Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini
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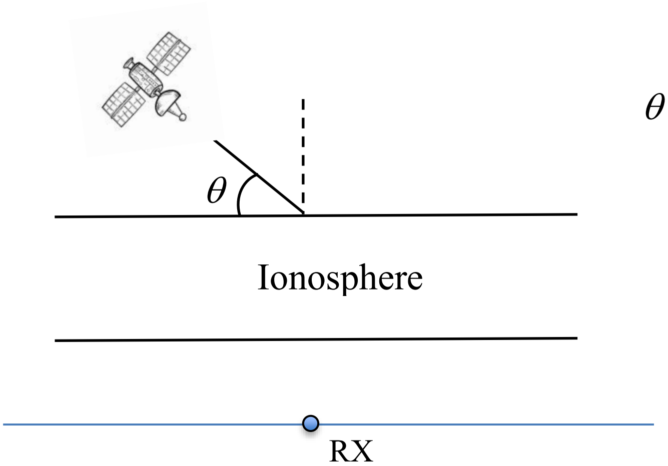
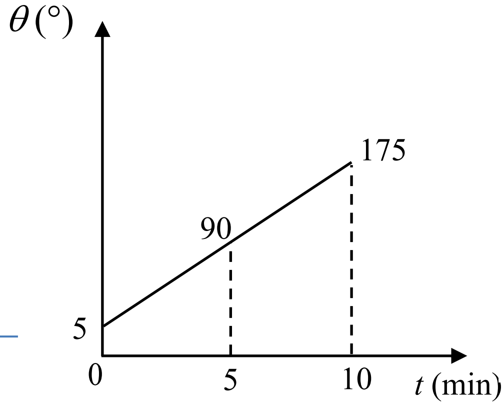
Problem 1

Referring to the figure below, a LEO satellite points at a ground station. The graph below (right side) shows the evolution of ψ during each satellite pass. The link operational frequency is $f = 25$ MHz and maximum electron content along the ionospheric profile (horizontal homogeneity) is fixed to $N_{max} = 2\times10^{12}$ e/m^3. Calculate the total time T (in minutes) during which the satellite-to-ground communication link is available for each pass. Propose a solution to guarantee uninterrupted communication between the satellite and the ground station for the entire duration of the satellite pass.





Solution

To determine if the ionosphere can be crossed, we need to invert the following equation to determine the critical angle θ_C given f and N_{max}:

$$\cos\theta_C = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2} \quad \Rightarrow \quad \theta_C = \cos^{-1}\left(\sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2}\right) = 30.6°$$

If $\theta_C \le \theta \le 180-\theta_C$, the wave crosses the ionosphere, otherwise it is completely reflected.

The first part of the trend of θ (up to 90°) in the figure is given by (t expressed in minutes):

$$\theta = 17t + 5$$

By imposing that $\theta_C = 30.6 \leq \theta \leq 90$:

$$1.5 \text{ min } \leq t \leq 5 \text{ min}$$

Considering that full pass, the total time T is:

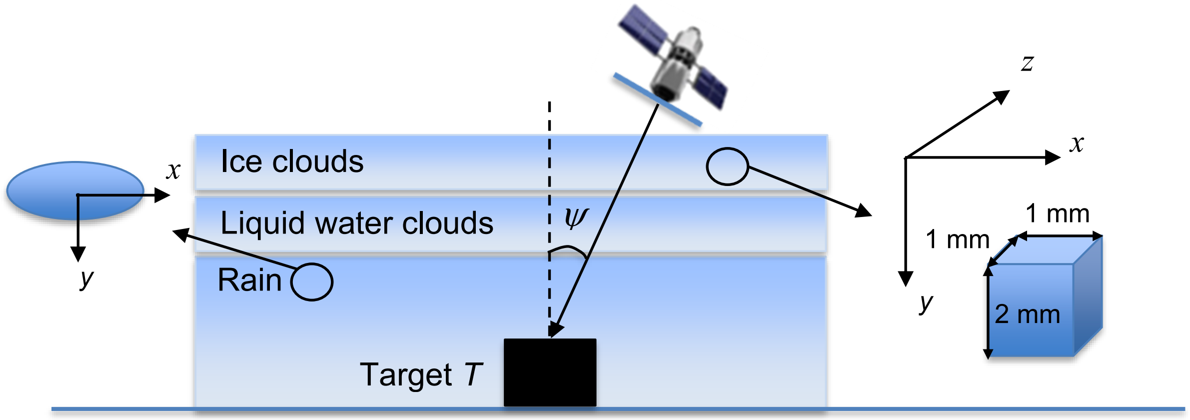
$$T = 2(5 - 1.5) = 7 \text{ min}$$

A possible solution is to increase the operational frequency beyond the one needed to cross the ionosphere when the satellite is at very low altitude ($\theta = 5°$ and $\theta = 175°$)

$$f = \frac{9\sqrt{N_{\max}}}{\sin\theta} = 146 \text{ MHz}$$

Problem 2

Referring to the figure below, a satellite is equipped with a full polarimetric sensor that transmits and receives both vertical and horizontal linear polarizations (oriented along x and z, respectively). The objective is to better characterize the target T, by exploiting the information on both polarizations. The radar operates at 20 GHz. The troposphere consists of three layers: 1) clouds composed by ice particles, whose dimension and shape are reported below (right side); 2) clouds composed by liquid water droplets; 3) rain, composed by drops whose dimension and shape are reported below (left side). For which observation angle(s) ψ does the radar achieve the highest accuracy in characterizing the target? Justify your answer: keep your answer brief and to the point (no essays).



Solution

$\psi = 0°$ is the best choice because:

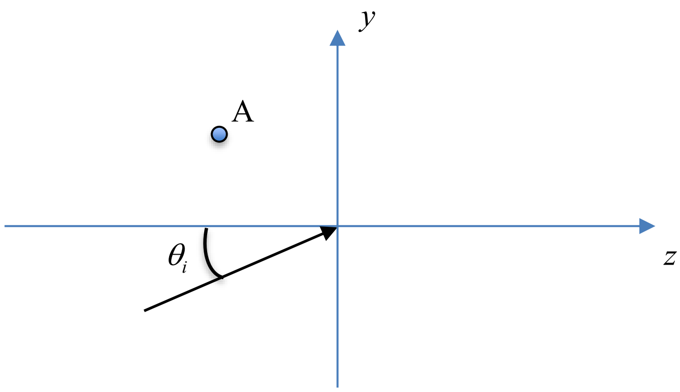
- the attenuation is minimized (shorter path through the troposphere)
- no depolarization due to ice particles because they appear as isotropic to the wave (same x and z dimension)
- no depolarization due to cloud droplets (whatever the value of ψ, as such particles are spherical)
- no depolarization due to rain drops because they appear as isotropic to the wave

Problem 3

A plane sinusoidal EM wave (incidence angle $\theta_i = 60°$) propagates from free space into a medium with $\varepsilon_{r2} = 3$, $\mu_{r2} = 1$. The expression for the incident electric field is:

$$\vec{E}_i = (\vec{\mu}_y - 1.7321\vec{\mu}_z + 2\vec{\mu}_x)e^{-j52.36z}e^{-j90.69y}\ \text{V/m}$$

1) Determine the polarization of the incident EM wave.
2) Determine the frequency of the EM wave.
3) Determine the electric field vector of the reflected wave at point A(1 m, 1 m, -1 m).
4) Draw the magnetic field of the reflected wave at point A(1 m, 1 m, -1 m) and calculate its absolute value.



Solution

1) The incident wave is composed by TE and TM components. The TM component, lying on the *yz* plane, has absolute value equal 2 V/m. The TE component, orthogonal to the *yz* plane, has absolute value equal 2 V/m. As the two components are in phase, the polarization is linear with a tilt angle of 45° with respect to the *x*-axis in the clockwise direction.

2) The frequency of the incident EM wave can be derived, for example, from the phase constant along *z*, $\beta_z = 52.36$ rad/m:

$$\beta_z = \beta\cos(\theta_i) \quad \text{and} \quad \beta = \frac{2\pi f}{c} \ \text{GHz}$$

By combining and inverting such expressions → $f = 5$ GHz.

3) Let's check the Brester angle, which is given by:

$$\sin\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r2}+\varepsilon_{r1}}}\right) = 60°$$

As the incident angle is equal to the Brester angle, the TM component will be completely transmitted in medium 2, so we need to consider only the reflected TE component.
The transmission angle is given by:

$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t$ → $\theta_t \approx 30°$

Let us first calculate the intrinsic impedances for the TE waves:

$$\eta_1^{TE} = \eta_0/\cos\theta_i = 754\ \Omega$$

$$\eta_2^{TE} = \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\eta_0/\cos\theta_t = 251.3\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.5$$

The full expression for the reflected electric field is given by:

$$\vec{E}_r = \vec{E}_i^{TE}(0,0,0)\Gamma^{TE}\ e^{-j\beta_1 \sin\theta_i y} e^{j\beta_1 \cos\theta_i z}\ \text{V/m}$$

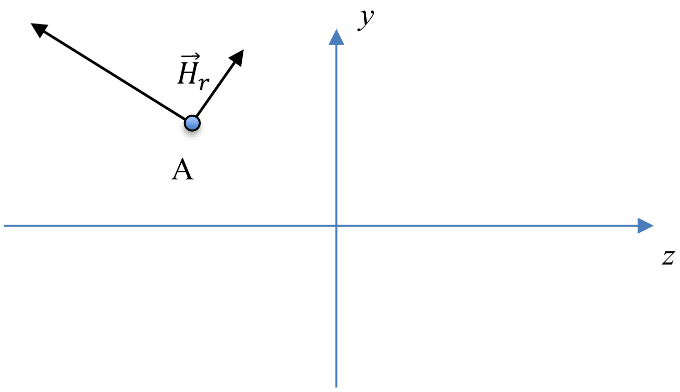
For point A, we obtain:

$$\vec{E}_r(A) = \vec{\mu}_x(-0.107 - \ j0.994)\ \text{V/m}$$

4) The absolute value of the magnetic field is

$$\left|\vec{H}_r(A)\right| = \frac{\left|\vec{E}_r(A)\right|}{\eta_0} = 2.65\frac{\text{mA}}{\text{m}}$$

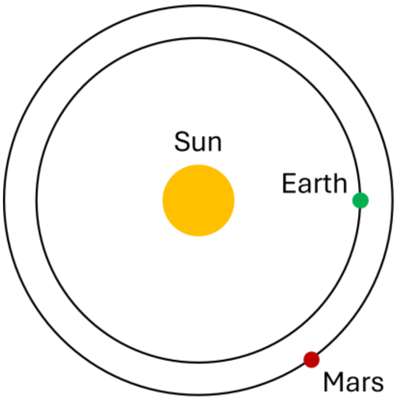
Given the direction of the reflected electric field (-$\vec{\mu}_x$), the direction of the associated magnetic field is the following



Problem 4

Referring to the figure below, consider a direct link from a ground station on the Earth (transmitter) to Perseverance rover on Mars (receiver). The link frequency is f = 20 GHz. The Earth's tropospheric attenuation along the slant path, A, is modelled using the following Complementary Cumulative Distribution Function:

$$P(A) = 100e^{-1.15A} \quad (P \text{ in \% and } A \text{ in dB})$$



Also, the atmosphere on Mars introduces a constant attenuation due to gaseous absorption of A_M = 6 dB and its average temperature is $T_M = -63$ °C. For this link:

1) What kind of polarization should be used?
2) Determine the maximum data rate achievable to guarantee a minimum SNR of 10 dB for 99.99% of the yearly time (QPSK modulation).

Use the following data:

- the gain of the transmitter antenna on the Earth is G_E = 70 dB
- the rover antennas is a Cassegrain reflector antenna with main reflector diameter D = 3 m and efficiency η = 0.7
- assume that both antennas are optimally pointed
- the transmitted power is P_T = 2 kW
- the distance between the ground station and the satellite is H = 110 million km
- the receiver LNA equivalent noise temperature is T_{LNA} = 109 K
- assume that there are no additional losses in the transmitter and receiver chains
- the Earth antenna has a perfect pointing accuracy, while the pointing accuracy of the rover antenna is P_C = 0.08°

Solution

1) Circular polarization is needed for geometric reasons.

2) To guarantee the target SNR for 99.99% of the time, the atmospheric attenuation needs to be lower than a given value. This value can be determined by setting $P(A)$ = 100%-99.99% = 0.01%. As a result, A = 8 dB = 0.1582. The SNR is given by:

$$SNR = \frac{P_T f_T L_{PTX} G_E \left(\frac{\lambda}{4\pi H}\right)^2 f_R L_{PRX} G_R A A_M}{k T_{sys} B}$$

where $\lambda = c/f$ = 0.015 m, f_S and f_R are set to 1 (antennas are optimally pointed). Also $L_{PTX} = 1$. As for L_{PRX} (pointing loss):

$$L_{PRX} = 12\left(\frac{DP_C}{70\lambda}\right)^2 = 0.63 \text{ dB} \rightarrow 0.8656$$

G_R is given by:

$$G_R = \frac{4\pi}{\lambda^2}\eta\left(\frac{D}{2}\right)^2 \pi = 27635 = 54.4 \text{ dB}$$

The noise power depends on the total system equivalent noise temperature, which, in this case, is given by:

$$T_{sys} = T_{LNA} + T_A = T_{LNA} + T_C A_M + T_M(1 - A_M) = 267\ \mathrm{K}$$

For the antenna equivalent noise temperature, any emission from the Earth is negligible due to the large distance; on the contrary, the effect of the attenuation due to the atmosphere on Mars on T_A is explicitly accounted for.

By inverting the *SNR*, the bandwidth B can be determined:

$B = 0.606$ MHz

Considering QPSK modulation, the data rate is $R = 2B = 1.212$ Mb/s.