Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini
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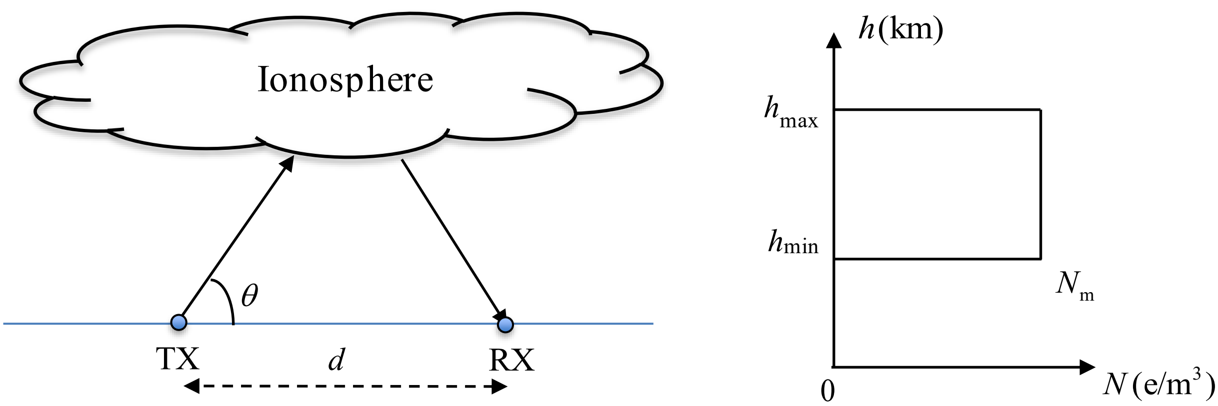
Problem 1

Referring to the figure below, the ionosphere is modelled with the sketched electron density profile, where $h_{\min}$ = 100 km, $h_{\max}$ = 400 km and the critical frequency is f_c = 12.728 MHz. For this scenario:
1) Determine N_m.
2) Assuming that TX transmits a signal with elevation angle of θ = 40°, determine the maximum frequency $f_{\max}$ to be used to reach RX by exploiting the reflection on the ionosphere.
3) Calculate the distance d between TX and RX, assuming the same conditions as in 2).
4) Keeping the same operational frequency $f_{\max}$, what happens if the elevation angle decreases to θ_2 = 30°?

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.



Solution

1) N_m is derived from $f_c = 9\sqrt{N_m}$ → $N_m = 2\times10^{12}$ e/m^3.
2) The maximum frequency $f_{\max}$ to be used to guarantee total reflection is:

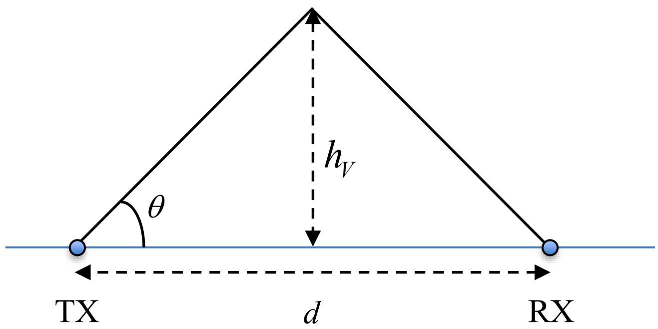
$$\cos\theta = \sqrt{1-\left(\frac{f_C}{f_{\max}}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_m}}{f_{\max}}\right)^2}$$

which yields (assuming $\theta = 40°$):

$$f_{\max} = \sqrt{\frac{81N_m}{1-\left[\cos(\theta)\right]^2}} \approx 19.8 \text{ MHz}$$

3) Exploiting the concept of virtual reflection height, d is given by:

$$d = \frac{2h_V}{\tan\theta} = \frac{2\,1.2\,h_{\min}}{\tan\theta} = 286 \text{ km}$$

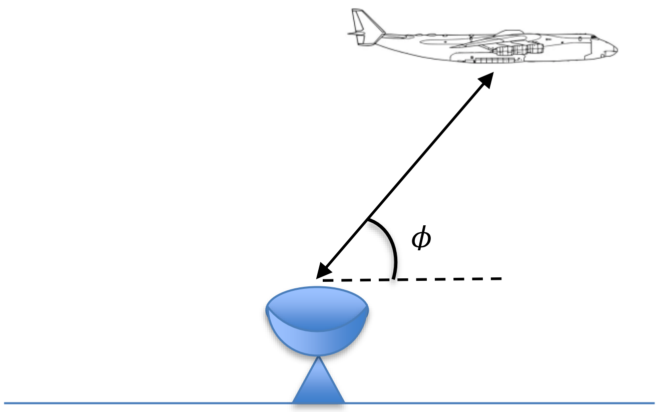


4) If the elevation angle decreases to 30°, keeping the same $f_{\max}$, the wave will still be reflected at $h_{\min}$.

Problem 2

An airport radar points zenithally and operates using electromagnetic pulses at f = 5 MHz to identify an aircraft seen at ϕ elevation angle and with backscattering cross section $\boldsymbol{\sigma_A}$ = 10 m^2. The antenna is a parabolic reflector with gain G = 30 dB and directivity function $f_{rad} = [\cos(\theta)]^2$, being θ the off-boresight angle.

1) Assuming $\phi = 60°$, calculate the altitude of the aircraft, given that $\Delta t = 2$ μs is the two-way time for the pulse to reach back the radar after reflection on the aircraft. To this aim, assume that the full atmosphere is simplistically represented by the following electromagnetic parameters (homogeneous vertically and horizontally): relative permittivity $\varepsilon_r = 16$, relative permeability $\mu_r = 1$, conductivity $\sigma = 10^{-3}$ S/m.
2) Keeping the same aircraft altitude, determine the minimum elevation angle for which the aircraft can be identified, considering that the maximum radar transmitted power P_T is 1142.4 W and that the minimum power density reaching back the radar needs to be at least S_R = 10 pW/m^2.



Solution
1) First, we need to characterize electromagnetically the propagation medium. In this case, the loss tangent is $\frac{\sigma}{\omega\varepsilon} = 0.2247$. Therefore, no approximations are possible. Therefore, the propagation constant is given by:
$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta = 0.0468 + j0.4218$ 1/m
The phase velocity is therefore:
$$v = \frac{\omega}{\beta} = 7.45 \times 10^7 \text{ m/s}$$
As a result, the distance to the aircraft is:
$L = v\, \Delta t/2 = 74.49$ m
Finally, the aircraft altitude is:
$h = L\, [\cos(\theta)]^2 = 64.5$ m
where $\theta = 90° - \phi = 30°$.

2) The power density reaching the aircraft is:
$S_{rad} = \frac{P_T}{4\pi L^2} G f_{rad} e^{-2\alpha L}$ W/m^2
The power density reaching back the radar after reflection is:
$S_R = \frac{S_{rad}\sigma_A}{4\pi L^2} e^{-2\alpha L}$ W
Expressing $f_{rad} = [\cos(\theta)]^2$ and $L = h/\cos(\theta)$, and inverting the equation:

$$[\cos(\theta)]^6 = \frac{S_R(4\pi h^2)^2}{G\sigma_A P_T e^{-4\alpha\frac{h}{\cos(\theta)}}}$$

The equation cannot be solved analytically. By trial and error, the resulting elevation angle is $\theta = 46.65°$.

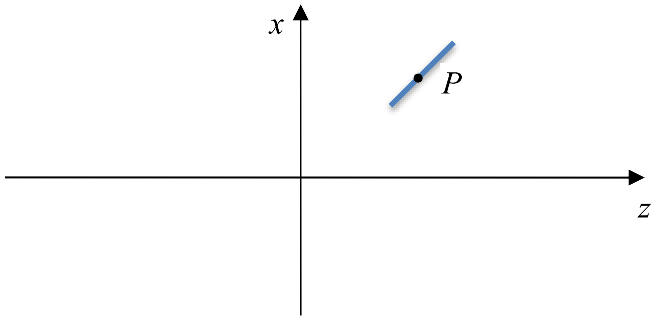
Problem 3

A uniform plane wave propagates in distilled water (ε_{r1} = 81 and μ_{r1} = 1) and impinges on the surface of a medium characterized by conductivity σ = 0.6 S/m, ε_{r2} = 9 and μ_{r2} = 1. The incident electric field is

$$\vec{E}_i = A(\vec{\mu}_x - j\vec{\mu}_y)e^{-j0.0377z} \text{ V/m}$$

Determine:

1. The polarization of the incident wave.
2. The value of A, knowing that the power absorbed by the dipole in P(0.5 m, 0.5 m, 0.5 m) is 1.6 mW. To this end, consider that following data for the dipole: directivity D = 4 dB, efficiency η = 0.3, it lies on the xz plane, 45° inclination with respect to both axes x and z.



Solution

1) The field has two components with same amplitude and phase difference of $\pi/2$ → the wave polarization if circular. The rotation can be determined by shifting to the space-time domain, which yields:

$\vec{E}_i(z=0,t) = A\cos(2\pi f t)\,\vec{\mu}_x - A\cos\,(2\pi f t + \pi/2)\vec{\mu}_y$ V/m → RHCP

2) The frequency of the wave is obtained from β_1 = 0.0377 rad/m:

$$f = \frac{\beta_1 c}{2\pi\sqrt{\varepsilon_{r1}}} = 200 \text{ kHz}$$

The loss tangent for this medium is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_2} \approx 6 \times 10^3$$

Thus, the good conductor approximations can be used. Therefore:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = 41.9\ \Omega$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma}} = (1+j)1.15\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.945 + j0.052$$

$$\alpha_2 = \beta_2 = \sqrt{\frac{\omega\mu_2\sigma}{2}} = 0.6883\ \ 1/\text{m}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = 9.13 \text{ m}$$

The gain of the dipole is:

$G = D\eta = 0.7536 = -1.229$ dB

The effective area of the dipole is:

$$A_e = G\frac{(\lambda_2)^2}{4\pi} = 5 \text{ m}^2$$

The dipole can receive only the x-component of the wave projected along the dipole orientation direction. The power absorbed is:

$P = SA_e$

where

$$S = \frac{1}{2}\frac{|E_2^x(z=0)\sin(45°)|^2}{|\eta_2|}e^{-2\alpha_2 z_P}\cos(\measuredangle\eta_2)$$

and

$E_2^x(z=0) = E_i^x(z=0)(1+\Gamma) = A(1+\Gamma)$

Setting P = 1.6 mW and inverting the equations → A = 1 V/m.

Problem 4

Consider the downlink from a LEO satellite to a ground user, which is seen at $\theta = 47°$ elevation angle at some point. The link is affected by rain and the carrier frequency is $f = 26$ GHz.

1) Define a suitable wave polarization.
2) Calculate the maximum data rate R achievable to guarantee a minimum $\text{SNR}_{\min} = 8$ dB. To this aim, consider BPSK-R modulation. Also, use the following data:
   - ground antenna: circular reflector, directivity $D_R = 20$ dB, efficiency is $\eta_R = 0.6$, radiation pattern $f_R = [\cos(\phi)]^2$ (ϕ off-boresight angle)
   - satellite antenna: circular reflector, directivity $D_S = 10$ dB, efficiency is $\eta_S = 0.7$, radiation pattern $f_S = [\cos(\phi)]^2$ (ϕ off-boresight angle)
   - the ground antenna points at the satellite, while the satellite antenna points at the Earth's center (assume ϕ=90-θ)
   - the power transmitted by the satellite is $P_T = 500$ W
   - the distance between the ground station and the satellite is $H = 1000$ km
   - the receiver LNA equivalent noise temperature is $T_{LNA} = 200$ K
   - the rain rate, assumed to be constant along the link, is $R = 10$ mm/h, the rain height is $h_R = 3.5$ km and the rain temperature is $T_{rain} = 10$ °C. For rain attenuation calculation use $k_R = 0.0956$ and $\alpha_R = 0.9933$
   - assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
   - assume that there are no additional atmospheric constituents inducing attenuation besides rain

Solution

1) Given the LEO satellite link, the wave polarization needs to be circular for geometrical reasons.

2) The wavelength is $\lambda = c/f = 0.0115$ m. The gains of the two antennas are:

$G_R = \eta_R D_R = 60$ (linear scale)

$G_S = \eta_S D_S = 7$ (linear scale)

The atmospheric attenuation is due to rain:

$A_R = k_R R^{\alpha_R} \dfrac{h_R}{\sin\theta} = 4.505$ dB → $A_R = 0.354$

The received power is:

$P_R = P_T G_S f_S \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A_R = 3.35 \cdot 10^{-14}$ W

where $f_R = 1$ (antenna optimally pointed) and $f_S = 0.5349$.

The noise power depends on the total system equivalent noise temperature:

$T_{sys} \approx T_A + T_{LNA}$

The equivalent antenna noise (sky noise) is given by:

$T_A = T_{rain}\,(1\text{-}A_R) + T_C\, A_R = 183.8$ K

Thus $T_{sys} \approx 383.8$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). By inverting the last equation, the maximum bandwidth can be determined.

$$SNR = \frac{P_R}{kT_{sys}B} > SNR_{\min} \rightarrow B < \frac{P_R}{kT_{sys}SNR_{\min}} = 1 \text{ MHz}$$

Assuming BPSK-R → $R = B = 1$ Mbit/s.