\pi}{\lambda}sin(\theta_p)\cdot ndx\right)$$

We have that

$$E(\theta) = sinc\left(\frac{sin(\theta) - sin(\theta_p)}{\lambda}Ndx\right)$$

This means that:

- The graph exhibits a peak when $\theta = \theta_p$

- The first zero after a peak corresponds to a shift in spatial frequency equal to $\Delta f_x = \frac{1}{Ndx}$. Given that $f_x = \frac{sin(\theta)-sin(\theta_p)}{\lambda}$ this corresponds to:

$$f_{x0} - f_{x_{peak}} = \frac{1}{Ndx}. => sin(\theta_0) = \frac{\lambda}{Ndx} + sin(\theta_p)$$

Moreover, the graph is not periodic for $dx \leq \frac{\lambda}{2}$. Otherwise, it exhibits multiple peaks with spatial frequency period $\frac{1}{dx}$. Referring to these additional peaks as "ambiguities", in the angular domain, this corresponds to:

$$f_{x\_amb} - f_{x_{peak}} = \frac{1}{dx} => sin(\theta_{amb}) = \frac{\lambda}{dx} + sin(\theta_p)$$

Graph 1:

- Main peak at 0 => $\theta_p = 0$
- There is a single peak, so one can safely assume $dx = \frac{\lambda}{2}$ (although other solutions are valid)
- $\theta_0 = 12.84°$. Using $sin(\theta_0) = \frac{\lambda}{Ndx} + sin(\theta_p)$ we get $0.22 = \frac{\lambda}{Ndx}$=>$N = \frac{2}{0{,}22} = 9$ antennas (rounding to the closest integer)

Graph 2 :

- Main peak at 0 => $\theta_p = 0$
- There are ambiguous peaks at +-30°. Using $sin(\theta_{amb}) = \frac{\lambda}{dx}$ we get that $dx = 2\lambda$.
- $\theta_0 = 4.1°$. Reasoning as in the previous case we get $0.07 = \frac{\lambda}{Ndx}$=>$N = \frac{1}{2 \cdot 0{,}07} = 7$ antennas (rounding to the closest integer)

Graph 3 :

- Main peak at 20° => $\theta_p = 20°$
- There is a single peak, so one can safely assume $dx = \frac{\lambda}{2}$ (although other solutions are valid)
- $\theta_0 = 34.35°$. Using $sin(\theta_0) = \frac{\lambda}{Ndx} + sin(\theta_p)$ we get $0.56 - 0.34 = \frac{\lambda}{Ndx}$=>$N = \frac{2}{0{,}22} = 9$ antennas (rounding to the closest integer)