

Electromagnetics and Signal Processing for Spaceborne Applications
September 9th, 2026

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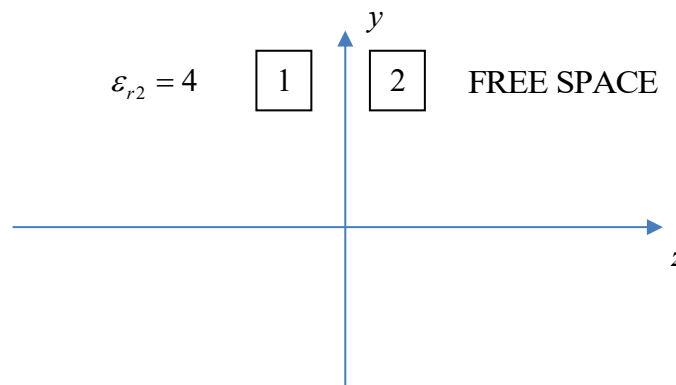
Problem 1

A plane sinusoidal EM wave propagates from a medium with electric permittivity $\epsilon_{r1} = 4$ ($\mu_{r1} = 1$) into free space, with incidence angle θ_i . The expression for the electric field in the first medium is:

$$\vec{E}_i(z, y) = \left[(\cos \theta_i \vec{\mu}_y + \sin \theta_i \vec{\mu}_z) + j \vec{\mu}_x \right] e^{-j \cos \theta_i 41.888 z} e^{j \sin \theta_i 41.888 y} \text{ V/m}$$

For this wave:

- 1) Determine the frequency of the EM wave.
- 2) Determine the direction of propagation and the polarization of the incident EM wave.
- 3) Determine θ_i to obtain a linearly polarized reflected wave.
- 4) Write the equation of the reflected electric field for the incidence angle determined at point 3)



Solution:

1) The frequency of the incident EM wave can be derived from the phase constant $\beta = 41.888$ rad/m:

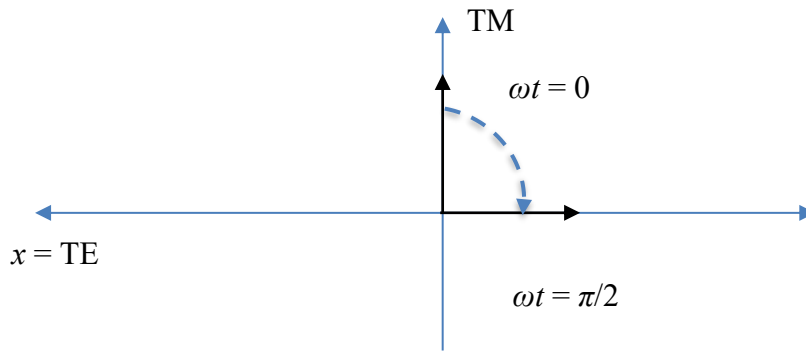
$$\beta = \frac{2\pi f}{c} \sqrt{\epsilon_{r1}} \Rightarrow f = \frac{c\beta}{2\pi\sqrt{\epsilon_{r1}}} = 1 \text{ GHz}$$

2) The wave propagates from the quadrant (+y,-z) towards the quadrant (-y,+z). The polarization of the incident wave is RHCP (right-hand circular polarization) because the two TE and TM components have the same amplitude and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re} \left\{ \left[(\cos \theta_i \vec{\mu}_y + \sin \theta_i \vec{\mu}_z) + j \vec{\mu}_x \right] e^{j\omega t} \right\} = \cos(\omega t) \vec{\mu}_{TM} + \cos\left(\omega t + \frac{\pi}{2}\right) \vec{\mu}_{TE} \text{ V/m}$$

Thus, referring to the figure below that shows the reference system as seen from behind the wave, for $t = 0 \rightarrow \vec{E}(0,0)|_{\omega t=0} = \vec{\mu}_{TM} \text{ V/m}$

Afterwards, for $\omega t = \pi/2 \rightarrow \vec{E}(0,0)|_{\omega t=\pi/2} = \vec{\mu}_{TE} \text{ V/m}$



3) The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. However, for the latter, the reflection is nullified when the incidence angle θ_i coincides with the Brewster angle θ_B , given by:

$$\theta_B = \tan^{-1} \left(\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \right) = 26.565^\circ$$

Thus, by imposing $\theta_i = \theta_B$, the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

4) First, we need to calculate the reflection coefficient for TE waves, which, in turn, requires calculating the refraction angle as:

$$\sqrt{\epsilon_{r1}\mu_{r1}} \sin \theta_i = \sqrt{\epsilon_{r2}\mu_{r2}} \sin \theta_t \rightarrow \theta_t = 63.435^\circ$$

$$\eta_1^{TE} = \frac{\eta_0}{\cos \theta_i \sqrt{\epsilon_{r1}}} = 210.7 \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos \theta_t \sqrt{\epsilon_{r2}}} = 843 \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.6$$

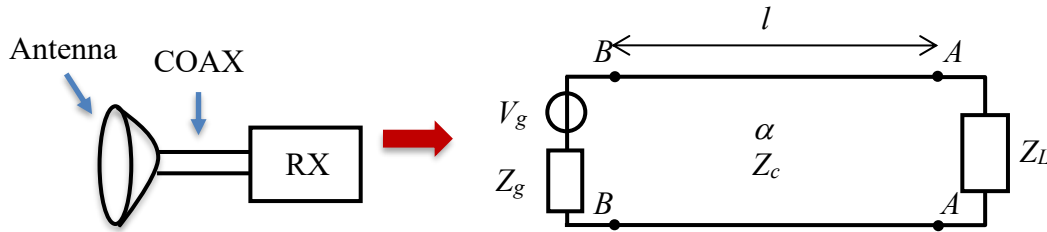
The full expression for the reflected electric field is given by:

$$\vec{E}_r(z,y) = \vec{E}_i^{TE}(0,0) \Gamma^{TE} e^{j \cos \theta_r 41.888z} e^{j \sin \theta_r 41.888y} = j0.6 \vec{\mu}_x e^{j37.466z} e^{j18.733y} \text{ V/m}$$

where: $\theta_r = \theta_i$

Problem 2

Consider the typical receiver sketched in the figure below. The antenna is modeled as a voltage generator $V_g = 50$ V (with internal impedance $Z_g = 50 \Omega$), which is connected to the receiver RX through a lossy coaxial cable (COAX) with characteristic impedance $Z_c = 50 \Omega$ and attenuation constant $\alpha = 30$ dB/km. The input impedance of the receiver is $Z_L = 50 \Omega$, and the frequency is $f = 3$ GHz. Determine the length of the line l to protect Z_L from damage, i.e. to guarantee that $|V_L| = 24$ V (V_L being the absolute value of the voltage on Z_L). After finding l , calculate the power absorbed by Z_L and by the line.



Solution

Given the value of Z_L , Z_c and Z_g , maximum power transfer is achieved. As a result, the power absorbed by the load is simply given by:

$$P_L = P_{AV} e^{-2\alpha l}$$

where:

$$P_{AV} = \frac{|V_g|^2}{8 \operatorname{Re}[Z_g]} = 6.25 \text{ W}$$

Also, $\alpha = 3.5 \times 10^{-3}$ Np/m.

The absorbed power P_L can also be expressed as:

$$P_L = \frac{1}{2} |V_L|^2 \operatorname{Re}\left(\frac{1}{Z_L}\right)$$

Therefore:

$$\frac{1}{2} |V_L|^2 \operatorname{Re}\left(\frac{1}{Z_L}\right) = P_{AV} e^{-2\alpha l}$$

By setting $|V_L| = 24$ V and solving for l :

$$l = \frac{\ln \left[\frac{\frac{1}{2} |V_L|^2 \operatorname{Re}\left(\frac{1}{Z_L}\right)}{P_{AV}} \right]}{-2\alpha} = 11.82 \text{ m}$$

Therefore, the power absorbed by the load is:

$$P_L = P_{AV} e^{-2\alpha l} = 5.76 \text{ W}$$

The power crossing section BB is given by P_{AV} . Therefore, the power absorbed by the line is:

$$P_{line} = P_{AV} - P_L = 0.49 \text{ W}$$

Problem 3

An automotive Radar transmits an electromagnetic pulse with a bandwidth of $B=500$ MHz centered about a carrier frequency $f_0 = 77$ GHz. Signal duration is $T=10$ microseconds. The signal is reflected by two targets, and the reflected echoes are then received by the Radar.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Assume that $g(t)$ is a chirp signal, $g(t) = \text{rect}\left(\frac{t}{T}\right) \exp(j\pi K t^2)$. Determine the value of the chirp rate K .
3. Assuming now that the two targets are found at distances $d_1 = 10$ m and $d_2 = 10.6$ m, draw the graphs of the magnitudes of the received complex signal. **Based on this graph**, is it possible to resolve the two targets?
4. Describe a general procedure to measure the distance of the targets from the Radar and evaluate the spatial resolution of such measurement.

Solution outline

1)

$$s_{Tx}(t) = g(t)e^{j2\pi f_0 t}$$

$$s_{Rx}(t) = g(t - t_1)e^{j2\pi f_0(t-t_1)} + g(t - t_2)e^{j2\pi f_0(t-t_2)}$$

$$s_{ce}(t) = g(t - t_1)e^{-j2\pi f_0 t_1} + g(t - t_2)e^{-j2\pi f_0 t_2}$$

2) Chirp bandwidth is $B = KT \Rightarrow K = B/T$

3) The duration of $g(t)$ is 10 microseconds, whereas the difference between the two delays is $2/c \cdot 0.6 = 4$ nanoseconds. It follows that the two echoes are practically perfectly overlapped. There is no way to resolve the targets based on this graph.

5) after cross-correlation with $g(t)$ the resulting signals are as large as $1/B = 2$ nanoseconds, which allows to detect (resolve) both targets by observing the positions where the signal presents high peaks. Spatial resolution is $c/2/B = 30$ cm.

Problem 4

A ground-based antenna array is used to determine the direction of arrival of signals emitted by spaceborne transmitters to within an angular resolution of 1° . Transmitters are assumed to operate at the frequency of 1 GHz.

1. Determine array length.
2. Determine the required number of elements assuming that the transmitters are located in the angular interval $(-90^\circ, 90^\circ)$.
3. Determine the required number of elements assuming that the transmitters are located in the angular interval $(-10^\circ, 10^\circ)$.
4. The array defined at point 3) is now used in transmission. Calculate the delay to be applied to each element to obtain the transmission of one beam at 5° .
5. Calculate the direction of the secondary beams that arises at point 4).

Solution

- 1) $\lambda = 30$ cm. Angular resolution $= 1/180 \cdot \pi$. Array length $= \lambda / \text{angular_res} = 17.2$ m
- 2) For unambiguous imaging we set the spacing between elements to $\lambda/2 \Rightarrow N_x = \text{Array_length} / (\lambda/2) = 115$ elements
- 3) The spatial frequency interval is now restricted to $\pm \sin(10^\circ)/\lambda$. The total spatial bandwidth is therefore $B_x = 2 \sin(10^\circ)/\lambda$. It follows from the sampling theorem that spatial sampling needs to be no coarser than $1/B_x = 0.86$ m. The number of elements is 20.
- 4) A beam at 5° is obtained by exciting the array elements with a signal $s(x) = \exp(-j2\pi f_{x0}x)$ with $f_{x0} = \frac{\sin(5^\circ)}{\lambda}$ and x the antenna position along the array. The same information can be given with reference to the delay applied to each antenna. Recalling that phase of a delayed RF signal is $2\pi f_0 \tau$, we get:

$$2\pi f_0 \tau = 2\pi \frac{\sin(5^\circ)}{\lambda} x$$

$$\text{Hence } \tau = \frac{\sin(5^\circ)}{f_0 \lambda} x = \frac{\sin(5^\circ)}{c} x$$

- 5) the spatial sampling frequency is the inverse of the sampling space, hence $f_s = 1/dx$. Accordingly, replicas arise in the frequency domain at:

$$f_r = f_{x0} + k f_s = \frac{\sin(5^\circ)}{\lambda} + \frac{k}{dx}$$

for any integer k .

Physical solutions are those for which $|\lambda f| \leq 1$. For these, the associated angles are obtained as

$$\theta = \text{asin}(\lambda f_r)$$

In this case we get: -72.6947 -37.4044 -15.0784 5.0000 25.7504 51.4209