Electromagnetics and Signal Processing for Spaceborne Applications
June 28th, 2023

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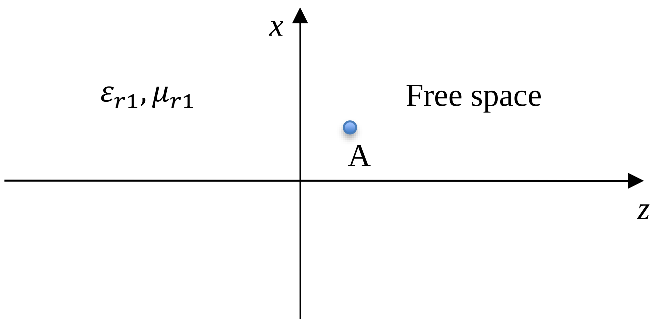
Problem 1

A uniform plane wave (frequency f = 300 MHz) propagates along z into free space from a medium with the following electromagnetic features: ε_{r1} = 3, μ_{r1} = 1 and σ_1 = 0.05 S/m. The incident electric field at the origin of the axis is $\vec{E}_i(z = 0\text{ m}) = jE_0\vec{\mu}_x$ V/m.

For this scenario:

1) What is the wave polarization?
2) Calculate the expression of the incident magnetic field in the first medium (left side) in the time domain.
3) Calculate the wavelength in medium 1.
4) Calculate E_0 knowing that power density power at point A(1,1,λ_2) is $S(A)$ = 3 mW/m^2.



Solution
1) The polarization is linear.

2) The propagation in medium 1 is regulated by the propagation constant. As no approximations are possible (the loss tangent is roughly 1):

$$\gamma_1 = \sqrt{j\omega\mu_1(\sigma_{1+}j\omega\varepsilon_1)} = 4.95 + j11.96 \text{ 1/m}$$

Therefore, the expression of the electric field in medium 1 is:

$\vec{E}_i = jE_0 e^{-\gamma_1 z}\vec{\mu}_x = jE_0 e^{-4.95z} e^{-j11.96z}\vec{\mu}_x$ V/m.

To find the magnetic field, we first need to calculate the intrinsic impedance of the medium:

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{(\sigma_1 + j\omega\varepsilon_1)}} = 169.1 + j70\ \Omega$$

In the phasor domain, the incident magnetic field is:

$$\vec{H}_i = \frac{E_0}{\eta_1} e^{-4.95z} e^{-j11.96z}\vec{\mu}_y = \frac{E_0}{|\eta_1|} e^{-4.95z} e^{-j11.96z} e^{-j\measuredangle(\eta_1)}\vec{\mu}_y$$

where $|\eta_1| = 183\ \Omega$ and $\measuredangle\ (\eta_1) = 0.3924$ rad.

Therefore, the expression of the incident magnetic field in the time domain is (considering also j, which translates into $\pi/2$ in the cosine argument):

$$\vec{H}_i(t) = \frac{E_0}{|\eta_1|} e^{-4.95z} \cos(2\pi f t - 11.96z - 1.18)\vec{\mu}_y$$

3) $\lambda_1 = \frac{2\pi}{\beta_1} = 0.525$ m

4) First, it is necessary to calculate the reflection coefficient, given by:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.359\ - j\ 0.174$$

where

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

The transmitted electric field, at $z = 0$ m, is:

$\vec{E}_t(z = 0\text{ m}) = jE_0 T\vec{\mu}_x = jE_0(1 + \Gamma)\vec{\mu}_x = E_0(0.174 + j1.359)\vec{\mu}_x$ V/m.

The power density reaching point A is (as there are no losses in the second medium, the power density does not change along z):

$$S(A) = \frac{1}{2}\frac{\left|\vec{E}_t(z = 0)\right|^2}{\eta_2} = \frac{1}{2}\frac{(E_0)^2|T|^2}{\eta_2} = 0.0025\ (E_0)^2 = 3\text{ mW/m}^2$$

Therefore:

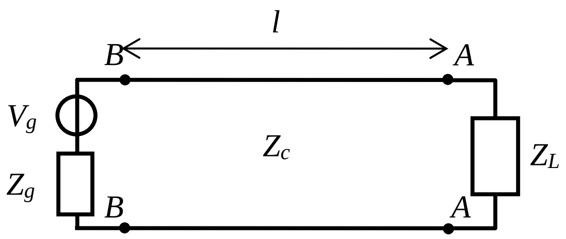
$E_0 = 1.1$ V/m

Problem 2

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a load Z_L = 50 Ω by a transmission line with characteristic impedance Z_C = 150 Ω. The line length is l = 6 m and the frequency is f = 400 MHz.

Calculate:

a) The power absorbed by the load
b) The temporal trend of the voltage at the beginning of the line (section BB below), V_B
c) The absolute value of the voltage at section AA below, V_A



Solution

a) The wavelength is:

$$\lambda = \lambda_0 = c/f = 0.75 \text{ m}$$

The reflection coefficient at section AA is:

$$\Gamma_A = \frac{Z_L - Z_C}{Z_L + Z_C} = -0.5$$

The reflection coefficient at section BB is:

$$\Gamma_B = \Gamma_A e^{-j2\beta l} = \Gamma_A e^{-j2\left(\frac{2\pi}{\lambda}\right)l} = -0.5$$

Therefore, the input impedance is:

$$Z_B = Z_C \frac{1+\Gamma_B}{1-\Gamma_B} = 50 \ \Omega = Z_L$$

There is perfect match at section BB, therefore

$$\Gamma_g = \frac{Z_B - Z_g}{Z_B + Z_g} = 0$$

As a result, all the power made available by the generator will completely cross section BB and reach the load:

$$P_L = P_{AV} = \frac{|V_g|^2}{8\,\mathrm{Re}(Z_g)} = 0.25 \text{ W}$$

b) The voltage at the beginning of the line is found as:

$$V_B = V_g \frac{Z_B}{Z_B + Z_g} = 5 \text{ V}$$

Therefore, the trend of V_B in time is given by:

$$v_B(t) = \mathrm{Re}\left[V_B e^{j\omega t}\right] = 5\cos(\omega t) \text{ V}$$

c) The absolute value of the voltage at the load section can be obtained by inverting the following equation:

$$P_L = \frac{1}{2}\frac{|V_A|^2}{Z_L}$$

Thus:

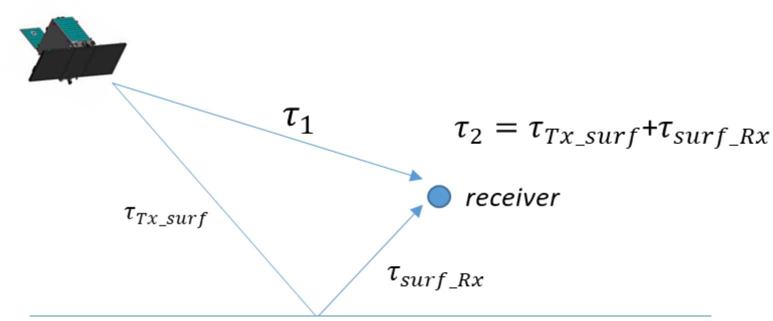
$$|V_A| = \sqrt{2P_L Z_L} = 5 \quad \text{V}$$

Problem 3

A spaceborne system transmits a Radio-Frequency signal:

$$s_{Tx}(t) = g(t) \cdot e^{j2\pi f_0 t}$$

where $g(t)$ is base-band signal with (two-sided) bandwidth equal to $B = 100$ MHz and $f_0 = 1$ GHz. The signal reaches a ground receiver along two distinct paths, as represented in the figure below:



The expression of the received signal is:

$$s_{Rx}(t) = s_{Tx}(t - \tau_1) + a \cdot s_{Tx}(t - \tau_2)$$

where difference between τ_1 and τ_2.is $\tau_2 - \tau_1 = 2.5$ nanoseconds and $a = 0.8$

1. Write the time domain expression of the complex envelope of the signal at the receiver.
2. Write the expressions of the Fourier Transform of the complex envelope of the signal at the receiver and draw the graph of its magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).
3. Discuss if it is possible to eliminate the effect of the reflection on the surface and describe a procedure to do it.
4. Write a short pseudo-code to implement the extraction of the complex envelope at the receiver.

Solution

1. Write the time domain expression of the complex envelope of the signal at the receiver.

The complex is obtained from the RF received signal as:

$$s(t) = s_{Rx}(t) \cdot e^{-j2\pi f_0 t}$$

Hence:

$$s(t) = g(t - \tau_1) \cdot e^{-j2\pi f_0 \tau_1} + a \cdot g(t - \tau_2) \cdot e^{-j2\pi f_0 \tau_2}$$

2. Write the expressions of the Fourier Transform of the complex envelope of the signal at the receiver and draw the graph of its magnitude (you can represent $|G(f)|$ as a rectangular pulse with bandwidth B).

$$S(f) = G(f) \cdot \left\{ e^{-j2\pi(f+f_0)\tau_1} + a \cdot e^{-j2\pi(f+f_0)\tau_2} \right\}$$
$$= G(f) \cdot e^{-j2\pi(f+f_0)\tau_1} \cdot \left\{ 1 + a \cdot e^{-j2\pi(f+f_0)(\tau_2-\tau_1)} \right\}$$

Hence:

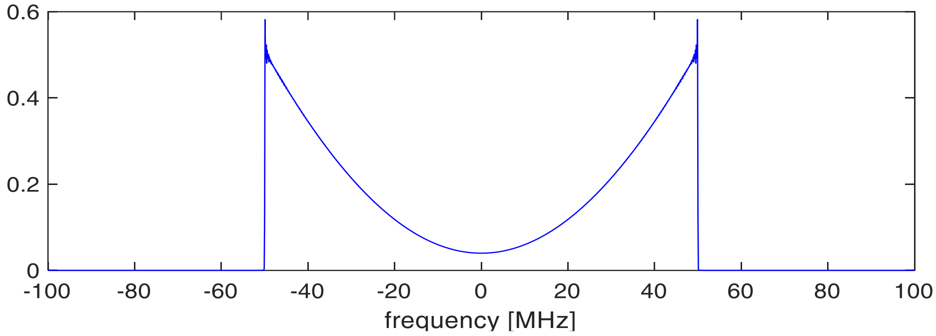
$$|S(f)| = rect\left(\frac{f}{B}\right)\sqrt{1 + a^2 + 2a \cdot cos\big(2\pi(f+f_0)(\tau_2-\tau_1)\big)}$$

The minimum is found when $2\pi(f + f_0)(\tau_2 - \tau_1) = k\pi$, for any odd integer k. Since $f_0(\tau_2 - \tau_1) = 2.5$, this condition for any value of frequency equal to:

$$f = \frac{k-5}{2(\tau_2 - \tau_1)}$$

The one value within the bandwidth B is obtained for k=5, hence for f=0.

The resulting graph of the squared magnitude ($|S(f)|^2$) is as follows:



3. Discuss if it is possible to eliminate the effect of the reflection on the surface and describe a procedure to do it.

   It is surely possible since the Fourier Transform of the signal combined with its reflection is never 0. Accordingly, the reflection is eliminated simply by filtering the received signal with a filter whose frequency response is:

$$H(f) = \left(1 + a \cdot e^{-j2\pi(f+f_0)(\tau_2-\tau_1)}\right)^{-1}$$

   The resulting signal in the frequency domain is:

$$S(f)H(f) = G(f) \cdot e^{-j2\pi(f+f_0)\tau_1}$$

   Which corresponds in the time domain to:

$$s(t) * h(t) = g(t-\tau_1) \cdot e^{-j2\pi f_0 \tau_1}$$

4. Write a short pseudo-code to implement the extraction of the complex envelope at the receiver.

Assuming that received signal is correctly sampled, the complex envelope is extracted as:

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h = sinc(B*t); % any filter with a flat response within the signal bandwidth will do

x = +conv(s_rx.*cos(2*pi*f0*t),h);

y = -conv(s_rx.*sin(2*pi*f0*t),h);

s = x + 1i*y;
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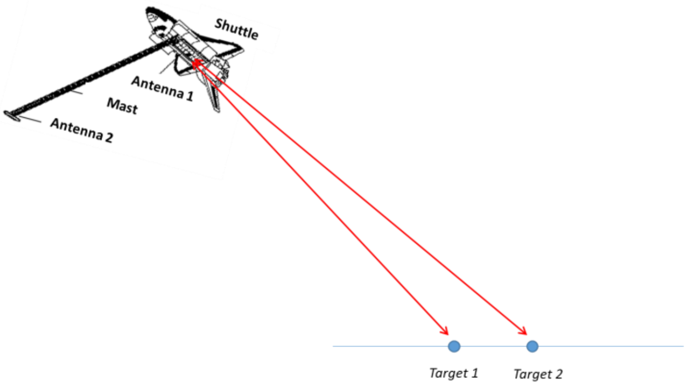
Note: this procedure works under the assumption that the signal is sampled with rate fs > 4*f0 (otherwise, the multiplication with cos and sin generates a replica centered about f=0). A more efficient implementation is possible, but not considered here.

Problem 4

A spaceborne Radar transmits an electromagnetic pulse with a total bandwidth of *B*=10 MHz centered about a carrier frequency f_0= 5 GHz: $s_{Tx}(t) = g(t) \cdot e^{j2\pi f_0 t}$.
The signal is reflected by two targets on the Earth surface at a distance of 300 Km $\pm$ 20 meters from the Radar, and the reflected echoes are then received by the Radar at times τ_1 and τ_2 as represented in the figure below:



Considering only a <u>single antenna</u>:

1. Write the time domain expression of the complex envelope of the signal received by the Radar.
2. Assume that $g(t)$ is a chirp signal, $g(t) = rect\left(\frac{t}{T}\right) exp(j\pi K t^2)$. Determine the value of the chirp rate *K* by assuming a total duration *T*=100 microseconds.
3. Write the expression of the cross-correlation between the complex envelope of the received signal and the waveform $g(t)$.
4. Can you distinguish the two targets and tell their distance from the Radar? How close can the two targets get before it is impossible to resolve them?

Discuss whether with the addition of the second antenna you could also tell the angle under which the two targets are seen by the Radar. ***Note***: *this was the idea behind the Shuttle Radar Topography Mission in 2001.*

Solution

1. Write the time domain expression of the complex envelope of the signal received by the Radar.

$$s(t) = g(t - \tau_1) \cdot e^{-j2\pi f_0 \tau_1} + g(t - \tau_2) \cdot e^{-j2\pi f_0 \tau_2}$$

With $\tau_1 = 2\frac{R_1}{c}$ and $\tau_2 = 2\frac{R_2}{c}$

2. Assume that $g(t)$ is a chirp signal, $g(t) = rect\left(\frac{t}{T}\right) exp(j\pi K t^2)$. Determine the value of the chirp rate *K* by assuming a total duration *T*=100 microseconds.

The bandwidth of a chirp signal is B = KT, hence K = B/T = 10^{11} Hz/s

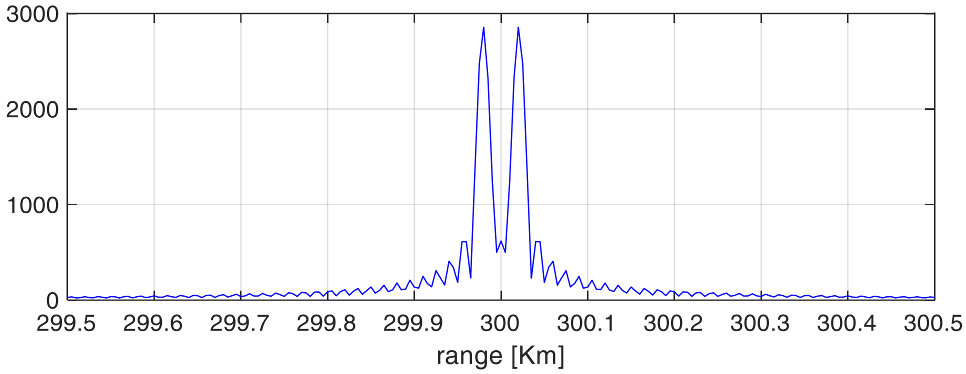
3. Write the expression of the cross-correlation between the complex envelope of the received signal and the waveform $g(t)$.

The autocorrelation of g(t) is a short pulse with duration 1/B, hence:

$$s(t) * g^*(-t) = R_g(t - \tau_1) \cdot e^{-j2\pi f_0 \tau_1} + R_g(t - \tau_2) \cdot e^{-j2\pi f_0 \tau_2}$$

4. Can you distinguish the two targets and tell their distance from the Radar? How close can the two targets get before it is impossible to resolve them?

   The cross-correlation ensures a temporal resolution dt=1/B=10-7 s. The interval between the two delays is $\tau_2 - \tau_1 = 2\frac{R_1 - R_2}{c} = 2.67 \cdot 10^{-7}$ seconds. Accordingly, the two targets appear as two well distinguished peaks. Their distance can be measured simply by taking the peak's positions.



Discuss whether with the addition of the second antenna you could also tell the angle under which the two targets are seen by the Radar. ***Note*****:** *this was the idea behind the Shuttle Radar Topography Mission in 2001.*

Of course, since having two (or more) antennas allows to measure the spatial frequency of the targets, which can be converted to their angular position via:

$$f_x = \frac{2}{\lambda} sin(\theta)$$

Operationally, the spatial frequency is obtained by comparing the phases of each peak in the signal acquired at the two antennas.