Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini
February 9th, 2026

1	2	3	4

do not write above

SURNAME AND NAME ______________________________

ID NUMBER ______________________________

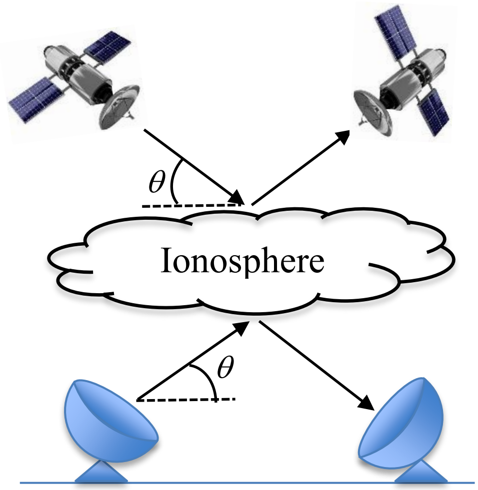
SIGNATURE ______________________________

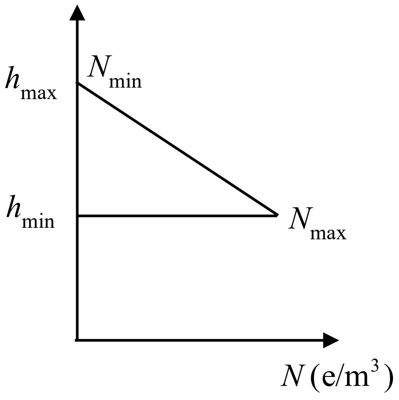
Problem 1

Referring to the figure below (left side), consider two bistatic radars, which aim at measuring the altitude of the lower layer (h_{min}) and of top layer (h_{max}) of the ionosphere – see the electron content profile depicted on the right side ($N_{\text{max}} = 4\times10^{12}$ e/m^3 and $N_{\text{min}} = 9\times10^{9}$ e/m^3). For both radars:

1. Determine the operational frequency to properly measure h_{max}, knowing that $\theta = 60°$.
2. Taking as reference $\theta = 60°$, assuming that the radars can change the operational frequency, should θ be increased or decreased to improve the accuracy in measuring h_{min}? Justify the answer.

Assumption: neglect tropospheric effects.





Solution

1) <u>Ground-based radar</u>

Let us calculate the frequency associated to N_{max}:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f_{\max}}\right)^2} \quad \Rightarrow \quad f_{\max} = \frac{9\sqrt{N_{\max}}}{\sin\theta} \approx 20.785\ \text{MHz}$$

The ground based radar cannot be used to measure $h_{\max}$: in fact, a wave associated to any frequency lower than or equal to $f_{\max}$ will be totally reflected at $h_{\min}$, while a wave associated to any frequency higher than $f_{\max}$ will completely cross the ionosphere.

Space-borne radar

Let us calculate the frequency associated to $N_{\min}$:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\min}}}{f_{\min}}\right)^2} \quad \Rightarrow \quad f_{\min} = \frac{9\sqrt{N_{\min}}}{\sin\theta} \approx 0.986\,\text{MHz}$$

The space-borne radar can be used to properly measured $h_{\max}$: in fact, a wave associated to any frequency lower than or equal to $f_{\min}$ will be totally reflected at $h_{\max}$.

2) Ground-based radar

Based on the discussion at point 1) above, the ground-based radar can measure $h_{\min}$ by using any frequency lower than or equal to $f_{\max}$ (total reflection at $h_{\min}$). Considering that any radar estimates the position of a target from the pulse travel time, neglecting tropospheric effects, the accuracy in measuring $h_{\min}$ does not depend on θ: in fact, the wave will not enter the ionosphere (no additional ionospheric delay).

Space-borne radar

The situation changes for the space-borne radar: as the wave will partially travel across the ionosphere, longer paths through it (i.e. smaller values of θ) imply a higher ionospheric delay and also a higher depolarization (for linearly polarized waves), i.e. increase uncertainty in measuring $h_{\min}$. Therefore, larger values of θ should be preferred to improve the radar accuracy.

Problem 2

Referring to the figure below, a plane EM wave at 30 GHz, propagating along the $-\vec{\mu}_z$ direction, crosses a layer of anisotropic particles (layer thickness h = 1 km), which is characterized by the following propagation constants:

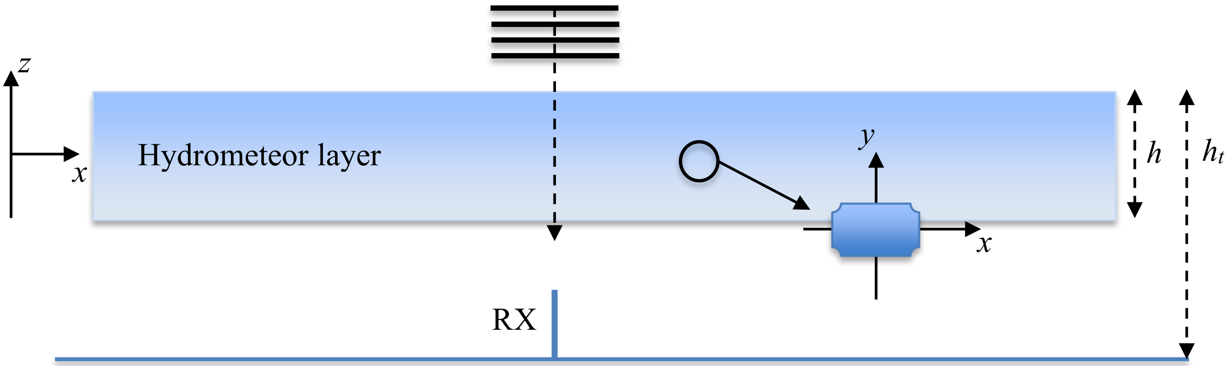
$\gamma_y = 0.6413 + j5658.7 = \alpha_y + j\beta_y$ 1/km

$\gamma_x = 2.9439 + j5658.7 = \alpha_x + j\beta_x$ 1/km

The electric field at the top of the layer is

$$\vec{E}(h_t) = \vec{\mu}_x + 0.1e^{j\frac{\pi}{2}}\vec{\mu}_y \text{ V/m}$$

Determine the best antenna at RX to maximize the received power.



Solution

1) First, it is necessary to determine the wave polarization at the top of the layer. In the space-time domain:

$$\vec{E}(h_t) = \cos(\omega t)\vec{\mu}_x + 0.1\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_y = E_x\vec{\mu}_x + E_y\vec{\mu}_y \text{ V/m}$$

For $\omega t = 0 \rightarrow E_x = 1$ V/m e $E_y = 0$ V/m

For $\omega t = \frac{\pi}{2} \rightarrow E_x = 0$ V/m e $E_y = -0.1$ V/m

Considering the electric field rotation direction and the differential amplitude for E_x and E_y, the wave has a RHEP.

2) After crossing the anisotropic layer, the wave is depolarized. As the phase constants along x and y are the same, the depolarization is totally ascribable to the differential attenuation.

The absolute value of the y component after the layer is:

$$|E_y(h_t - h)| = |E_y(h_t)|e^{-\alpha_y h} = 0.1e^{-\alpha_y h} = 0.0527 \text{ V/m}$$

The absolute value of the x component after the layer is:

$$|E_x(h_t - h)| = |E_x(h_t)|e^{-\alpha_x h} = e^{-\alpha_x h} = 0.0527 \text{ V/m}$$

As the amplitude of the two components is the same, the wave reaching RX has RHCP: the best antenna must be designed to receive that kind of waves.

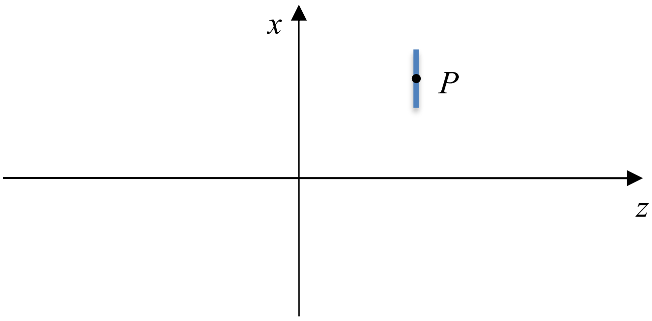
Problem 3

A uniform plane wave propagates in distilled water (ε_{r1} = 81 and μ_{r1} = 1) and impinges on the surface of a medium characterized by conductivity σ = 0.6 S/m, ε_{r2} = 9 and μ_{r2} = 1. The incident electric field is

$$\vec{E}_i = (\vec{\mu}_x + j\vec{\mu}_y)e^{-j0.0377z}\ \text{V/m}$$

Determine:

1. The wavelength in the second medium.
2. The power absorbed by the dipole in P(1,1,1 m), lying on the xz plane. To this end, consider that following data for the dipole: directivity D = 6 dB and efficiency η = 0.3.



Solution

1) First, it is necessary to calculate the frequency. Based on β_1 = 0.0377 rad/m:

$$\boldsymbol{f} = \frac{\boldsymbol{\beta_1 c}}{\boldsymbol{2\pi\sqrt{\varepsilon_{r1}}}} = \boldsymbol{200}\ \text{kHz}$$

The wavelength in the second medium depends on the propagation constant. The loss tangent for this medium is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_2} \approx 6\times 10^3$$

Thus, the good conductor approximations can be used:

$$\gamma_2 = \alpha_2 + j\beta_2 = \sqrt{\frac{\omega\mu_2\sigma}{2}} + j\sqrt{\frac{\omega\mu_2\sigma}{2}} = 0.688(1+j)\ \ 1/\text{m}$$

The wavelength in the second medium is:

$$\lambda_2 = \frac{2\pi}{\beta_2} = 9.13\ \text{m}$$

2) The electric field reaching point P depends on the reflection coefficient, which, in turn, depends on the intrinsic impedance of the medium. Using the same kind of approximation:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = 41.9\ \Omega$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma}} = (1+j)1.15\ \Omega$$

$$\Gamma = \frac{\boldsymbol{\eta_2 - \eta_1}}{\boldsymbol{\eta_2 + \eta_1}} = -0.945 + j0.052$$

The gain of the dipole is:

$G = D\eta = 1.19 = 0.77$ dB

The effective area of the dipole is:

$$A_e = G\frac{\lambda^2}{4\pi} = 7.92\ \mathrm{m}^2$$

The dipole can receive only the *x*-component of the wave. The power absorbed is:
$P = SA_e = 0.39\ \mathrm{W}$
where:
$$S = \frac{1}{2}\frac{|E_2^x(z=0)|^2}{|\eta_2|}e^{-2\alpha_2 z_P}\cos(\measuredangle\eta_2) = 0.0493\ \frac{\mathrm{W}}{\mathrm{m}^2}$$
and
$|E_2^x(z=0)| = |E_i^x(z=0)||\Gamma| = 0.9467\ \mathrm{V/m}$

Problem 4

Consider the uplink at 20 GHz from a gateway to a broadcast geostationary satellite. The link has $\theta = 40°$ elevation angle and is affected by the tropospheric attenuation A_{dB}, whose statistics are modelled using the following CCDF (probability expressed in percentage values, A_{dB} expressed in dB and representing the value along the zenith):

$$P(A_{dB}) = 100e^{-2.3A_{dB}} \text{ (\%)}$$

The satellite antenna points at the gateway, positioned in an area with brightness temperature $T_G = 250$ K. Calculate the data rate R achieved for 99.999% of the time, associated to the minimum SNR = 14 dB (consider 2PSK modulation). Use the following data:

- the directivity of the ground antenna is $D_R = 34$ dB and its efficiency is $\eta_R = 0.6$
- the directivity of the satellite antenna is $D_S = 24$ dB and its efficiency is $\eta_S = 0.7$
- assume that both antennas are optimally pointed
- the power transmitted by the gateway is $P_T = 1$ kW
- the distance between the ground station and the satellite is $H = 40000$ km
- the receiver LNA equivalent noise temperature is $T_{LNA} = 163$ K
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- mean radiating temperature $T_{mr} = 298$ K.

Solution

1) The wavelength is $\lambda = c/f = 0.01$ m. The gains of the two antennas are:

$G_R = \eta_R D_R = 1507.1$ (converted to linear scale)

$G_S = \eta_S D_S = 175.8$ (converted to linear scale)

The zenithal atmospheric attenuation is obtained by inverting the CCDF after setting $P = 0.001\%$:

$A_{dB} = 5$ dB

The slant path attenuation (in linear scale) is:

$A = 10^{-A_{dB}/(10\sin(\theta))} = 0.196$

The received power is:

$$P_R = P_T G_S f_S \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A = 4.624 \times 10^{-14} \text{ W}$$

where $f_S = f_R = 1$ (antennas optimally pointed). The noise power depends on the total system equivalent noise temperature:

$T_{sys} \approx T_A + T_{LNA}$

The equivalent antenna noise (sky noise) is given by:

$T_A = T_G A + T_{mr}(1-A) = 284.6$ K

Thus $T_{sys} \approx 444.6$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K).

Imposing SNR = 14 dB and inverting the last equation, the maximum bandwidth can be determined.

$$B = \frac{P_R}{kT_{sys}SNR} = 250 \text{ kHz}$$

As a result, *R* = 250 kbit/s.