Electromagnetics and Signal Processing for Spaceborne Applications
July 18th, 2022

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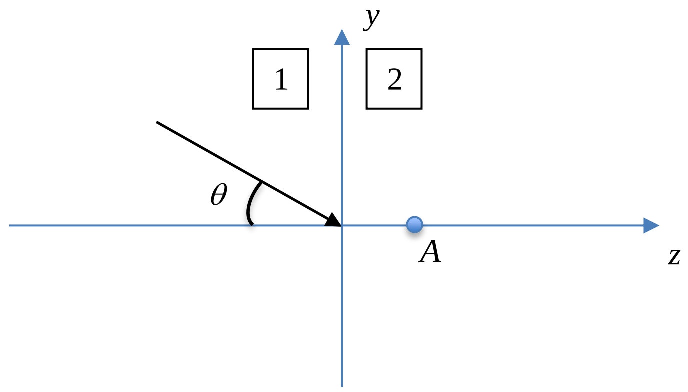
Problem 1

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 4 (assume μ_r = 1 for both media) into free space. The expression of the incident electric field is:

$$\vec{E}_i(z,y) = -\vec{\mu}_x\, e^{-j\frac{\sqrt{2}}{2}\beta_1 z} e^{j\frac{\sqrt{2}}{2}\beta_1 y} \text{ V/m}$$

1) What is the polarization of the incident field (specify the details of the polarization)?
2) Determine the value of the electric field in A(z = 1 cm, y = 0 m).



Solution

1) The wave polarization is linear, specifically a TE component (along -x).

2) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta_1 \sin(\theta) = \beta_1\sqrt{2}/2 \rightarrow \sin(\theta) = \sqrt{2}/2 \rightarrow \theta = 45°$

To determine the transmitted wave, it is first necessary to calculate the refraction angle, which is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = \sin^{-1}(\sqrt{2}) \approx \sin^{-1}(1.4142)$$

This is the sign of an evanescent wave: this wave is totally reflected, but the electric field will penetrate in the second medium. The expression of the transmitted field will be:
$\vec{E}_t(z,y) = -\vec{\mu}_x(1+\Gamma_{TE})\, e^{-j\beta_{2z}z}e^{j\beta_{2y}y}$ V/m

As in A, y = 0 m → $\vec{E}_t(z, y = 0\text{ m}) = -\vec{\mu}_x\Gamma_{TE}\, e^{-j\beta_{2z}z}$ V/m

Let us calculate β_{2z}:

$$\beta_{2z} = \beta_2\cos(\theta_2) = \beta_2\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{1-\sqrt{2}^2} = \beta_0\sqrt{-1}$$
$$= \pm j\beta_0 = -\,j\beta_0$$

The negative sign is chosen to obtain a physical solution:

$e^{-j\beta_{2z}z} = e^{-j(-\,j\beta_0)z} = e^{-\beta_0 z}$

The reflection coefficient can be calculated as:

$$\eta_1 = \frac{\eta_0}{\cos(\theta)\sqrt{\varepsilon_{r1}}} = 266.6\ \Omega$$

$$\eta_2 = \frac{\eta_0}{\cos(\theta_2)\sqrt{\varepsilon_{r2}}} = \frac{\eta_0}{-\,j} = j377\ \Omega$$

The choice of the negative sign in η_2 is consistent with the one in β_{2z}.

$$\Gamma = \frac{\eta_2-\eta_2}{\eta_2+\eta_1} = 0.334 + j0.943$$

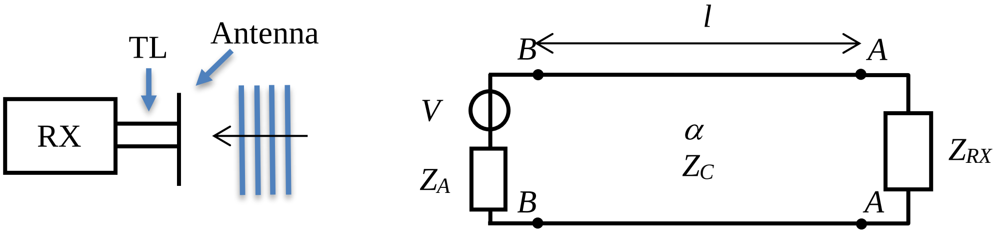
Therefore:
$\vec{E}_t(A) = \vec{E}_t(z = 0.01\text{ m}, y = 0\text{ m}) = -\vec{\mu}_x(1.334 + j0.943)e^{-0.01\,\beta_0} = -\vec{\mu}_x(0.202\ +\ j0.143)$ V/m

with $\beta_0 = 188.62$ rad/m.

Problem 2

A plane wave (linear vertical polarization), propagating in free space and whose absolute value of the electric field is $|\vec{E}| = 1\ \text{V/m}$, is received by a linear vertical antenna (same direction as the wave polarization, gain G = 6 dB). The power available at the generator section, assumed to be equal to the power received by the antenna, is conveyed into the receiver RX via a lossy coaxial cable (attenuation constant α = 30 dB/km), with intrinsic impedance Z_C = 50 Ω. The antenna acts as an equivalent generator with internal impedance Z_A = 100 Ω. The RX, which acts as a load, is matched to the transmission line. The frequency is f = 600 MHz. The line length is l = 5.2 m.

1. Determine the power absorbed by RX, P_{RX}.
2. Calculate the absolute value of the voltage at section BB.



Solution

1) The wavelength is $\lambda = c/f$ = 0.5 m. The available power is calculated from the power density of the incident wave and the antenna effective area. The former is:

$$S = \frac{1}{2}\frac{|E|^2}{\eta_0} = 1.3\ \text{mW/m}^2$$

The latter is:

$$A_{RX} = \frac{\lambda^2}{4\pi}G = 0.0792\ \text{m}^2$$

Therefore, the available power is:

$$P_{AV} = SA_{RX} = 1.05 \times 10^{-4}\ \text{W}$$

Having a look at the impedances, there is match at the load section, but not at the generator one. In this case, there will be only one reflection in the circuit, specifically at section BB. Therefore, the power absorbed by the load is:

$$P_L = P_{AV}\left(1 - |\Gamma_g|\right)e^{-2\alpha l}$$

The reflection coefficient at section AA is given by:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0$$

Therefore, at section BB:

$$\Gamma_{BB} = \Gamma_L e^{-2\alpha l} e^{-2j\beta l} = 0 \rightarrow Z_{BB} = Z_C$$

Finally:

$$\Gamma_g = \frac{Z_{BB} - Z_A}{Z_{BB} + Z_A} = \frac{Z_C - Z_A}{Z_C + Z_A} = -0.333$$

The attenuation coefficient is converted in Np/m as:

$$\alpha_l = \frac{\alpha}{8.686 \times 1000} = 0.0035\ \text{Np/m}$$

Therefore:

$$P_L = P_{AV}\left(1 - |\Gamma_g|\right)e^{-2\alpha l} = 9 \times 10^{-5}\ \text{W}$$

2) First, we need to calculate the voltage of the equivalent generator as:

$$|V| = \sqrt{P_{AV} 8 Z_A} = 0.29 \text{ V}$$

Recalling that Z_{BB} = Z_C, the voltage the can be partitioned as follows:

$$|V_{BB}| = |V| \frac{Z_{BB}}{Z_A + Z_{BB}} = |V| \frac{Z_C}{Z_A + Z_C} = 0.0967 \text{ V}$$

Problem 3

An automotive Radar transmits an electromagnetic pulse with a total bandwidth of B = 1 GHz centered about a carrier frequency f_0 = 77 GHz. The signal is reflected by a pedestrian passing by at a distance of 15 meters from the Radar, and the reflected echo is then received by the Radar after a time τ.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Assume that g(t) is a chirp signal, $g(t) = rect\left(\frac{t}{T}\right) exp(j\pi K t^2)$. Determine the value of the chirp rate K assuming a total duration T=100 microseconds.
3. Describe a procedure to measure the distance of the pedestrian from the Radar.
4. Evaluate the temporal and spatial resolution of such measurement.
5. Comment on why you can achieve a temporal resolution much better than pulse length.

Solution

1)

$$s_{tx}(t) = g(t) \cdot e^{j2\pi f_0 t}$$

$$s_{rx}(t) = s_{Tx}(t-\tau) = g(t-\tau) \cdot e^{j2\pi f_0 (t-\tau)}$$

$$z(t) = s_{rx}(t) \cdot e^{-j2\pi f_0 t} = g(t-\tau) \cdot e^{-j2\pi f_0 \tau}$$

2) For chirp B = KT, hence K = B/T = $\frac{10^9}{10^{-4}} = 10^{13}$ Hz/s

3) cross-correlating the complex envelope with the transmitted pulse g yields a sharp peak centred about τ :

$$z(t) * conj\big(g(-t)\big) = T sinc\big((t-\tau)B\big) \cdot e^{-j2\pi f_0 \tau}$$

the delay can therefore be measured by the peak position. Once the delay is known, it is converted into distance as $R = \frac{c}{2}\tau$

4) $\Delta\tau = \frac{1}{B}$, $\Delta R = \frac{c}{2}\Delta\tau = \frac{c}{2B}$,

5) What matters is pulse bandwidth, not pulse length.

Problem 4

A signal $s(t)$ with total (two-sided) bandwidth B = 2 MHz arrives at two receivers following two distinct paths. The signals output at the two receivers are modeled as:

$$d_1(t) = s(t) + s(t - \tau_1)$$
$$d_2(t) = s(t) + s(t - \tau_2)$$

Where $\tau_1 =$ 1 microsecond and $\tau_2 =$ 1.5 microseconds.

3. Calculate the expression of the Fourier Transforms of $d_1(t)$ and $d_2(t)$ and draw the graphs of their absolute values, having care to highlight the positions where they are null (for this point, you can approximate $S(f)$ to a rectangular pulse in the frequency domain).
4. Propose a procedure to restore s(t) using only $d_1(t)$. Is it possible to obtain $s(t)$ with no errors?
5. Propose a procedure to restore s(t) using $d_1(t)$ <u>*and*</u> $d_2(t)$. Is it now possible to obtain $s(t)$ with no errors?

Solution

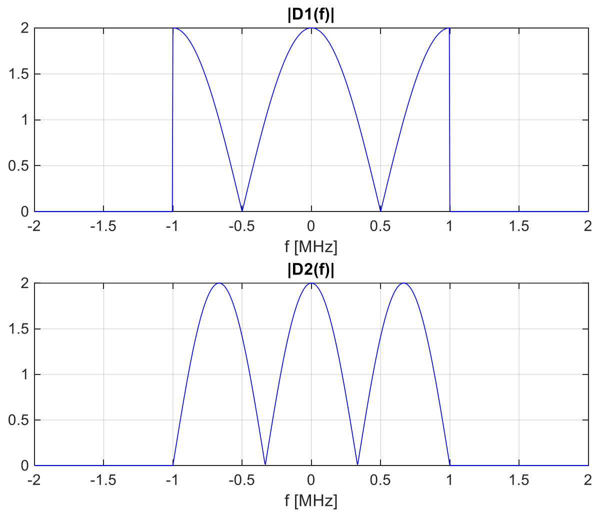
1) Taking the FT:

$$d_1(f) = s(f) \cdot \left(1 + exp(-j2\pi f\tau_1)\right) = s(f)H_1(f)$$
$$d_2(f) = s(f) \cdot \left(1 + exp(-j2\pi f\tau_2)\right) = s(f)H_2(f)$$

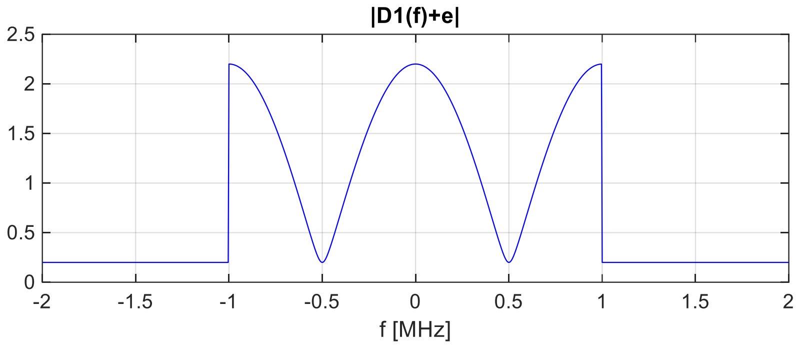
$d_1(f) = 0$ when $exp(-j2\pi f\tau_1) = -1$, so when $2\pi f\tau_1 = (2k+1)\pi$ => $f = \frac{(2k+1)}{2\tau_1}$

With $\tau_1 =$ 1 microsecond one has $f = 0.5$ MHz for k = 0 and $f = -0.5$ MHz for k = -1. The other values of k are not important since they result in values of frequency outside the spectral support of the signal.

Similarly for signal d2 (for k = -1, 0, 1). The graphs are shown here below



2) The procedure would be to convolve d_1 for a filter u whose frequency response is the inverse of $H_1(f) = 1 + exp(-j2\pi f\tau_1)$, but this cannot be implemented exactly since the frequency response cannot be inverted when it is 0. In this case, one could compute the inverse of a modified version of $H_1(f)$ that shows no null values, such as $H_1(f) + \epsilon$, with ϵ a conveniently small number. See the graph below for an example:



3) Many answers are possible, all leveraging the idea to use both d_1 and d_2 to avoid null values in the frequency domain. A simple solution could be to form a new signal $d_3 = d_1 + d_2$.
In the frequency domain one has

$$d_3(f) = s(f) \cdot \left(2 + exp(-j2\pi f\tau_1) + exp(-j2\pi f\tau_2)\right) = s(f)H_3(f)$$

$H_3(f)$ is never 0 within the signal bandwidth (see graph below), hence it can be inverted at all frequencies.