

**EM wave propagation for space-borne systems – Prof. L. Luini,
September 1st, 2026**

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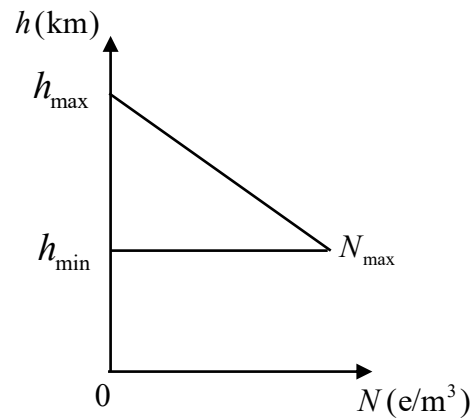
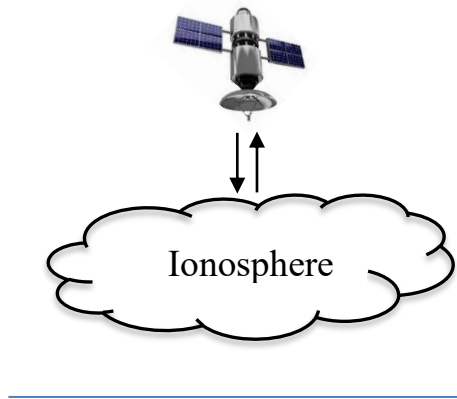
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Problem 1

Referring to the figure below, a Low Earth Orbit (LEO) satellite, pointing to the Earth center, features a pulsed radar intended to measure the ground altitude. The electron content profile is shown in the figure below, where the ionosphere thickness is $W = 400$ km (right side). Calculate $N_{\max}$ and the satellite altitude H , knowing that the radar operates at two frequencies $f_1 = 600$ MHz and $f_2 = 1$ GHz, and that the pulse round trip time for the pulses at f_1 and f_2 are $T_1 = 4.0015$ ms and $T_2 = 4.00054$ ms, respectively.

Assumption: absence of the troposphere.



Solution

The total time it takes for a pulse to propagate to the ground and back to the radar is:

$$T_i = 2 \left(\frac{H}{c} + \frac{1}{2c} \frac{81}{f_i^2} TEC \right) = 2 \left(T^{FS} + T_i^{IONO} \right)$$

where $i = 1$ or 2 , H is the satellite altitude, c is the speed of light and TEC is the total electron content. Taking the difference of T_1 and T_2 , we obtain:

$$\Delta T = T_1 - T_2 = \frac{1}{c} \frac{81}{f_1^2} TEC - \frac{1}{c} \frac{81}{f_2^2} TEC$$

Inverting the equation, we can derive the TEC :

$$TEC = \frac{c \Delta T}{81} \left/ \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \right. = 2 \times 10^8 \text{ e/m}^2$$

Considering the profile:

$$TEC = \int_{h_{\min}}^{h_{\max}} N(h) dh = \frac{N_{\max} W}{2} \rightarrow N_{\max} = 10^{13} \text{ e/m}^3$$

The satellite altitude is calculated by inverting:

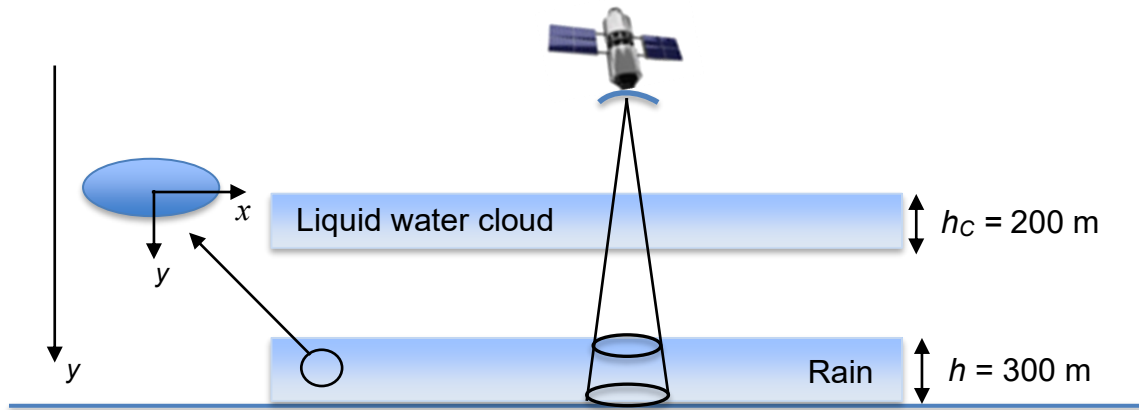
$$T_1 = 2 \left(\frac{H}{c} + \frac{1}{2c} \frac{81}{f_1^2} TEC \right) \rightarrow H = 600 \text{ km}$$

Problem 2

Referring to the figure below, the pulsed radar onboard the satellite, operating at 50 GHz with zenithal pointing, aims at estimating the rain intensity during wintertime (rain height $h = 300$ m). The pulse duration is $1 \mu\text{s}$. The satellite altitude is $H = 500$ km and the parabolic reflector has diameter $D = 6$ m and efficiency $\eta = 0.7$. The rain drop density is 500 drops/m^3 and the backscatter section of each drop is $\sigma = 2 \text{ mm}^2$. For this scenario:

1. Choose the best polarization to maximize the radar sensitivity and accuracy.
2. Calculate the received power P_R , knowing that: the transmit power is $P_T = 1 \text{ kW}$, the specific attenuation of the liquid water cloud is $\gamma = 6 \text{ dB/km}$ (constant vertically and horizontally) and the cloud thickness is $h_C = 200$ m.

Assume pencil beam for the radar antenna.



Solution

1) Due to the geometry of the link, all atmospheric particles are seen as isotropic from the wave standpoint: any polarization is equivalent.

2) Let us first calculate the attenuation induced by the cloud, given by:

$$A_C = \gamma h_C = 1.2 \text{ dB} \rightarrow A_C = 0.7586$$

The effective area of the antenna is:

$$A_R = \left(\frac{D}{2}\right)^2 \pi \eta = 19.79 \text{ m}^2$$

The gain is:

$$G = \frac{4\pi}{\lambda^2} A_R = 6908723 = 68.4 \text{ dB}$$

The power density reaching the rain layer is:

$$S_R = \frac{P_T}{4\pi H^2} G A_C = 1.7 \text{ mW/m}^2$$

The volume of rain drops causing the radar reflection is given by:

$$V = h a = h \Omega H^2$$

where:

$$\Omega = \frac{4\pi}{G} = 1.82 \times 10^{-6}$$

The power reirradiated by the drops is:

$$P_D = S_R \sigma N V = 227.6 \text{ W}$$

Finally, the power received by the radar is:

$$P_R = \frac{P_D}{4\pi H^2} A_C A_R = 1.1 \text{ nW}$$

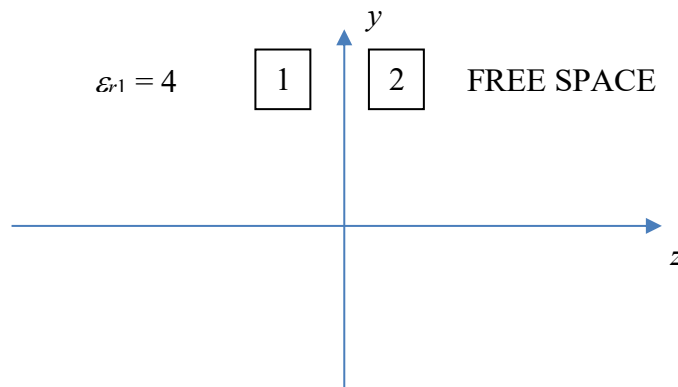
Problem 3

A plane sinusoidal EM wave propagates from a medium with electric permittivity $\epsilon_{r1} = 4$ ($\mu_{r1} = 1$) into free space, with incidence angle θ_i . The expression for the electric field in the first medium is:

$$\vec{E}_i(z, y) = \left[(\cos \theta_i \vec{\mu}_y + \sin \theta_i \vec{\mu}_z) + j\vec{\mu}_x \right] e^{-j \cos \theta_i 41.888z} e^{j \sin \theta_i 41.888y} \text{ V/m}$$

For this wave:

- 1) Determine the frequency of the EM wave.
- 2) Determine the direction of propagation and the polarization of the incident EM wave.
- 3) Determine θ_i to obtain a linearly polarized reflected wave.
- 4) Write the equation of the reflected electric field for the incidence angle determined at point 3)



Solution:

1) The frequency of the incident EM wave can be derived from the phase constant $\beta = 41.888$ rad/m:

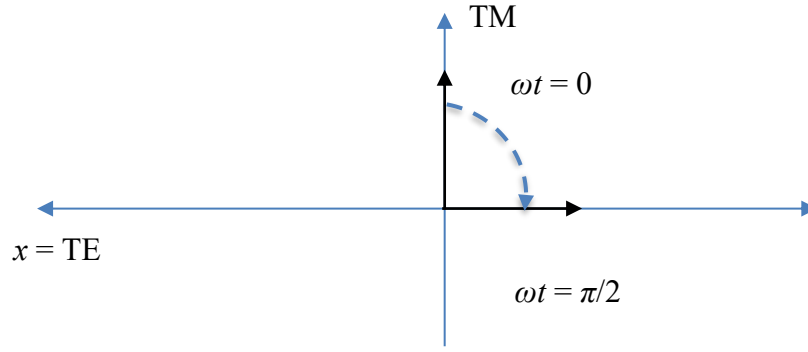
$$\beta = \frac{2\pi f}{c} \sqrt{\epsilon_{r1}} \Rightarrow f = \frac{c\beta}{2\pi\sqrt{\epsilon_{r1}}} = 1 \text{ GHz}$$

2) The wave propagates from the quadrant $(+y, -z)$ towards the quadrant $(-y, +z)$. The polarization of the incident wave is RHCP (right-hand circular polarization) because the two TE and TM components have the same amplitude and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re} \left\{ \left[(\cos \theta_i \vec{\mu}_y + \sin \theta_i \vec{\mu}_z) + j\vec{\mu}_x \right] e^{j\omega t} \right\} = \cos(\omega t) \vec{\mu}_{TM} + \cos\left(\omega t + \frac{\pi}{2}\right) \vec{\mu}_{TE} \text{ V/m}$$

Thus, referring to the figure below that shows the reference system as seen from behind the wave, for $t = 0 \rightarrow \vec{E}(0,0)|_{\omega t=0} = \vec{\mu}_{TM} \text{ V/m}$

Afterwards, for $\omega t = \pi/2 \rightarrow \vec{E}(0,0)|_{\omega t=\pi/2} = \vec{\mu}_{TE} \text{ V/m}$



3) The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. However, for the latter, the reflection is nullified when the incidence angle θ_i coincides with the Brewster angle θ_B , given by:

$$\theta_B = \tan^{-1} \left(\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \right) = 26.565^\circ$$

Thus, by imposing $\theta_i = \theta_B$, the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

4) First, we need to calculate the reflection coefficient for TE waves, which, in turn, requires calculating the refraction angle as:

$$\sqrt{\epsilon_{r1}\mu_{r1}} \sin \theta_i = \sqrt{\epsilon_{r2}\mu_{r2}} \sin \theta_t \rightarrow \theta_t = 63.435^\circ$$

$$\eta_1^{TE} = \frac{\eta_0}{\cos \theta_i \sqrt{\epsilon_{r1}}} = 210.7 \, \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos \theta_t \sqrt{\epsilon_{r2}}} = 843 \, \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.6$$

The full expression for the reflected electric field is given by:

$$\vec{E}_r(z, y) = \vec{E}_i^{TE}(0,0) \Gamma^{TE} e^{j \cos \theta_r 41.888z} e^{j \sin \theta_r 41.888y} = j0.6 \vec{\mu}_x e^{j37.466z} e^{j18.733y} \, \text{V/m}$$

where: $\theta_r = \theta_i$

Problem 4

Consider a downlink from a spacecraft to a ground station. Determine whether it is more convenient to use $f_1 = 20$ GHz or $f_2 = 30$ GHz as a carrier frequency for the link (i.e. which one will provide the highest SNR for the same target yearly availability of the link), considering that the statistics of the atmospheric attenuation A can be modelled using the following Complementary Cumulative Distribution Functions (probability expressed in percentage values, A expressed in dB) for the two frequencies:

$$P(A) = 100e^{-1.15A} \text{ at 30 GHz}$$

$$P(A) = 100e^{-2.3A} \text{ at 20 GHz}$$

Use the following data:

- the target yearly availability of the link is $P_{AV} = 99.999\%$
- transmitter and receiver antennas are parabolic with diameter of $D = 2$ m
- the efficiency of transmitter and receiver antennas is $\eta = 0.6$ (for both frequencies)
- transmitter and receiver antennas are optimally pointed
- the transmitted power is $P_T = 5$ W
- the distance between the ground station and the satellite is $H = 36000$ km
- the receiver LNA equivalent noise temperature is $T_{LNA} = 120$ K
- the mean radiating temperature of the troposphere is $T_{mr} = 278$ K
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- the system bandwidth is $B = 50$ MHz

Solution

To guarantee the target availability for the link (99.999% of the time), we need to set $P(A) = 100\% - 99.999\% = 0.001\%$. As a result, we obtain $A_{30} \approx 10$ dB = 0.1 and $A_{20} \approx 5$ dB = 0.316, at 30 GHz and 20 GHz, respectively. The SNR is calculated as:

$$SNR = \frac{P_T G_T \left(\frac{\lambda}{4\pi H} \right)^2 G_R A}{k T_{sys} B}$$

where f_T and f_R (radiation pattern) are assumed to be 1, as antennas are optimally pointed.

Several terms in such equation depend on the frequency, specifically:

Gain

Starting from the key formula $\frac{A_{eff}}{G} = \frac{\lambda^2}{4\pi}$, and using the available data, we obtain:

$$G = \frac{4\pi\eta}{c^2} A_{phy} f^2 = \frac{4\pi\eta}{c^2} \left(\frac{D}{2} \right)^2 \pi f^2 = \frac{\pi^2 \eta D^2}{c^2} f^2$$

Therefore $G_{20} = 105275 = 50.2$ dB and $G_{30} = 236870 = 53.7$ dB

Wavelength

$\lambda_{20} = 0.015$ m and $\lambda_{30} = 0.01$ m

Equivalent antenna noise power

$$T_A = T_{mr} (1 - A) + T_C A$$

Thus ($T_C = 2.73$ K):

$$T_{A20} = 191 \text{ K and } T_{A30} = 250.5 \text{ K}$$

Therefore, the total system equivalent noise temperature is:

$$T_{\text{sys}} = T_A + T_{LNA}$$

Thus:

$$T_{\text{sys}20} = 311 \text{ K and } T_{\text{sys}30} = 370.5$$

Using these data in the SNR equation, we obtain:

$$SNR_{20} = 89.6 \text{ dB and } SNR_{30} = 53.5 \text{ dB} \rightarrow \text{The link at 20 GHz offers better performance.}$$