Electromagnetics and Signal Processing for Spaceborne Applications
February 16th, 2024

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SURNAME AND NAME ______________________________

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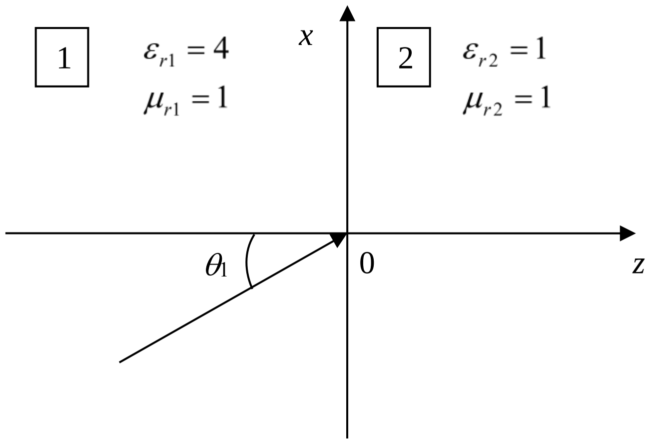
Problem 1

Consider the plane sinusoidal wave below f = 1 GHz, whose incident electric field is:

$$\vec{E}(0,0,0) = j\vec{\mu}_y + 0.819\vec{\mu}_x - 0.574\vec{\mu}_z \text{ (V/m)}$$



Calculate:

1) The incident angle θ_1.
2) The polarization of the incident wave.
3) The reflection coefficient of the TE component.
4) The power density in the second medium in the z direction.

Solution

1) The wave consists of a TE component ($j\vec{\mu}_y$) and a TM one ($0.819\vec{\mu}_x - 0.574\vec{\mu}_z$). The absolute value of the TE component is clearly 1 V/m, that of the TM component is:

$$\left|\vec{E}_{TM}\right| = \sqrt{0.819^2 + 0.574^2} = 1 \text{ V/m}$$

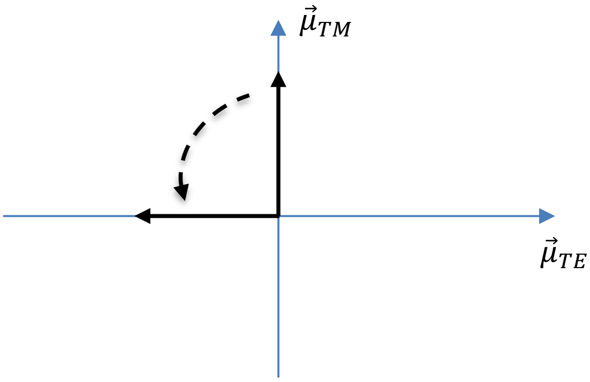
Considering the x component:

$$E_x = \left|\vec{E}_{TM}\right| \cos(\theta_1) = \cos(\theta_1) \;\rightarrow\; \theta_1 = 35°$$

2) Let us consider the temporal evolution of the total field:

$\vec{E}(0,0,0,t) = \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} + \cos(\omega t)\vec{\mu}_{TM}$ V/m

Looking from the back of the wave:



Setting $t = 0$ → $\vec{E}(0,0,0,t) = \vec{\mu}_{TM}$ V/m

Setting $\omega t = \pi/2$ → $\vec{E}(0,0,0,t) = -\vec{\mu}_{TE}$ V/m

As a result, the wave has LHCP.

3) Let us calculate the refraction angle:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \Rightarrow \quad \sin\theta_2 = 1.1472 \quad \Rightarrow \quad$ evanescent wave

The reflection coefficient of the TE component is calculated as:

$$\eta_{TE}^1 = \frac{\eta_0}{\sqrt{4}} \frac{1}{\cos\theta_1} = 230.1\ \Omega$$

$$\eta_{TE}^2 = \frac{\eta_0}{\cos\theta_2} = \frac{\eta_0}{\sqrt{1-(\sin\theta_2)^2}} = \frac{\eta_0}{\pm j0.5622} \Rightarrow \eta_{TE}^2 = j670.6\,\Omega$$

The sign of the latter intrinsic impedance is linked to the need of obtaining an exponentially decreasing trend of the electric field in the second medium (negative solution for the square root at the denominator).

$$\Gamma = \frac{\eta_{TE}^2 - \eta_{TE}^1}{\eta_{TE}^2 + \eta_{TE}^1} = 0.789 + j0.614$$

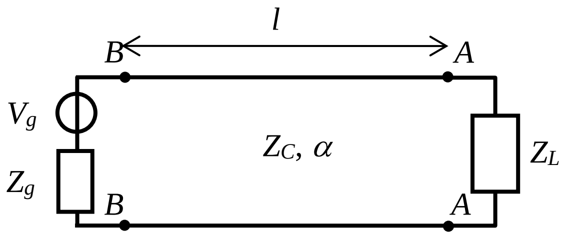
4) Evanescent wave → total reflection → no power density in the second medium

Problem 2

A source with voltage V_g = 100 V and internal impedance Z_g = 100 Ω is connected to a load Z_L = 200 Ω by a transmission line with characteristic impedance Z_C = 50 Ω and attenuation constant α = 8 dB/m. The line length is l = 0.375 m and the frequency is f = 200 MHz.

Calculate:

1) The power absorbed by the load
2) The trend of V_A in time



Solution

1) The wavelength is:

$$\lambda = \frac{c}{f} = 1.5 \text{ m}$$

The reflection coefficient at section AA is:

$$\Gamma_A = \frac{Z_L - Z_C}{Z_L + Z_C} = 0.6$$

The reflection coefficient at section BB is:

$$\Gamma_B = \Gamma_A e^{-j2\beta l} e^{-2\alpha l} = -0.3007$$

Therefore, the input impedance is:

$$\mathrm{Z}_B = \mathrm{Z}_C \frac{1 + \Gamma_B}{1 - \Gamma_B} = 26.88\ \Omega$$

The reflection coefficient for the source is:

$$\Gamma_g = \frac{Z_B - Z_g}{Z_B + Z_g} = -0.5763$$

Therefore, the power crossing section BB is:

$$\mathrm{P}_B = \mathrm{P}_{AV}\left(1 - \left|\Gamma_g\right|^2\right) = 8.3 \text{ W}$$

This power is partially absorbed by the line and partially by the load.

The voltage at the beginning of the line is:

$$\mathrm{V}_B = \mathrm{V}_g \frac{Z_B}{Z_B + Z_g} = 21.2 \text{ V}$$

The progressive wave on the right side of section BB is:

$$V_B^+ = \frac{V_B}{1 + \Gamma_B} = 30.3 \text{ V}$$

The voltage at section AA is:

$$V_A = V_B^+ e^{-j\beta l} e^{-\alpha l}(1 + \Gamma_A) = -j34.3 \text{ V}$$

The power absorbed by the load is:

$$\mathrm{P}_L = \frac{1}{2}\frac{|V_A|^2}{Z_L} = 0.0858 \text{ W}$$

2) The trend of V_A in time is given by:

$$v_A(t) = 34.3\cos(2\pi f t - \pi/2)\ \mathrm{V}$$

Problem 3

An automotive Radar transmits a signal with a total bandwidth of *B*=3 GHz centered about a carrier frequency f_0 = 25 GHz. The signal is reflected by a single target at the distance of 50 m. The transmitted signal is modeled as $s_{tx}(t) = g(t)cos(2\pi f_0 t)$, with $g(t)$ a real-valued signal.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. A discrete-time signal s_n is obtained by sampling the real-valued received signal $s_{rx}(t)$ with sampling frequency $f_s = 20\ GHz$. Write the expressions of the Fourier Transform of s_n and draw a graph of the magnitude.
3. Describe a procedure to recover the complex envelope of the received signal from the sequence s_n.

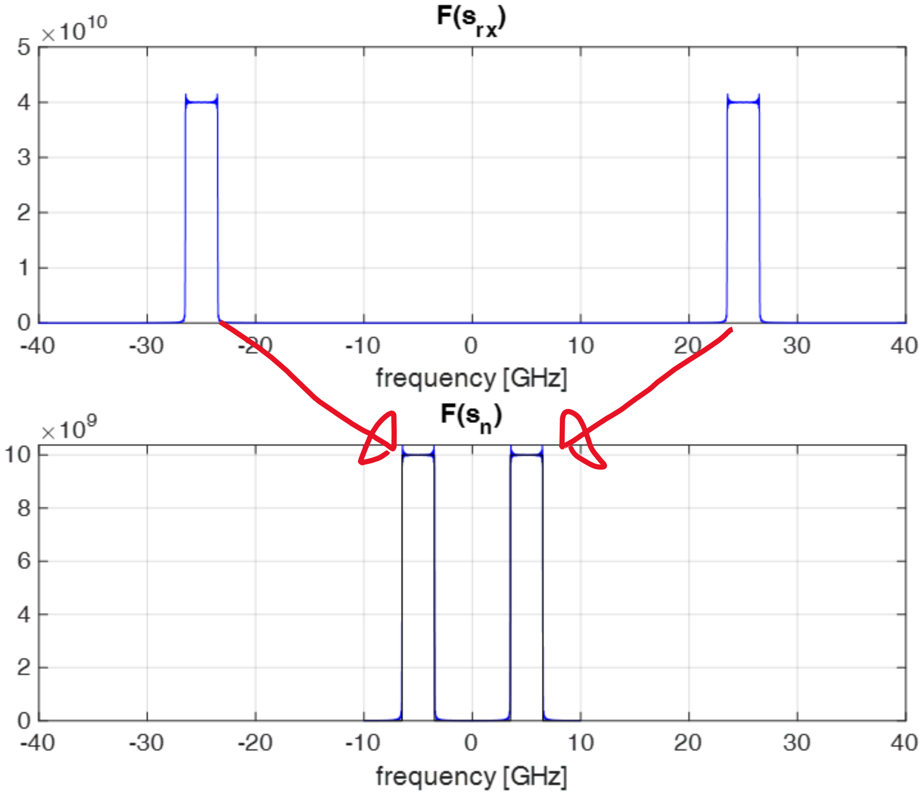
Solution

Complex representation

$$s_{tx}(t) = g(t)exp(j2\pi f_0 t)$$
$$s_{rx}(t) = g(t-\tau)exp\big(j2\pi f_0(t-\tau)\big)$$
$$s_{rx,compl\ env}(t) = g(t-\tau)exp(-j2\pi f_0 \tau)$$

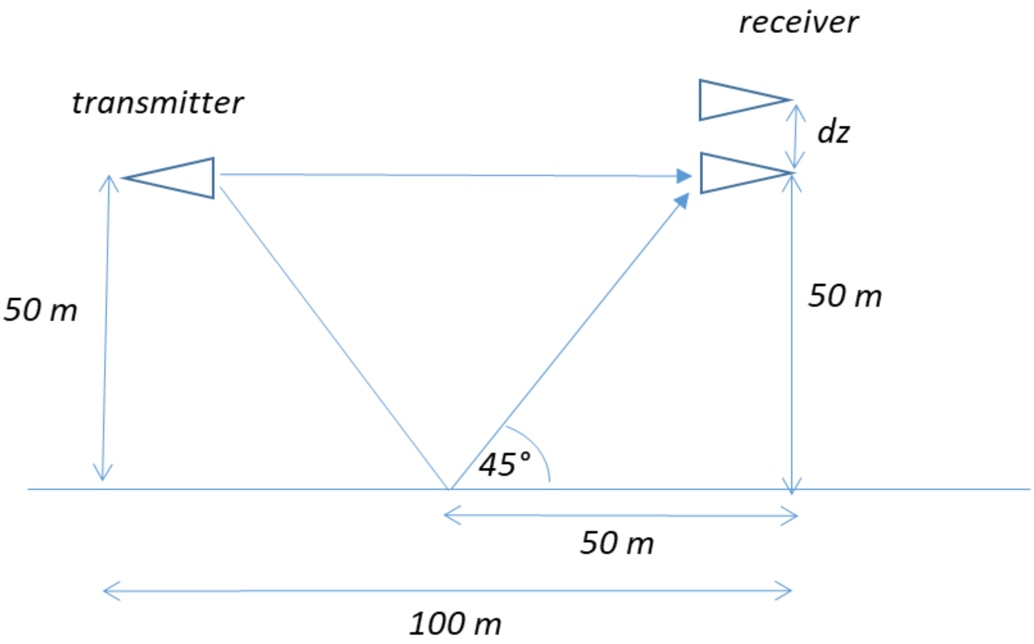
Sampling and complex envelope

The Fourier transform of the receive signal before and after being sampled are reported in the figure below. The two components at +-25 GHz are mapped to +- 5 GHz. The two components are well separated, so the complex envelope can be retrieved simply by filtering out the component at – 5 GHz and shifting the other to base-band.



Problem 4

A receiver with two antennas receives a signal from a transmitter plus its reflection onto the ground, as depicted in the figure below. The carrier frequency is $f_0 = 5$ GHz. The antenna spacing dz is set equal to half a wavelength. The complex transmitted signal can be modeled as $s_{tx}(t) = g(t)exp(j2\pi f_0 t)$, with $g(t)$ a signal with a bandwidth $B = 10$ MHz.



1. Write the expression of the complex envelope of the signals received at the two antennas.
2. Propose a procedure to restore the direct signal (that is, to cancel out the reflected signal) using the two signals from the two receiving antennas.
3. Optimize the value of the antenna spacing dz to maximize the amplitude of the restored direct signal (as obtained according to the procedure at point 2).
4. Discuss whether it would be possible (or advisable) to restore the direct signal using just one receiving antenna.

Solution

Direct signal at the two receivers (complex envelope):

$$s_{1,dir}(t) = g(t - \tau_{dir})exp(-j2\pi f_0 \tau_{dir})$$
$$s_{2,dir}(t) = s_{1,dir}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{dir})dz\right)$$

Where $\psi_{dir} = 0$

Reflected signal at the two receivers (complex envelope):

$$s_{1,ref}(t) = g(t - \tau_{ref})exp(-j2\pi f_0 \tau_{ref})$$
$$s_{2,ref}(t) = s_{1,ref}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right)$$

Where $\psi_{dir} = \frac{\pi}{4}$.

Total signals:

$$s_1(t) = s_{1,dir}(t) + s_{1,ref}(t)$$
$$s_2(t) = s_{2,dir}(t) + s_{2,ref}(t)$$

Cancellation of the reflected signal:

$$s(t) = s_1(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - s_2(t)$$
$$= s_{1,dir}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - s_{2,dir}(t) + s_{1,ref}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - s_{2,ref}$$
$$= s_{1,dir}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - s_{1,dir}(t) +$$
$$s_{1,ref}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - s_{1,ref}(t)exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right)$$
$$= s_{1,dir}(t)\left[exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) - 1\right]$$

The optimal value of dz is the one that maximizes the amplitude of the restored signal, hence $exp\left(-j\frac{2\pi}{\lambda}sin(\psi_{ref})dz\right) = -1 \Rightarrow \frac{2\pi}{\lambda}sin(\psi_{ref})dz = \pi \Rightarrow dz = \frac{\lambda}{2sin(\psi_{ref})}$

Using a single antenna, the signal can be characterized in the frequency domain as:

$$S_1(f) = G(f)\left[exp(-j2\pi(f+f_0)\tau_{dir}) + exp\left(-j2\pi(f+f_0)\tau_{ref}\right)\right]$$
$$= G(f)exp(-j2\pi(f+f_0)\tau_{dir})\left[1 + exp\left(-j2\pi(f+f_0)\left(\tau_{ref} - \tau_{dir}\right)\right)\right]$$
$$= S_{1,dir}(f)H(f)$$

With $H(f) = 1 + exp\left(-j2\pi(f+f_0)\left(\tau_{ref} - \tau_{dir}\right)\right)$.

The filter can be inverted as long as its magnitude presents no zeroes. Otherwise, prefect inversion is not possible. Accordingly, the procedure is not as robust as in the case of two antennas.