Electromagnetics and Signal Processing for Spaceborne Applications
July 19th, 2024

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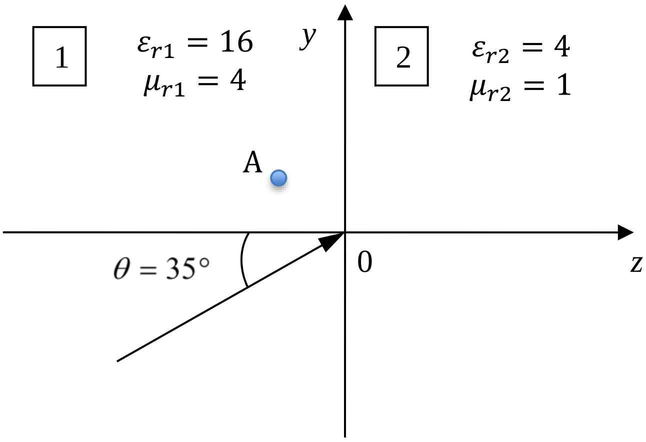
Problem 1

Consider the plane sinusoidal wave below, with f = 1 GHz and incident electric field given by:

$$\vec{E}_i(0,0,0) = j\mu_x \quad \text{(V/m)}$$



Calculate the electric field in point A(x = -5 m, y = 1 m, z = -1 m).

Solution

The total electric field is given by the summation between the incident field and the reflected field (if any). To determine the reflected field, it is first necessary to calculated the refraction angle:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\mu_{r1}\varepsilon_{r1}}{\mu_{r2}\varepsilon_{r2}}}\right) \approx \sin^{-1}(2.294)$$

This result indicates an evanescent wave, which means total reflection. For the complete expression of the reflected field though, it is necessary to calculate the reflection coefficient. To this aim, let us calculate β_{2z}:

$$\beta_{2z} = \beta_2\cos(\theta_2) = \beta_2\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{\mu_{r2}\varepsilon_{r2}}\sqrt{1-[\sin(\theta_2)]^2} = \pm j4.13\beta_0 = -j4.13\beta_0$$

It is necessary to pick the negative sign to obtain a physical solution. The reflection coefficient can now be calculated:

$$\eta_1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}} \frac{1}{\cos(\theta)}} = 266.6\ \Omega$$

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}} \frac{1}{\cos(\theta_2)}} = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}} \frac{1}{\cos(\theta_2)}} = j91.3\ \Omega$$

The choice of the negative sign for $\cos(\theta_2)$ in η_2 is consistent with the one in β_{2z}.

$$\Gamma = \frac{\eta_2 - \eta_2}{\eta_2 + \eta_1} = -0.728 + j0.686$$

The expression of the total electric field in medium 1 is therefore:

$$\vec{E}(y,z) = \vec{E}_i(y,z) + \vec{E}_r(y,z) = j\mu_x e^{-j\beta_1 \cos(\theta) z} e^{-j\beta_1 \sin(\theta) y} + \Gamma j\mu_x e^{j\beta_1 \cos(\theta) z} e^{-j\beta_1 \sin(\theta) y}\ \text{V/m}$$

The value of the electric field in A is:

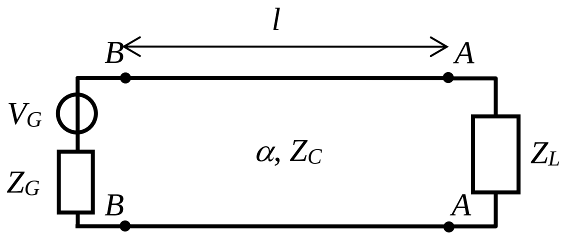
$\vec{E}(A) = (-0.697 - j0.893)\mu_x$ V/m

with $\beta_1 = \beta_0\sqrt{\mu_{r1}\varepsilon_{r1}} = \frac{2\pi}{\lambda_0}\sqrt{\mu_{r1}\varepsilon_{r1}} = 167.5\ \text{rad/m}$

Problem 2

A source with voltage V_G = 100 V and internal impedance Z_G = 60 Ω is connected to a load Z_L = 60 Ω through a lossy transmission line with characteristic impedance Z_C = 100 Ω and specific attenuation α = 30 dB/km. The line length is l = 20 m, the frequency is f = 300 MHz.
For this circuit:

1) Calculate the power absorbed by Z_L.
2) Calculate the absolute value of the voltage at section, $|V_{AA}|$.
3) Calculate the trend in time of the voltage at section, $V_{AA}(t)$.



Solution

1) First, let us calculate the reflection coefficient at AA:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.25$$

The reflection coefficient at BB is:

$$\Gamma_{BB} = \Gamma_L e^{-j2\beta l} e^{-2\alpha l} = -0.2177$$

where $\beta = 6.283 \text{ rad/m}$

The equivalent input impedance is:

$$\mathrm{Z}_{BB} = 64.24\ \Omega$$

Let us now calculate:

$$\Gamma_G = 0.034$$

The power absorbed by what is beyond section BB (load plus line) is:

$$P_{BB} = P_{AV}(1 - |\Gamma_G|^2) = 20.8 \text{ W}$$

with

$$P_{AV} = \frac{|V_G|^2}{8Z_G} = 20.83 \text{ W}$$

This is not yet the power reaching the load, as we need to discriminate what part of the power is absorbed by the line. As question 3 requires the calculation of $V_{AA}(t)$, we can proceed by using voltage transfer. To this aim:

$$V_{BB} = V_G \frac{Z_{BB}}{Z_G + Z_{BB}} = 51.7 \text{ V}$$

Then, the progressive wave at BB is:

$$V_{BB}^{+} = \frac{V_{BB}}{1 + \Gamma_{BB}} = 66.1 \text{ V}$$

Finally:

$$V_{AA} = V_{BB}^{+} e^{-j\beta l} e^{-\alpha l}(1 + \Gamma_L) = 46.3 \text{ V}$$

The power absorbed by the load is:

$$P_L = \frac{1}{2}\frac{|V_{AA}|^2}{Z_L} = 17.8 \text{ W}$$

2) From $V_{AA} \rightarrow |V_{AA}| = 46.3$ V

3) Finally, as $\measuredangle V_{AA} = 0$ rad, $V_{AA}(t) = 46.3\cos(1.88 \times 10^9 t)$ V

Problem 3

A signal $s(t)$ and its reflection are received at two antennas. The two received signals can be modelled as:

$$s_1(t) = s(t) + s(t - \tau_1)$$
$$s_2(t) = s(t) + s(t - \tau_2)$$

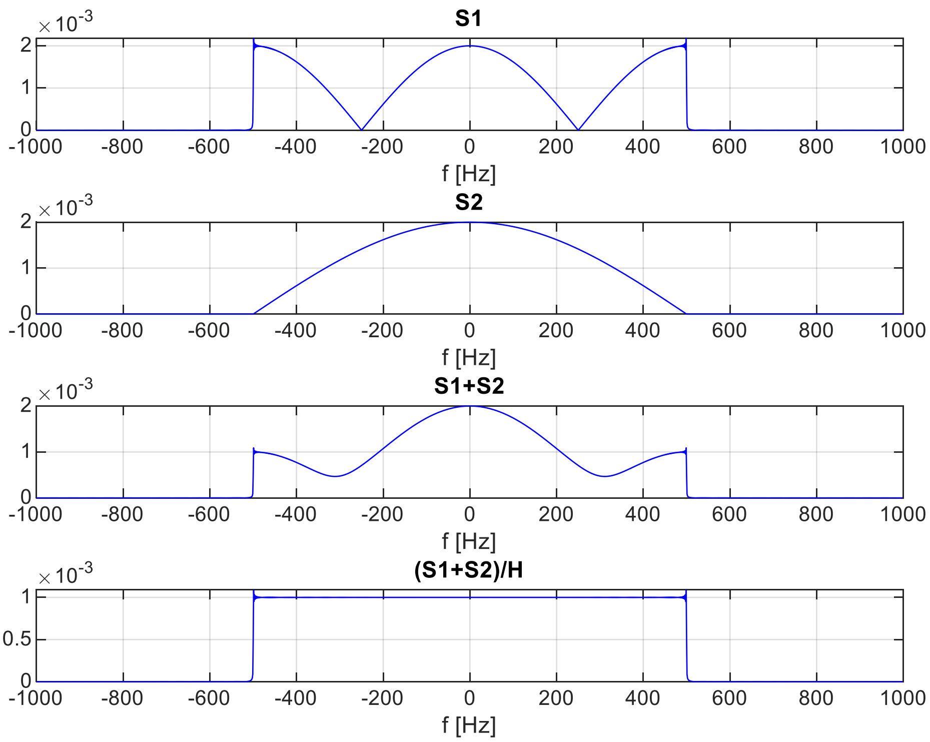
The transmitted signal $s(t)$ is modeled a real-valued base-band signal with a two-sided bandwidth of B = 1 KHz. The two delays are $\tau_1 = 2$ ms and $\tau_2 = 1$ ms

1. Find the expression of the Fourier Transform of the two signals and plot the graphs of their magnitudes (N.B.: two graphs are required).
2. Propose a method to cancel the reflection working on signal $s_1(t)$ alone. Is it possible to retrieve $s(t)$ with no error?
3. Propose a method to cancel the reflection working using a combination of $s_1(t)$ and $s_2(t)$. Is it possible to retrieve $s(t)$ with no error?

Solution

$$S_1(f) = S(f)(1 + \exp{(-j2\pi f\tau_1)}) = S(f)H_1(f)$$
$$S_2(f) = S(f)(1 + \exp{(-j2\pi f\tau_2)}) = S(f)H_2(f)$$

The first three graphs below show the Fourier Transforms of s1, s2, s1+s2 (assuming that S(f) is a rectangular pulse in the frequency domain with bandwidth of 1 KHz). It is immediate to see that both S1(f) and S2(f) present zeroes, from which it follows that the original signal S(f) cannot be reconstructed without errors. In the case of s1+s2 the transfer function is H1+H2, which does not present any zeros. Accordingly, the signal can be restored with no error by applying the inverse filter 1/(H1+H2)



Problem 4

A ground-based antenna array is used to determine the direction of arrival of signals emitted by spaceborne transmitters to within an angular resolution of 0.2°. Transmitters are assumed to operate at the frequency of 10 GHz.

1. Determine array length.
2. Determine the required number of elements assuming that the transmitters are located in the angular interval (-90°,90°).
3. Determine the required number of elements assuming that the transmitters are located in the angular interval (-30°,30°).
4. The array defined at point 3) is now used in transmission. Calculate the delay to be applied to each element to obtain the transmission of one beam at 45°.
5. Calculate the direction of the secondary beam that arises at point 4).

Solution

1) Lambda = 3 cm. Angular resolution = 0.2/180*pi = 0.0035 rad. Array length = lambda/angular_res = 8.6 m
2) For unambiguous imaging we set the spacing between elements to lambda/2 => Nx = Array_length/(lambda/2) = 573 elements
3) The spatial frequency interval is now restricted to +- sin(30°)/lambda = +-16.67 m^-1. The total spatial bandwidth is therefore 33.33 m^-1. It follows from the sampling theorem that spatial sampling needs to be no coarser than 1/33.33 = 3 cm. The number of elements is 287.
4) A beam at 45° is obtained by exciting the array elements with a signal $s(x) = \exp\left(-j2\pi f_{x0}x\right)$ with $f_{x0} = \frac{sin(45°)}{\lambda}$ =23.57 m^-1 and x the antenna position along the array. The same information can be given with reference to the delay applied to each antenna. Recalling that phase of a delayed RF signal $2\pi f_0\tau$, we get:

$$2\pi f_0\tau = 2\pi\frac{sin(45°)}{\lambda}x$$

Hence $\tau = \frac{sin(45°)}{f_0\lambda}x = \frac{sin(45°)}{c}x$
5) the spatial sampling frequency is the inverse of the sampling space, hence f_s =1/0.03 = 33.33 m^-1. Accordingly, replicas arise the frequency domain at spatial frequencies for any integer k:

$$f_r = f_{x0} + kf_s$$

Computing five replicas we have
-43.0964   -9.7631   23.5702   56.9036   90.2369
This is converted to angles via

$$\theta = asin(\lambda f_r)$$

Physical solutions are only found for fr = [-9.7631   23.5702], which correspond to 17° and 45°