Electromagnetics and Signal Processing for Spaceborne Applications – EM part
January 24th, 2023

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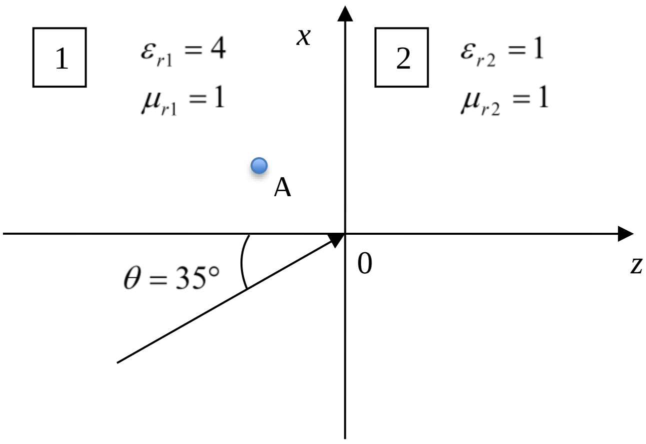
Problem 1

Consider the plane sinusoidal wave depicted below hitting the interface between two lossless media, whose EM characteristics are reported in the figure. The incidence angle is $\theta = 35°$ as shown below. The wave frequency is $f = 1$ GHz. The incident electric field in medium 1 at the origin of the system is:

$$\vec{E}_i(0,0,0) = j\vec{\mu}_y \text{ V/m}$$



For this scenario, calculate:

a) The reflection coefficient at the interface.
b) The power density flowing in the z direction in the second medium.
c) The absolute value of the total electric field in point A($x = 1$, $y = 2$, $z = -1$).

Solution:

a) The wave is TE. To calculate the reflection coefficient, first, let us calculate the refraction angle, which is obtained from the Snell's law:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \Rightarrow \quad \sin\theta_2 = 1.1472 \quad \Rightarrow \quad$ evanescent wave

The wave is evanescent, which implies that the absolute value of the reflection coefficient is 1, i.e. all the power density is reflected. For the detailed calculation of the reflection coefficient, we start from the intrinsic impedances:

$$\eta_{TE}^{1} = \frac{\eta_0}{\sqrt{4}} \frac{1}{\cos\theta_1} = 230.1\ \Omega$$

$$\eta_{TE}^{2} = \frac{\eta_0}{\cos\theta_2} = \frac{\eta_0}{\sqrt{1-(\sin\theta_2)^2}} = \frac{\eta_0}{\pm\, j0.5622} \Rightarrow \eta_{TE}^{2} = j670.6\,\Omega$$

Note that this mathematical solution is picked to achieve a physical solution for the wave equation in the second medium. Finally, the reflection coefficient is:

$$\Gamma = \frac{\eta_{TE}^{2} - \eta_{TE}^{1}}{\eta_{TE}^{2} + \eta_{TE}^{1}} = 0.789 + j0.614$$

b) As the wave is evanescent wave, no power density will travel in the z direction in the second medium.

c) The total electric field in the first medium is the combination of the incident wave and of the reflected wave, i.e.:

$$\vec{E}(x,y,z) = \vec{E}_i(x,y,z) + \vec{E}_r(x,y,z) = j\vec{\mu}_y e^{-j\frac{2\pi}{\lambda}(\cos(\theta)z+\sin(\theta)x)} + \Gamma\left(j\vec{\mu}_y\right)e^{-j\frac{2\pi}{\lambda}(-\cos(\theta)z+\sin(\theta)x)} \quad \text{V/m}$$

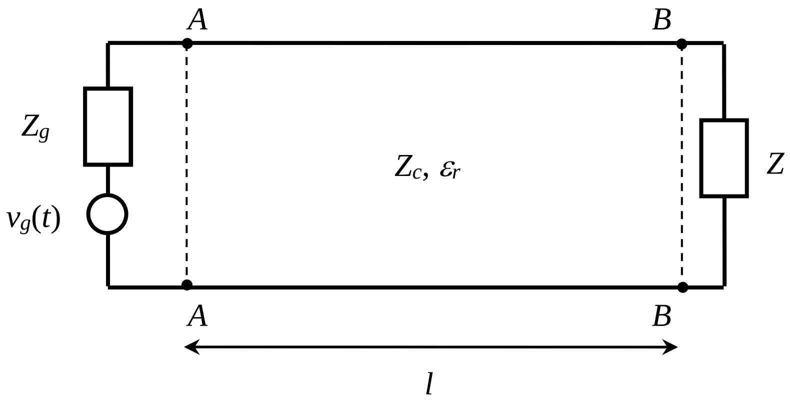
with $\lambda = c/\left(\sqrt{4} f\right) = 0.15$ m.

The absolute value of the electric field in A can be obtained by setting x to 1 and z to -1 in the equation above and by taking the absolute value. This leads to:

$$\vec{E}_t(A) = (1.6626 - j0.2231)\vec{\mu}_y \quad \text{V/m} \quad \rightarrow \quad \left|\vec{E}_t(A)\right| = 1.6775 \quad \text{V/m}$$

Problem 2

Making reference to the figure below, determine the maximum value for the specific attenuation α that can be accepted to guarantee that the load Z receives at least 1/3 of the available power at the generator P_{AV}. For that case, calculate the power lost along the transmission line.



$v_g(t) = 10\cos\left(1.59\pi 10^9 t\right)$ V
Z_g = 50 Ω
Z_c = 50 Ω
Z = 100+j100 Ω
l = 100 m

Solution:
From the equation expressing the temporal trend of the generator voltage, we get:
V_g = 10 V
f = 0.795 GHz
Given the generator load, the intrinsic impedance and the load at the end of the line, the power absorbed by the line is:

$$P_L = P_d\, e^{-2\alpha l}\left(1-\left|\Gamma_{BB}\right|^2\right)$$

where:

$$\Gamma_{BB} = \frac{Z-Z_c}{Z+Z_c} = 0.54 + j0.31$$

Setting $P_L = P_d/3$, we get:

$$e^{-2\alpha l} = \frac{1}{3\left(1-\left|\Gamma_{BB}\right|^2\right)} = 0.542$$

Si ottiene dunque:

$$\alpha = \frac{\ln(0.542)}{(-2l)} = 0.0031 \text{ Np/m}$$

For the power lost along the line, let us first calculate the absolute value of the reflection coefficient at section AA:

$$\left|\Gamma_{AA}\right| = \left|\Gamma_{BB}\, e^{-2\alpha l}\right| = 0.336$$

As $Z_g = Z_c$:

$$\left|\Gamma_{AA}\right| = \left|\Gamma_g\right|$$

The power absorbed by all the elements beyond section AA is:

$$P_{AA} = P_d\left(1-\left|\Gamma_g\right|^2\right) = 0.222 \text{ W}$$

con

$$P_d = \frac{\left|V_g\right|^2}{8\,\mathrm{Re}\left(Z_g\right)} = 0.25 \text{ W}$$

Therefore, the power lost along the line is:
$P_{diss} = P_{AA} - P_L = 0.139$ W
after setting:
$P_L = P_d / 3 = 0.083$ W

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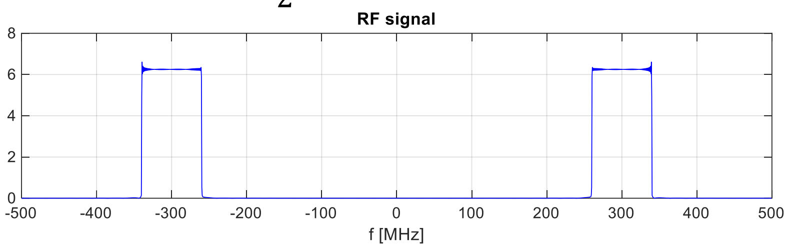
Problem 3

Let $s(t)$ be a RF signal: $s(t) = g(t)cos(2\pi f_0 t)$, where $g(t)$ is a short pulse with a (two-sided) bandwidth of 80 MHz and the carrier frequency is $f_0 = 300$ MHz.

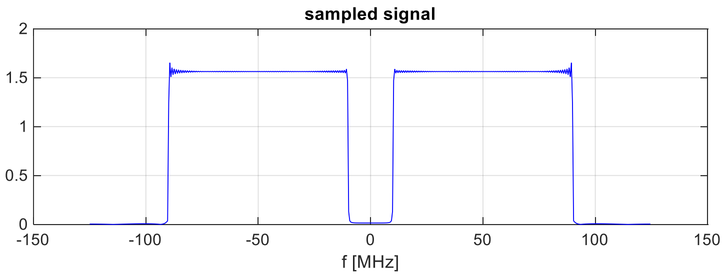
1. Write the expressions of the Fourier Transform of $s(t)$ and draw a graph of the magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).
2. A discrete-time signal s_n is obtained by sampling $s(t)$ with sampling frequency $f_s = 250\ MHz$. Write the expressions of the Fourier Transform of s_n and draw a graph of the magnitude.
3. Propose a procedure for the extraction of the complex envelope (I & Q components) of $s(t)$ based on the sampled signal s_n.
4. Write a short pseudo-code to implement the procedure at point 3.

1: $S(f) = G(f) * \mathcal{F}\big(cos(2\pi f_0 t)\big) = G(f) * \frac{1}{2}\big(\delta(f - f_0) + \delta(f + f_0)\big) =$

$$=> S(f) = \frac{1}{2}\big(G(f - f_0) + G(f + f_0)\big)$$



2) Just replicate the graph above with a period $f_s = 250\ MHz$



The analytical expression is:

$$S_s(f) = \frac{f_s}{2}\left(G(f - f_1) + G(f + f_1)\right)$$

With $f_1 = 50\ MHz$

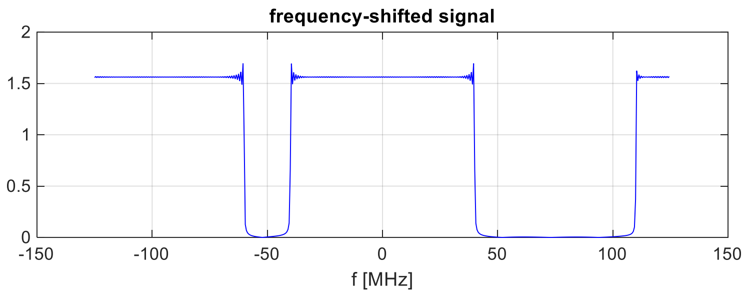
3) The complex envelope of s(t) is g(t) (assuming no delay). A simple procedure to recover $G(f)$ is to apply a shift in frequency followed by a low-pass filter:

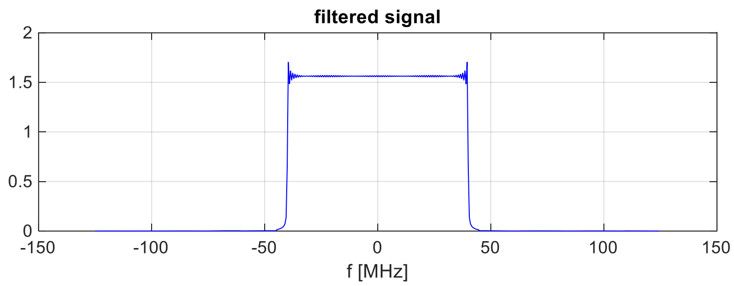
$$G(f) = H(f)S_s(f + f_1)$$

In the time domain this is obtained as:

$$g_n = h_n * \left(s_n exp\left(-j2\pi\frac{f_1}{f_s}n\right)\right)$$

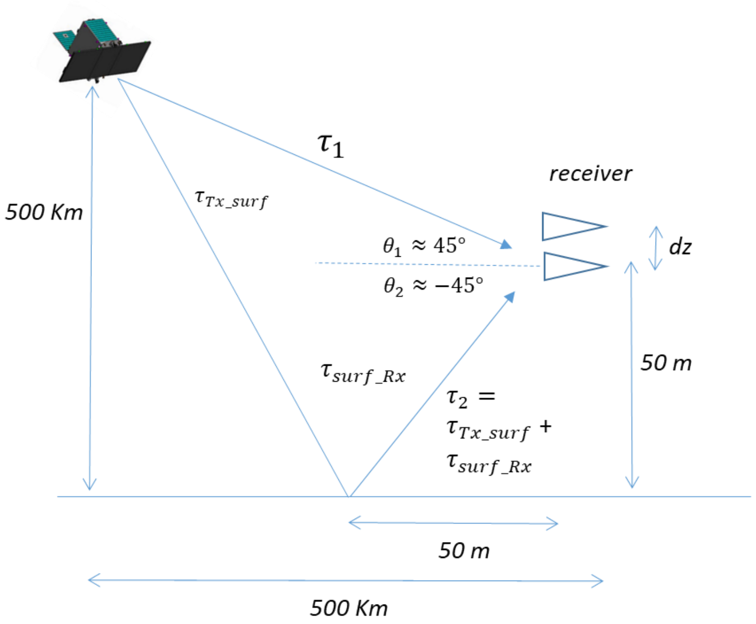
any low-pass filter with bandwidth 80<B<100 MHz will do. Notice that n/f_s represent the time instants of the sampled signal s_n.





Problem 4

A ground station equipped with an array of 2 antennas receives the signal from a spaceborne transmitter and its reflection onto the surface. The transmitted signal is modeled as $s(t) = g(t)exp(j2\pi f_0 t)$, with $g(t)$ a short pulse with a bandwidth of 100 MHz and the carrier frequency is $f_0 = 5$ GHz. The vertical spacing between the two antennas is $dz = \frac{\lambda}{2}$.



1. Write the expression of the complex envelope of the signals received at the two antennas, $d_1(t)$ and $d_2(t)$. In doing that, make explicit use of the two angles of arrival θ_1 and θ_2. ***Tip****: no need for long calculations, just remember the role of the angle of arrival in antenna array theory.*
2. Write the expressions of the Fourier Transform of the two signals at point 1 and draw a graph of the magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).
3. Propose a procedure to separate the direct and reflected signals using $d_1(t)$ and $d_2(t)$. Is it possible to achieve a perfect separation?

1) The signal at the first antenna is the sum of the direct and reflected signals:

$$d_1(t) = s_1(t) + s_2(t)$$

where $s_1(t) = g(t - \tau_1)exp(-j2\pi f_0 \tau_1)$ and $s_2(t) = g(t - \tau_2)exp(-j2\pi f_0 \tau_2)$.

The signal at the second antenna experience an extra-phase rotation depending on their angle of arrival:

$$d_2(t) = s_1(t)exp(ja) + s_2(t)exp(-ja)$$

With

$$a = \frac{2\pi}{\lambda} sin(\theta)dz$$

2) The Fourier transform is obtained as:

$$S_1(f) = G(f)exp(-j2\pi f \tau_1)exp(-j2\pi f_0 \tau_1) = G(f)exp(-j2\pi(f + f_0)\tau_1)$$
$$S_2(f) = G(f)exp(-j2\pi(f + f_0)\tau_2)$$

Hence:

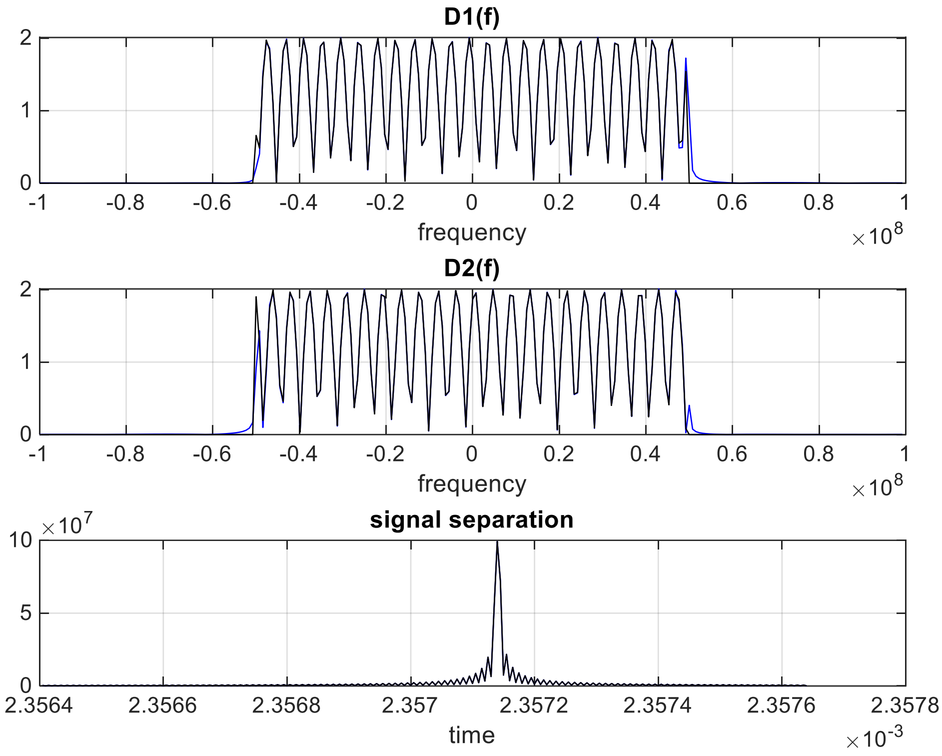
$$D_2(f) = G(f)[exp(-j2\pi(f + f_0)\tau_1)exp(ja) + exp(-j2\pi(f + f_0)\tau_2)exp(-ja)]$$

The graph of the magnitude can be obtained by evaluating $|D_2(f)|^2$:

$$|D_2(f)|^2 = |G(f)|^2[1 + 1 + 2cos(2\pi(f + f_0)(\tau_1 - \tau_2) - 2a)]$$

So that:

$$|D_2(f)| = 2|G(f)|[1 + cos(2\pi(f + f_0)(\tau_1 - \tau_2) - 2a)]$$



3)

$$d_1(t) = s_1(t) + s_2(t)$$
$$d_2(t) = s_1(t)exp(ja) + s_2(t)exp(-ja)$$

$$d_2(t) - d_1(t)exp(ja) = s_2(t)exp(-ja) - s_2(t)exp(ja)$$

$$= -2js_2(t)sin(a)$$

So $s_2(t)$ is obtained as:

$$s_2(t) = \frac{d_2(t) - d_1(t)exp(ja)}{-2js_2(t)sin(a)}$$

And likewise for $s_1(t)$. Notice that no separation is possible if a=0, which indeed corresponds to the situation where the two antennas are in the same position.