Electromagnetics and Signal Processing for Spaceborne Applications
January 23rd, 2026

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SURNAME AND NAME ______________________________

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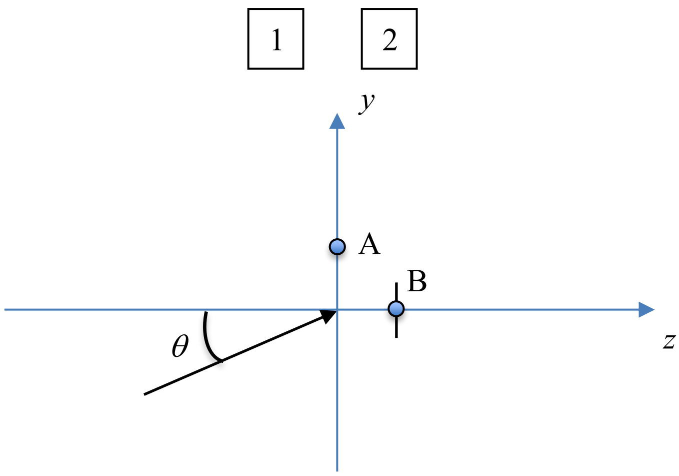
Problem 1

A plane sinusoidal EM wave (frequency f = 2 GHz, incidence angle θ = 20°) propagates from a medium with $\varepsilon_{r1} = 2$, $\mu_{r1} = 2$ into free space. The incident electric field is TE and has absolute value $|\vec{E}_i| = 1\ \mathrm{V/m}$, while its phase is zero in (0,0,0). Moreover, considering the figure below and the refence system therein, $\vec{E}_i$ has a negative x-component.

1) Write the expression of the incident magnetic field $\vec{H}_i$.
2) Calculate the electric field at point A(x = 10 m, y = 1 m, z = 0 m).
3) Calculate the power absorbed by the dipole positioned in B(0,0,1) and lying on the yz plane.



Solution

1) The first step is to calculate the phase constant β in the first medium. As medium 1 is lossless:

$\beta_1 = \omega\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r} = 83.83\ \mathrm{rad/m}$

The incident electric field is:

$\vec{E}_i(z,y) = -\vec{\mu}_x\, e^{-j\beta_1 \cos\theta\, z} e^{-j\beta_1 \sin\theta\, y} = -\vec{\mu}_x\, e^{-j78.78\, z} e^{-j28.67\, y}\ \mathrm{V/m}$

The intrinsic impedance for medium 1 is:

$$\eta_1 = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = 377\ \Omega$$

Therefore, the incident magnetic field is:

$$\vec{H}_i(z,y) = \frac{\left(sin\theta\vec{\mu}_z - cos\theta\vec{\mu}_y\right)}{\eta_1} e^{-j78.78\ z} e^{-j28.67\ y}\ \text{A/m}$$
$$= \left(0.91\vec{\mu}_z - 2.5\vec{\mu}_y\right) e^{-j78.78\ z} e^{-j28.67\ y}\ \text{mA/m}$$

2) The electric field in A can be calculated by knowing the reflected electric field. To this aim, the transmission angle is given by:

$$\sqrt{\varepsilon_{r1}\mu_{r1}}\, sin\,\theta \quad = \sqrt{\varepsilon_{r2}\mu_{r2}}\, sin\,\theta_t \rightarrow \theta_t \approx 43.2°$$

Let us calculate the intrinsic impedances for the TE waves, as well as the reflection coefficient:

$$\eta_1^{TE} = \frac{\eta_1}{cos\,\theta} = 401.2\ \Omega$$
$$\eta_2^{TE} = \frac{\eta_2}{cos\,\theta_t} = \frac{\eta_0}{cos\,\theta_t} = 516.8\ \Omega$$
$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.126$$

The expression for the electric field in A is:

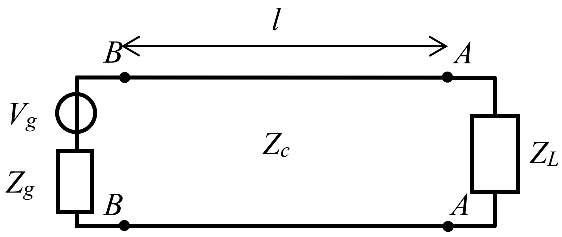
$$\vec{E}(A) = \vec{E}_i(A) + \vec{E}_r(A) = (1 + \Gamma^{TE})\vec{E}_i(A) = (1 + \Gamma^{TE})\vec{E}_i(10,1,0) = -\vec{\mu}_x(-1.04\ +\ j0.44)\ \text{V/m}$$

3) As the transmitted electric field is orthogonal to the *yz* plane (TE wave) and the dipole lies on such a plane, the absorbed power is zero.

Problem 2

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a transmission line with characteristic impedance Z_C = 75 Ω, the frequency is f = 600 MHz and the length of the line is l = 4.5 m.

a) Determine the value of the load Z_L to maximize the absorbed power.
b) Calculate the power absorbed by the load using the value of Z_L determined at point a)
c) Calculate the power absorbed by the internal impedance Z_g = 50 Ω using the value of Z_L determined at point a)



Solution

a) The wavelength is:

$$\lambda = \lambda_0 = 0.5 \text{ m}$$

As a consequence, the length of the line normalized to the wavelength is:

$$l / \lambda = 9$$

In other terms, the line is a multiple of λ. In this case:

$$Z_{BB} = Z_L$$

Therefore, in order to maximize the power absorbed by the load → $Z_L = Z_g$ = 50 Ω. In fact:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_L - Z_g}{Z_L + Z_g} = 0$$

b) Therefore, the power absorbed by the load is simply:

$$P_L = P_{AV} = \frac{|V_g|^2}{8\,\mathrm{Re}[Z_g]} = 0.25 \text{ W}$$

c) At section BB, the voltage on Z_g is given by:

$$V_{Zg} = V_g \frac{Z_g}{Z_g + Z_L} = \frac{V_g}{2} \text{ V}$$

Therefore, the power absorbed by Z_g is:

$$P_g = \frac{1}{2}|V_{Zg}|^2 \,\mathrm{Re}\left[\frac{1}{Z_g}\right] = 0.25 \text{ W}$$

As expected in the case of perfect matching, the power absorbed by Z_g and by Z_L is the same.

Problem 3

An automotive Radar transmits an electromagnetic pulse with a total bandwidth of *B*=500 MHz centered about a carrier frequency f_0= 28 GHz. The signal is reflected by two targets at a distance of 30 and 31 meters from the Radar, and the reflected echoes are then received by the Radar.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Assume that g(t) is a chirp signal, $g(t) = rect\left(\frac{t}{T}\right) exp(j\pi K t^2)$. Determine the value of the chirp rate *K* assuming a total duration *T*=30 microseconds.
3. Describe a procedure to measure the distance of the targets from the Radar.
4. Evaluate the spatial resolution of such measurement.
5. Comment on why you can achieve a temporal resolution much better than pulse length.

Solution <u>outline</u>

1)

$$s_{Tx}(t) = g(t)e^{j2\pi f_0 t}$$

$$s_{Rx}(t) = g(t - t_1)e^{j2\pi f_0 (t-t_1)} + g(t - t_2)e^{j2\pi f_0 (t-t_2)}$$

$$s_{ce}(t) = g(t - t_1)e^{-j2\pi f_0 t_1} + g(t - t_2)e^{-j2\pi f_0 t_2}$$

2)
Bandwidth is B = KT => K = B/T

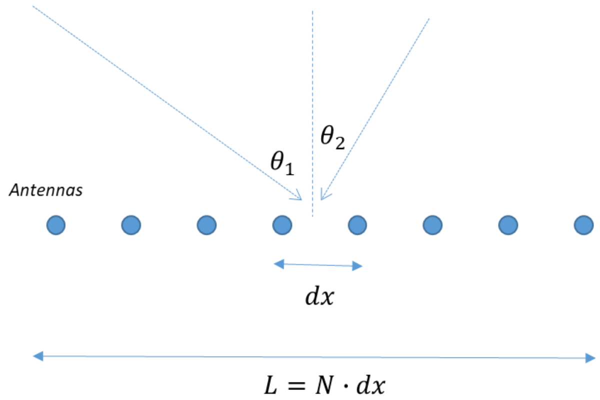
3) cross-correlate with g(t) to find the peaks. Peak positions indicate delays, which are converted to distances as r=c/2*delay

4) dr = c/2/B

5) What matters is bandwidth. Indeed, after cross-correlation the signal gets much shorter.

Problem 4

Multiple electromagnetic waves radiated from distant sources at the frequency $f_0 = 435$ MHz impinge simultaneously on an antenna array as represented in the figure below:



Each antenna is equipped with its own circuity to generate the complex envelope of the received signal.

1. Describe a procedure to measure the directions of arrival $\theta_1, \theta_2, \dots$ based on the N signals (complex envelope) output by the array.
2. Discuss how the set antenna spacing dx
3. After fixing antenna spacing dx, determine the number of antennas required to obtain an angular resolution $\Delta\theta = 3°$ (you can assume an incident direction of 0° as a reference in the calculation of resolution)
4. How would you change your answers at points 3 and 4 if you had knowledge that the direction of arrival of all impinging waves is limited in the interval (-10°,+10°)? (you don't necessarily have to use too much time on calculations, just highlight the rationale, and proceed to details only if you have time left)

Solution outline

1) The signal at the n-th antenna can be expressed as:

$$s_n = exp\left(j\frac{2\pi}{\lambda}sin(\theta)x_n\right)$$

Accordingly, the direction of arrival is mapped into the spatial frequency $f_x = \frac{sin(\theta)}{\lambda}$. It follows that it suffices to compute the Fourier Transform of the received signal to detect spatial sinusoids and measure their frequencies.

2) if no information is given, the safest choice is $dx = \frac{\lambda}{2}$ to avoid ambiguities

3) $\Delta\theta = \frac{\lambda}{Ndx}$

4) if the direction of incoming waves is limited in the interval (-θ_{max}, θ_{max}), then spatial frequencies are limited in the interval $\left(-\frac{sin(\theta_{max})}{\lambda}, \frac{sin(\theta_{max})}{\lambda}\right)$, so that the total spatial bandwidth is $\frac{2sin(\theta_{max})}{\lambda}$. The sampling theorem is then met by setting $dx \le \frac{2sin(\theta_{max})}{\lambda}$. It follows that spacing between antenna elements can be made larger, so that we need fewer antennas for the same angular resolution.