Electromagnetics and Signal Processing for Spaceborne Applications – EM part
September 11th, 2024

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SURNAME AND NAME ____________________

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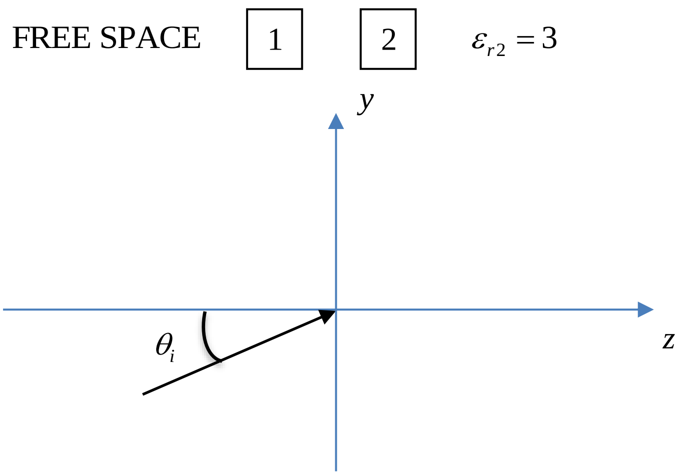
Problem 1

A plane sinusoidal EM wave propagates from free space into a medium with electric permittivity $\varepsilon_{r2} = 3$ ($\mu_{r2} = 1$), with incidence angle θ_i. The expression for the electric field in the first medium is:

$$\vec{E}_i(z,y) = \left[\left(\frac{1}{2}\vec{\mu}_y - \frac{\sqrt{3}}{2}\vec{\mu}_z\right) + j2\vec{\mu}_x\right] e^{-j104.72z} e^{-j181.38y} \text{ V/m}$$

For this wave:

1) Determine θ_i.
2) Determine the frequency.
3) Determine the polarization of the incident and reflected EM wave.
4) Write the expression of the TE component of the refracted field.



Solution:

1) The incident angle can be obtained from the TM component of the field. For example:

$$\left|\vec{E}_{TM}\right| = \sqrt{\left(E_{TM}^{y}\right)^2 + \left(E_{TM}^{z}\right)^2} = 1$$

$$E_{TM}^{y} = \cos\theta_i \left|\vec{E}_{TM}\right| = \frac{1}{2} \rightarrow \theta_i = 60°$$

2) The frequency of the incident EM wave can be derived from the phase constant. For example:

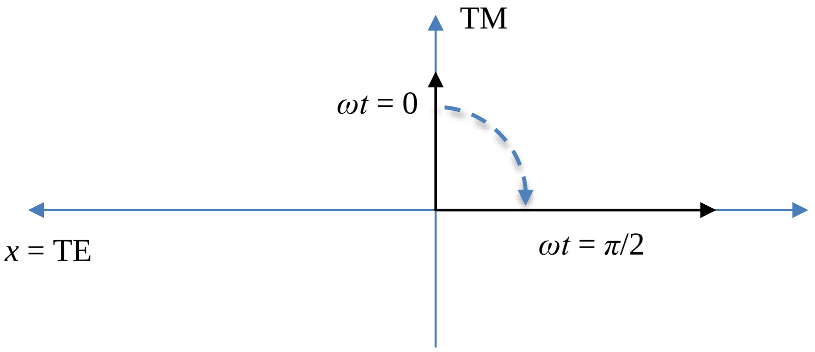
$$\beta_z = \beta\cos\theta_i = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}}\cos\theta_i \quad \rightarrow \quad f = \frac{c\beta_z}{2\pi\sqrt{\varepsilon_{r1}}\cos\theta_i} = 10 \text{ GHz}$$

3) The polarization of the incident wave is RHEP (right-hand elliptical polarization) because the two TE and TM components have different amplitudes and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re}\left\{\left[\left(\frac{1}{2}\vec{\mu}_y - \frac{\sqrt{3}}{2}\vec{\mu}_z\right) + j2\vec{\mu}_x\right]e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_{TM} + 2\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} \text{ V/m}$$

Thus, making reference to the figure below that shows the reference system as seen from behind the wave, for $t = 0 \rightarrow \left.\vec{E}(0,0)\right|_{\omega t=0} = \vec{\mu}_{TM}$ V/m

Afterwards, for $\omega t = \pi/2 \rightarrow \left.\vec{E}(0,0)\right|_{\omega t=\pi/2} = -2\vec{\mu}_{TE}$ V/m



The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. As there is a TM component, it is worth checking the Brewster angle θ_B:

$$\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 60°$$

As $\theta_i = \theta_B$, the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

4) First, we need to calculate the reflection coefficient for TE waves, which, in turn, requires calculating the refraction angle as:

$$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t \rightarrow \theta_t = 30.54°$$

$$\eta_1^{TE} = \frac{\eta_0}{\cos\theta_i \sqrt{\varepsilon_{r1}}} = 754\ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t \sqrt{\varepsilon_{r2}}} = 252.7\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.497$$

The full expression for the refracted TE component is given by:

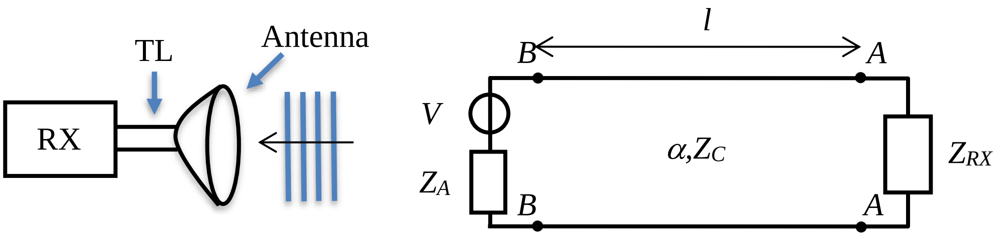
$$\vec{E}_t^{TE}(z,y) = \vec{E}_t^{TE}(0,0)(1+\Gamma^{TE})e^{-j\beta_2 cos\theta_t z}e^{-j\beta_2 sin\theta_t y} = -j0.994\vec{\mu}_x e^{-j312.4z}e^{-j184.3y} \quad \text{V/m}$$

where $\beta_2 = 362.76$ rad/m.

Problem 2

A plane EM wave carries a power density of $S = 10$ mW/m^2 in front of an antenna. The power received by such antenna is conveyed into the receiver RX via a lossy coaxial cable ($\alpha_{dB} = 30$ dB/km), with intrinsic impedance $Z_C = 75\ \Omega$. The antenna, whose gain is $G = 10$ dB, acts as an equivalent generator with voltage V and internal impedance $Z_A = 75\ \Omega$; the RX, which acts as a load, has impedance $Z_{RX} = 100+j50\ \Omega$. The frequency is $f = 600$ MHz. The line length is $l = 40$ m.

1. Calculate the power received by the antenna.
2. Calculate the power absorbed by RX, P_{RX}.
3. With the aim of maximizing P_{RX}, is it better to have a capacitor-like or an inductor-like reactive part in Z_{RX}?



Solution

1) The power received by the antenna is the power available to the generator. The wavelength is $\lambda = c/f = 0.5$ m, and the antenna effective area is:

$$A_E = G\frac{\lambda^2}{4\pi} = 0.1989 \text{ m}$$

The received power is:

$$P_{AV} = S\,A_E = 2 \text{ mW}$$

2) First, let us calculate the reflection coefficient at section AA:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0.2075 + j0.2264$$

The solution is simplified by the partial match at section BB, so the power absorbed by the load can be simply calculated as (only one reflection at the load section):

$$P_{RX} = P_{AV}e^{-2\alpha l}(1 - |\Gamma_L|^2) = 1.4 \text{ mW}$$

where $\alpha = 3.5\times10^{-3}$ Np/m.

3) It is easy to verify that any reactive part in Z_{RX} will generate an increase in Γ_L: the same absolute value of the reactive part will generate the same effect. To maximize the received power, such reactive part needs to be nullified.

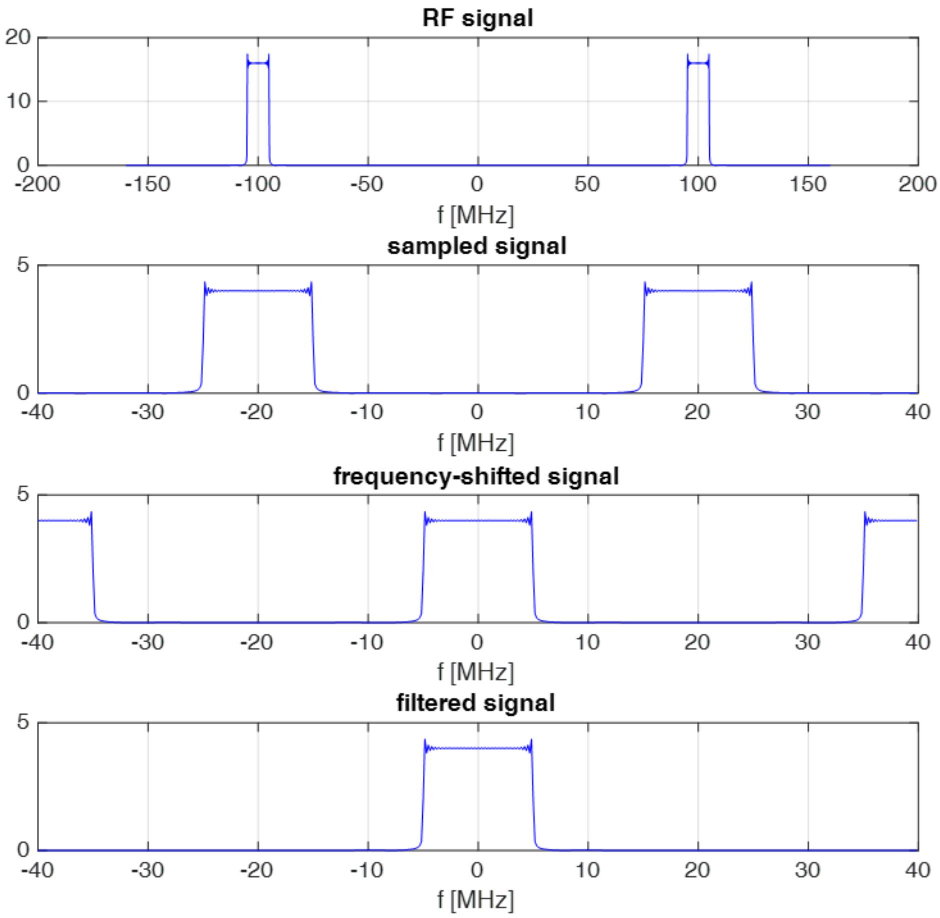
Problem 3

Let $s(t)$ be a RF signal: $s(t) = g(t)cos(2\pi f_0 t)$, where $g(t)$ is a short pulse with a (two-sided) bandwidth of 10 MHz and the carrier frequency is $f_0 = 100$ MHz.

1. Write the expressions of the Fourier Transform of $s(t)$ and draw a graph of the magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).
2. A discrete-time signal s_n is obtained by sampling $s(t)$ with sampling frequency $f_s = 80\ MHz$. Write the expressions of the Fourier Transform of s_n and draw a graph of the magnitude.
3. Propose a procedure for the extraction of the complex envelope (I & Q components) of $s(t)$ based on the sampled signal s_n.

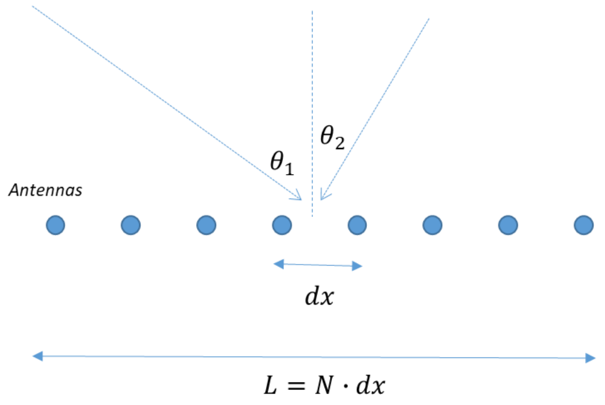
Solution

1) $S(f) = \frac{1}{2}G(f - f_0) + \frac{1}{2}G(f + f_0)$, as resulting from the convolution between the Fourier Transform of g(t) (=> G(f)) and $cos(2\pi f_0 t)$ (=> $\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$). See the figure below, top row.
2) Replicating the signal with a period of $f_s = 80\ MHz$ yields $S(f) = \frac{f_s}{2}G(f - f_1) + \frac{f_s}{2}G(f + f_1)$, with $f_1 = 20\ MHz$. See the second row in the figure below.
3) As the two replicas do not overlap, it is possible to correctly extract the complex envelope simply by shifting the signal by 20 MHz and applying a low pass filter. In the time domain this can be done as follows: $z_n = h_n * \left(s_n e^{-j2\pi f_1 nT}\right)$, with $T = \frac{1}{f_s}$. See third and fourth rows.


RF signal
f [MHz]
sampled signal
f [MHz]
frequency-shifted signal
f [MHz]
filtered signal
f [MHz]

Problem 4

Multiple electromagnetic waves radiated from distant sources at the frequency $f_0 = 435$ MHz impinge simultaneously on an antenna array as represented in the figure below:



Each antenna is equipped with its own circuity to generate the complex envelope of the received signal.

1. Describe a procedure to measure the directions of arrival $\theta_1, \theta_2, \ldots$ based on the N signals (complex envelope) output by the array.
2. Discuss how the set antenna spacing dx
3. After fixing antenna spacing dx, determine the number of antennas required to obtain an angular resolution $\Delta\theta = 6°$ (you can assume an incident direction of 0° as a reference in the calculation of resolution)
4. How would you change your answers at points 3 and 4 if you had knowledge that the direction of arrival of all impinging waves is limited in the interval (-30°,+30°)?
5. The antenna array at point 4 is now used to transmit a wave by letting all antennas radiate simultaneously. Calculate the angular width of the transmitted beam and the angular position of secondary beams.

Solution

1) A signal coming from direction θ determines a phase variation across antenna elements equal to $\varphi(x) = 2\pi \frac{sin(\theta)}{\lambda} x$ with. Running a Fourier Transform along the array produces a peak at spatial frequency $f_x = \frac{sin(\theta)}{\lambda}$, allowing for the detection of the direction if arrival of individual signals.
2) With no prior information on the range of angular direction of impinging signals one must choose $dx \leq \frac{\lambda}{2}$, to guarantee no ambiguous peak.
3) Angular resolution is obtained as $\Delta\theta = \frac{\lambda}{Lx} = \frac{\lambda}{Ndx}$, with $Lx = Ndx$ array length. Hence $N = \frac{\lambda}{\Delta\theta dx} = \frac{2}{\Delta\theta}$=19.1 => 20 antennas
4) The prior knowledge about the angular interval converts into the minimum and maximum possible spatial frequencies to be $\pm\frac{sin(\theta)}{\lambda}$=$\pm\frac{1}{2\lambda}$. The total spatial bandwidth is therefore $Bx = \frac{1}{\lambda}$, and by the sampling theorem this requires $dx \leq \frac{1}{Bx} = \lambda$. The number of antennas required to have a resolution of 6° is therefore $N = \frac{\lambda}{\Delta\theta dx} = \frac{1}{\Delta\theta}$= 9.5 => 10 antennas

5) When used in transmission, the array will produce a main beam in the direction $\theta = 0°$ (orthogonal to the array) and beamwidth of 6°. The main beam corresponds to spatial frequency fx=0. Ambiguous beams will be produced at spatial frequencies $\frac{k}{dx} = \frac{k}{\lambda}$, with k any integer (except 0, where we have the main beam). Those spatial frequencies are converted to angles as $\theta = asin(f_x \lambda)$, yielding $\theta = [-90° \;\; 90°]$.