Electromagnetics and Signal Processing for Spaceborne Applications
June 26th, 2026

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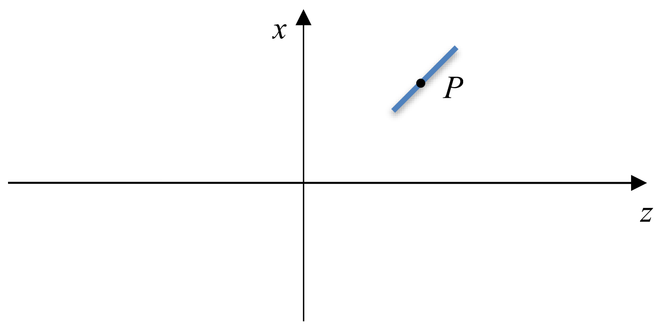
Problem 1

A uniform plane wave propagates in distilled water (ε_{r1} = 81 and μ_{r1} = 1) and impinges on the surface of a medium characterized by conductivity σ = 0.6 S/m, ε_{r2} = 9 and μ_{r2} = 1. The incident electric field is

$$\vec{E}_i = A(\vec{\mu}_x - j\vec{\mu}_y)e^{-j0.0377z} \text{ V/m}$$

Determine:

1. The polarization of the incident wave.
2. The value of A, knowing that the power absorbed by the dipole in P(0.5 m, 0.5 m, 0.5 m) is 1.6 mW. To this end, consider that following data for the dipole: directivity D = 4 dB, efficiency η = 0.3, it lies on the xz plane, 45° inclination with respect to both axes x and z.



Solution

1) The field has two components with same amplitude and phase difference of $\pi/2$ → the wave polarization is circular. The rotation can be determined by shifting to the space-time domain, which yields:

$\vec{E}_i(z=0,t) = A\cos(2\pi f t)\,\vec{\mu}_x - A\cos(2\pi f t + \pi/2)\vec{\mu}_y$ V/m → RHCP

2) The frequency of the wave is obtained from $\beta_1 = 0.0377$ rad/m:

$$f = \frac{\beta_1 c}{2\pi\sqrt{\varepsilon_{r1}}} = 200 \text{ kHz}$$

The loss tangent for this medium is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_2} \approx 6 \times 10^3$$

Thus, the good conductor approximations can be used. Therefore:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = 41.9\ \Omega$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma}} = (1+j)1.15\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.945 + j0.052$$

$$\alpha_2 = \beta_2 = \sqrt{\frac{\omega\mu_2\sigma}{2}} = 0.6883\ \ 1/\text{m}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = 9.13 \text{ m}$$

The gain of the dipole is:

$$G = D\eta = 0.7536 = -1.229 \text{ dB}$$

The effective area of the dipole is:

$$A_e = G\frac{(\lambda_2)^2}{4\pi} = 5 \text{ m}^2$$

The dipole can receive only the *x*-component of the wave projected along the dipole orientation direction. Therefore, the power absorbed is:

$$P = SA_e$$

where

$$S = \frac{1}{2}\frac{|E_2^x(z=0)\cos(45°)|^2}{|\eta_2|}e^{-2\alpha_2 z_P}\cos(\measuredangle\eta_2)$$

and

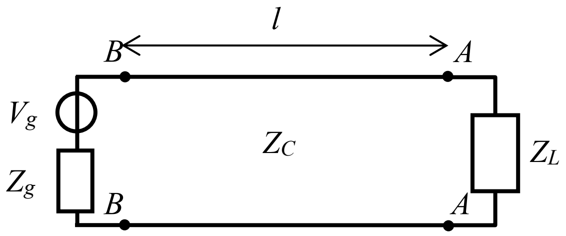
$$E_2^x(z=0) = E_i^x(z=0)(1+\Gamma) = A(1+\Gamma)$$

Setting $P = 1.6$ mW and inverting the equations → $A = 1$ V/m.

Problem 2

A source with voltage $V_g = 100\cos(600\pi 10^6 t + 0.7854)$ V and internal impedance $Z_g = 100\ \Omega$ is connected to a load $Z_L = 100\ \Omega$ by a lossless transmission line with characteristic impedance $Z_C = 50\ \Omega$ and length $l = 4.15$ m.
Calculate:

a) The power absorbed by the load
b) The power absorbed by the line
c) The temporal trend of the voltage at the beginning of the line (section BB below), V_{BB}
d) (OPTIONAL) Assuming that the impedances cannot be changed, what is the simplest way of maximizing the power transfer from the generator to the load?



Solution

a) First, let us calculate the reflection coefficient at the load section:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{1}{3}$$

The reflection coefficient at the generator section is:

$$\Gamma_{BB} = \Gamma_L e^{-2j\beta l} = -0.103 - j\,0.317$$

where $\beta = 6.2832$ rad/m.
As a result, the load at section BB can be calculated as:

$$Z_{BB} = Z_L \frac{1 + \Gamma_{BB}}{1 - \Gamma_{BB}} = 33.7\ - j24.1\ \Omega$$

The reflection coefficient for the generator is:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \text{-0.4485 - j0.2607}$$

Therefore, the power absorbed by the load is:

$$P_L = P_{AV}\left(1 - \left|\Gamma_g\right|^2\right) = \frac{\left|V_g\right|^2}{8\,\mathrm{Re}\left[Z_g\right]}\left(1 - \left|\Gamma_g\right|^2\right) = 9.14\ \text{W}$$

b) As the line is not lossy, no power is dissipated along the line.

c) V_{BB} is calculated as:

$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Zg} = 28.7\ + j10.3\ \text{V}$$

Therefore, the trend of V_{BB} in time is given by:

$$v_{BB}(t) = \mathrm{Re}\left[V_{BB}\, e^{j\omega t}\right] = \left|V_{BB}\right|\cos(\omega t + \measuredangle V_{BB}) = 30.5\cos(\omega t + 0.344)\ \ \text{V}$$

d) Any value of l that is multiple of $\lambda/2$ will allow transferring all the available power to the load.

Problem 3

Consider the signal $s(t) = A_1 cos(2\pi f_1 t) + A_2 cos(2\pi f_2 t)$, with $f_1 = 150$ Hz and $f_2 = 200$ Hz.

1. Write the expression of the Fourier Transform of s(t) and draw its graph.
2. For how long do you need to observe the signal to be able to measure the amplitudes A_1 and A_2 ?
3. The signal s(t) is sampled with a sampling frequency $f_s = 175$ Hz to generate the numerical sequence s_n. Write the expression of the Fourier Transform of s_n and draw its graph. Is it possible to measure A_1 and A_2 based on s_n?
4. Repeat point 3 assuming a sampling frequency $f_s = 180$ Hz.

Solution

1) $S(f) = \frac{A_1}{2}\delta(f - f_1) + \frac{A_1}{2}\delta(f + f_1) + \frac{A_2}{2}\delta(f - f_2) + \frac{A_2}{2}\delta(f + f_2)$

2) To resolve the two sinusoids the observation time has to be (much) longer than the inverse of the frequency difference, $T_{obs} \gg \frac{1}{50}$

3) Sampling at 175 Hz results in:

- The sinusoid at +-150 Hz to be mapped at -+ 25 Hz
- The sinusoid at +-200 Hz to be mapped at +-25 Hz

This means that after sampling the two sinusoids are perfectly overlapped, hence it is no longer possible to measure their amplitudes (one can only measure the sum). The FT of the sampled signal is:

$$S(f) = f_s\left\{\frac{A_1 + A_2}{2}\delta(f - 25) + \frac{A_1 + A_2}{2}\delta(f + 25)\right\}$$

4) Sampling at 180 Hz results in:

- The sinusoid at +-150 Hz to be mapped at -+ 30 Hz
- The sinusoid at +-200 Hz to be mapped at +- 20 Hz

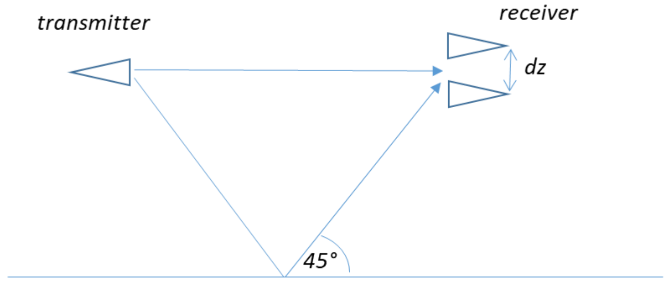
This means that after sampling the two sinusoids are separated, hence it is possible to measure their amplitudes. The FT of the sampled signal is:

$$S(f) = f_s\left\{\frac{A_1}{2}\delta(f - 30) + \frac{A_1}{2}\delta(f + 30) + \frac{A_2}{2}\delta(f - 20) + \frac{A_2}{2}\delta(f + 30)\right\}$$

Note that in this case the frequency separation is reduced to 10 Hz, so the requirement about the observation time is changed to $T_{obs} \gg \frac{1}{10}$

Problem 4

A receiver with two antennas receives a signal from a transmitter plus its reflection onto the ground, as depicted in the figure below.



System parameters are as follows:

- The complex transmitted signal can be modeled as $s_{tx}(t) = g(t)exp(j2\pi f_0 t)$, with $g(t)$ a signal with a bandwidth $B = 1$ MHz.
- The carrier frequency is $f_0 = 1$ GHz
- The antenna spacing dz is set equal to half a wavelength
- τ_{dir} - τ_{ref} = 2 microseconds.

with τ_{dir} and τ_{ref} denoting the delays of the direct and reflected signals from the transmitter to the lower antenna.

1. Write the expression of the complex envelope of the signals received at the two antennas.
2. Write the expression of the Fourier Transform of the complex envelope of the signal at the **lower** antenna. Draw the graph of the magnitude of the Fourier Transform, having care to indicate for which frequency it goes to 0. Is it possible to cancel out the effect of the reflected signal using the signal from a single antenna?
3. Propose a procedure to restore the direct signal (that is, to cancel out the reflected signal) using the **two** signals from the two receiving antennas.

Solution

1) The complex envelope of the signal at the lower antenna is:

$$s_1(t) = g(t - \tau_{dir})exp(-j2\pi f_0 \tau_{dir}) + g(t - \tau_{ref})exp(-j2\pi f_0 \tau_{ref})$$

The complex envelope of the signal at the upper antenna is simply obtained by noting that the delay difference is given by $\frac{sin(\psi_{dir})dz}{c} = 0$ for the direct signal and $\frac{sin(\psi_{ref})dz}{c} \neq 0$ for the reflected signal. Hence:

$$s_2(t) = g(t - \tau_{dir})exp(-j2\pi f_0 \tau_{dir}) + g\left(t - \tau_{ref} - \frac{sin(\psi_{dir})dz}{c}\right)exp(-j2\pi f_0 \tau_{ref})exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right)$$

2)

$$S_1(f) = G(f)\{exp(-j2\pi(f + f_0)\tau_{dir}) + exp(-j2\pi(f + f_0)\tau_{ref})\}$$

So one has:

$$S_1(f) = G(f)exp(-j2\pi(f + f_0)\tau_{dir})\{1 + exp(-j2\pi(f + f_0)\Delta)\}$$

With $\Delta = 2$ microseconds.

To compute the position of null values it suffices to note that the FT is null when $exp(-j2\pi(f + f_0)\Delta)) = -1$, that is when:

$2\pi(f + f_0)\Delta = \pi + 2k\pi$

for any integer k. This entails that:

$f\Delta = \frac{1}{2} + k - f_0\Delta$

For this problem one has that $f_0\Delta$ is an integer number, so the solution is

$$f = \frac{1}{2\Delta} + \frac{k}{\Delta}$$

So we have zeros for f = 250 kHz and -250 kHz.

The presence of zeroes indicates that the spectrum **cannot** be restored by applying an inverse filter.

3) For the signal at the upper antenna: reasoning exactly as in point 2 one finds that zeroes occur for

$$f = \frac{1}{2\Delta'} + \frac{k}{\Delta'} - f_0 \frac{sin(\psi_{ref})dz}{c\Delta'}$$

With $\Delta' = \Delta + \frac{sin(\psi_{ref})dz}{c}$.

Accordingly, we have that zeroes occur at different frequencies for the signals received at the two antennas. This means that the two signals can be combined (for example, they can be summed) to obtain a new signal with no zeroes, which can be afterwards inverted with an inverse filter.