Electromagnetics and Signal Processing for Spaceborne Applications
June 27th, 2022

1	2	3	4

do not write above

SURNAME AND NAME ______________________________

ID NUMBER ______________________________

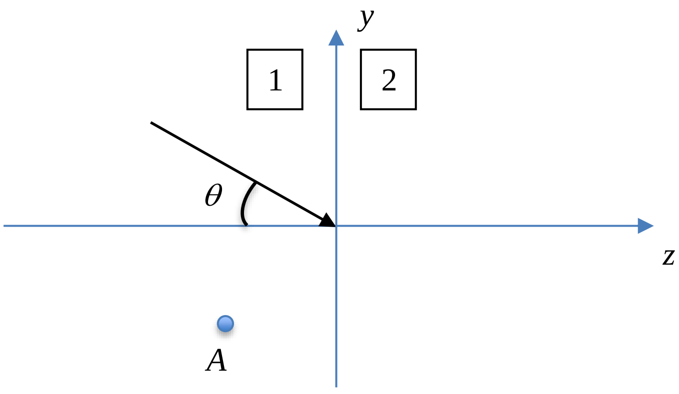
SIGNATURE ______________________________

Problem 1

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 3 into free space (assume μ_r = 1 for both media). The expression of the incident electric field is:

$$\vec{E}_i(z,y) = E_0\left(\frac{\sqrt{3}}{2}\vec{\mu}_y + \frac{1}{2}\vec{\mu}_z + \frac{j}{2}\vec{\mu}_x\right)e^{-j\beta\frac{\sqrt{3}}{2}z}e^{j\frac{\beta}{2}y}\ \text{V/m}$$

1) 1 Determine the incidence angle θ.
2) 2 Determine the polarization of the incident field $\vec{E}_i$ (it is sufficient to state whether the polarization is elliptical, circular or linear; additional details, such as rotation direction or tilt angle, are not required).
3) 3 Determine the polarization of the reflected field $\vec{E}_r$.
4) 4 Calculate the absolute value of E_0 by knowing that the power density of the electric field in A(x = -1 m, y = -1 m, z = -1 m) is S_T = 0.5 W/m^2 (consider only the contribution of the reflected field).



Solution
1) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta\sin(\theta) = \beta/2 \rightarrow \sin(\theta) = 1/2 \rightarrow \theta = 30°$

2) The wave has two components: a TE one, along x, and a TM one, given by the combinations of the two fields along y and z. The absolute value of the components is E_0 V/m (TM) and $E_0/2$ V/m (TE), and the differential phase shift is $\pi/2$

3) Considering that the wave also has a TM component, it is worth checking the Brewster angle:

$$\theta_B = \mathrm{tg}^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As $\theta = \theta_B \rightarrow$ the TM component is totally transmitted into the second medium. As a result, the polarization of the reflected wave will be linear (along x).

4) To determine E_0, it is first necessary to calculate the reflection coefficient for the TE component. To this aim, the transmission angle is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 60°$$

The TE reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) - \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})}{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) + \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})} = 0.5$$

The reflected field is therefore given by:

$$\vec{E}_r(z, y) = j\frac{E_0}{2}\Gamma\vec{\mu}_x e^{j\beta\frac{\sqrt{3}}{2}z} e^{j\frac{\beta}{2}y} \text{ V/m}$$

The power density reaching A will be:

$$S_T = \frac{1}{2}\frac{\left|\vec{E}_r(A)\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = \frac{1}{2}\frac{\left|\frac{E_0}{2}\Gamma\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = \frac{\Gamma^2 E_0^2\sqrt{\varepsilon_{r1}}}{8\eta_0}$$

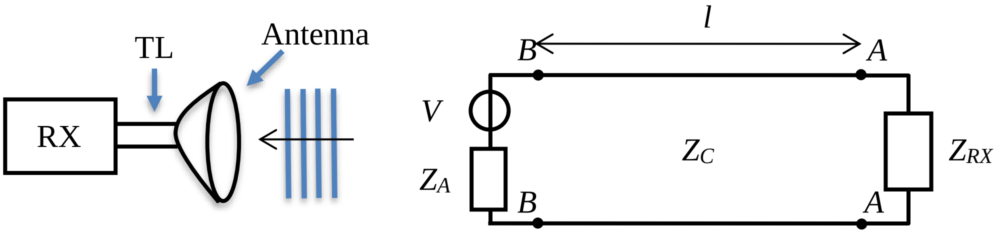
Solving for E_0:

$E_0 = 59$ V/m

Problem 2

The power received by an antenna is conveyed into the receiver RX via a lossless coaxial cable, with intrinsic impedance Z_C = 50 Ω. The antenna acts as an equivalent generator with voltage V = 10^{-3} V and internal impedance Z_A = 50 Ω; the RX, which acts as a load, has impedance Z_{RX} = 150 Ω. The frequency is f = 600 MHz. The line length is l = 5.2 m.

1. Determine the power absorbed by RX, P_{RX}.
2. Using the same coaxial cable, propose changes to the circuit to achieve maximum power transfer from the antenna to RX: how much would P_{RX} be in that case?



Solution

1) The wavelength is $\lambda = c/f$ = 0.5 m. The reflection coefficient at section AA is given by:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0.5$$

The solution is simplified by the partial match at section BB, so the power absorbed by the load can be simply calculated as (only one reflection at the load section):

$$P_{RX} = P_{AV}(1 - |\Gamma_L|^2) = \frac{|V|^2}{8Z_A}(1 - |\Gamma_L|^2) = 1.875 \text{ nW}$$

2) Maximum power transfer is achieved by perfect matching, i.e. $Z_A = Z_{RX} = Z_C = 50\ \Omega$. In this case, no reflections occur and the power transferred to RX is the whole available power. In this case:

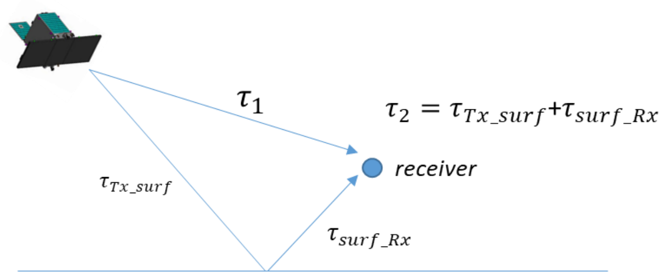
$$P_{AV} = \frac{|V|^2}{8Z_C} = 2.5 \text{ nW}$$

Problem 3

A spaceborne Radar is designed to transmit a Radio-Frequency signal:

$$s_{Tx}(t) = g(t) \cdot e^{j2\pi f_0 t}$$

where $g(t)$ is base-band signal with (two-sided) bandwidth equal to $B = 6$ MHz and $f_0 = 435$ MHz. The signal reaches a ground receiver along two distinct paths, as represented in the figure below:



The total delays along the two paths are τ_1 and τ_2.

1. Write the expressions of the Fourier Transform of the transmitted signal $s_{Tx}(t)$ and draw a graph of the magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).
2. Write the time domain expression of the complex envelope of the signal at the receiver.
3. Draw a graph of the magnitude of the complex envelope at point 2 after it is cross-correlated with $g(t)$ (i.e.: convolution with $g^*(-t)$), considering two cases:
   3.1. $|\tau_1 - \tau_2| \gg \frac{1}{B}$.
   3.2. $|\tau_1 - \tau_2| \ll \frac{1}{B}$.

   What is the role played by the phase terms in the two cases?

4. Write a short pseudo-code to implement the extraction of the complex envelope at the receiver and the cross-correlation with $g(t)$.

Solution

Point 1)

By the properties of the FT we have
$s_{Tx}(t) = g(t) \cdot e^{j2\pi f_0 t}$ => $s_{Tx}(tf) = g(f - f_0)$

The graph of |the magnitude is therefore a rectangular pulse centered about f0 and with total width (bandwidth) of 6 MHz (so it goes from 432 to 438 MHz)|

Point 2)

$$s_{rx}(t) = s_{Tx}(t - \tau_1) + s_{Tx}(t - \tau_2) =$$
$$= g(t - \tau_1) \cdot e^{j2\pi f_0 (t-\tau_1)} + g(t - \tau_2) \cdot e^{j2\pi f_0 (t-\tau_2)}$$

When working in complex notation the complex envelope is simply obtained as

$$z(t) = s_{rx}(t) \cdot e^{-j2\pi f_0 t} =$$

$$= g(t-\tau_1)\cdot e^{-j2\pi f_0\tau_1} + g(t-\tau_2)\cdot e^{-j2\pi f_0\tau_2}$$

Point 3)

Cross-correlation:

$$z_{cr}(t) = z(t) * g^*(-t) =$$
$$= \left\{g(t-\tau_1)\cdot e^{-j2\pi f_0\tau_1} + \ g(t-\tau_2)\cdot e^{-j2\pi f_0\tau_2}\right\} * g^*(-t)$$
$$= R_g(t-\tau_1)\cdot e^{-j2\pi f_0\tau_1} + R_g(t-\tau_2)\cdot e^{-j2\pi f_0\tau_2}$$

Where $R_g = g(t) * g^*(-t)$ is the autocorrelation function of g(t).

By the properties of the autocorrelation function R_g can be represented as a short signal of effective duration equal to the inverse of the bandwidth.

Therefore, if $|\tau_1 - \tau_2| \gg \frac{1}{B}$.we have two well-separate peaks. In this case the phases have no effect on magnitude.

Otherwise, the two peaks overlap and interfere. In this case the interference can be constructive or destructive, depending on the value of the difference $\tau_1 - \tau_2$ in the expression of the phases.

Point 4)

Perfect answer: start from the <u>real-valued</u> received signal

```
dt = 1/(2*f0); % to be sure you have no alias when representing the real-valued signal
t = (0:N-1)*dt;
A = s_rx.*cos(2*pi*f0*t);
B = s_rx.*sin(2*pi*f0*t);
h = fir_filter(B); % filter with bandwidth >= B
I = filter(A,h); % In phase component
Q = filter(B,h); % In quadrature component
Z = I + 1i*Q;
Zrc = conv2(Z,conj(fliplr(g)));
```

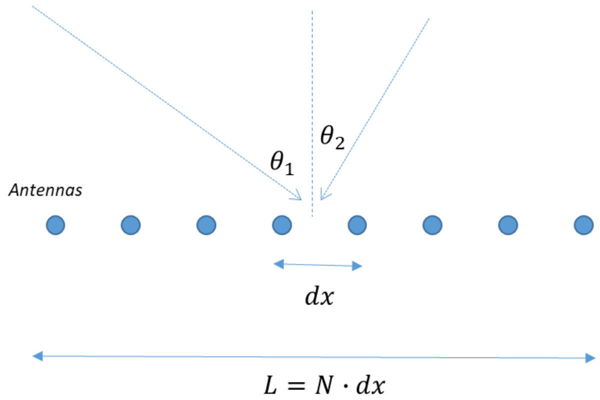
Frequency domain implementations were accepted as well.

Totally acceptable answer: start from the <u>complex</u> received signal

```
dt = 1/B; % notice the much less stringent requirement w.r.t. the previous case
t = (0:N-1)*dt;
Z = s_rx.*exp(-1i*2*pi*f0*t)
Zrc = conv2(Z,conj(fliplr(g)));
```

Problem 4

Multiple electromagnetic waves radiated from distant sources at the frequency $f_0 = 1.3$ GHz impinge simultaneously on an antenna array as represented in the figure below:



Each antenna is equipped with its own circuity to generate the complex envelope of the received signal.

1. Describe a procedure to measure the directions of arrival $\theta_1, \theta_2, \ldots$ based on the N signals (complex envelope) output by the array.
2. Write a short pseudo-code to implement the procedure at point 1.
3. Discuss how the set antenna spacing dx
4. After fixing antenna spacing dx, determine the number of antennas required to obtain an angular resolution $\Delta\theta = 5°$ (you can assume an incident direction of 0° as a reference in the calculation of resolution)
5. How would you change your answers at points 3 and 4 if you had knowledge that the direction of arrival of all impinging waves is limited in the interval (-9°,+9°)? (you don't necessarily have to use too much time on calculations, just highlight the rationale, and proceed to details only if you have time left)

Solution

Point 1)

Under the far-field assumption the signal output at each element is expressed as a sum of sinusoids $s_n = \sum_i \; exp\left(j2\pi \frac{sin(\theta i)}{\lambda} n \cdot dx\right)$

The problem is then analogous to frequency detection and estimation, which is implemented by FT.

Accordingly, we define : $S(\theta) = \sum_{n=1}^{N} s_n \cdot exp\left(-j2\pi \frac{sin(\theta)}{\lambda} n \cdot dx\right)$, which by construction will produce a peak at the values of θ corresponding to any direction of arrival.

Point 2)

Nf = 2*N;

fx = (-Nf/2:Nf/2-1)/Nf/dx; % axis of spatial frequencies

S = fftshift(fft(s,Nf)); % with s = vector of the signals from the array

f_peak = detect_peaks(abs(S));

teta_est = asin(lambda*f_peak);

Point 3)

The spatial frequency is $\frac{sin(\theta)}{\lambda}$. If any direction is possible then the min and max frequencies are $\pm\frac{1}{\lambda}$, hence the total bandwidth is $\frac{2}{\lambda}$. Accordingly, the condition not to have replicas is that $dx \leq \frac{\lambda}{2}$

Point 4)

Angular resolution is (approximately) obtained as $\Delta\theta = \frac{\lambda}{L} = \frac{5}{180}\pi\ rad$ => L = 2.644 m

Setting $dx = \frac{\lambda}{2}$ the number of antennas is L/dx = 22.9 => 23 antennas

Point 5)

If I know that no source exists outside a given interval, then it is no problem if I allow replicas <u>outside</u> such interval. Accordingly, I can increase the spacing dx, and use this degree of freedom to improve angular resolution (hence larger array length L) and/or to reduce the number of antennas.