Electromagnetics and Signal Processing for Spaceborne Applications
June 28th, 2024

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SURNAME AND NAME ______________________________

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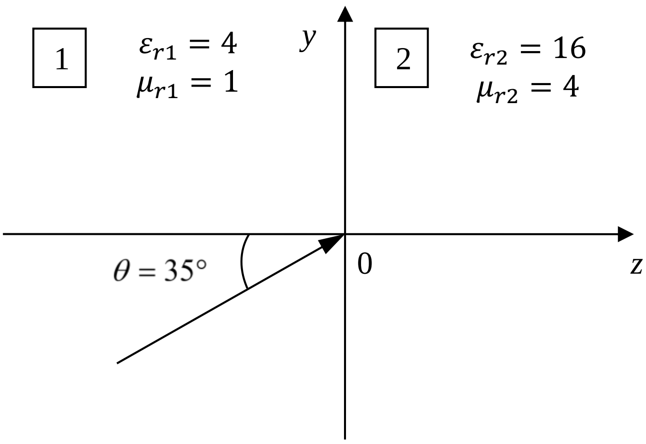
Problem 1

Consider the plane sinusoidal wave below, with f = 1 GHz and incident electric field given by:

$$\vec{E}_i(0,0,0) = j\mu_x \quad \text{(V/m)}$$



Calculate:

1) What is the wave polarization?
2) The total electric field in the first medium.
3) The power density in the second medium in the z direction.
4) What happens if the incidence angle becomes 0°?

Solution:

1) The wave is TE.
2) For the total electric field in medium 1, the reflection coefficient is needed. The refraction angle is:

$$\sqrt{\mu_{r1}\varepsilon_{r1}}\sin(\theta) = \sqrt{\mu_{r2}\varepsilon_{r2}}\sin(\theta_t) \rightarrow \theta_t = 8.24°$$

$$\eta_{TE}^{1} = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}\frac{1}{\cos(\theta)}} = 230.1\ \Omega$$

$$\eta_{TE}^{2} = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} \frac{1}{\cos(\theta_t)} = 190.5\ \Omega$$

$$\Gamma = \frac{\eta_{TE}^{2} - \eta_{TE}^{1}}{\eta_{TE}^{2} + \eta_{TE}^{1}} = -0.094$$

$$\vec{E}_t(x,y,x) = \vec{E}_i(x,y,x) + \vec{E}_r(x,y,x) = j\mu_x e^{-j\frac{2\pi}{\lambda_1}[\cos(\theta)z+\sin(\theta)y]} + \Gamma j\mu_x e^{-j\frac{2\pi}{\lambda_1}[-\cos(\theta)z+\sin(\theta)y]}\text{V/m}$$

where

$$\lambda_1 = \frac{c}{f\sqrt{\mu_{r1}\varepsilon_{r1}}} \approx 0.15\ \text{m}$$

3) The power density propagating in the second medium in the *z* direction is:

$$S_z^2 = S_z^1(1-|\Gamma|^2) = \frac{1}{2}\frac{|\vec{E}|^2}{\eta^1}\cos(\theta)\,(1-|\Gamma|^2) = 2.2\ \text{mW/m}^2$$

where

$$\eta^1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 188.5\ \Omega$$

4) For orthogonal incidence:

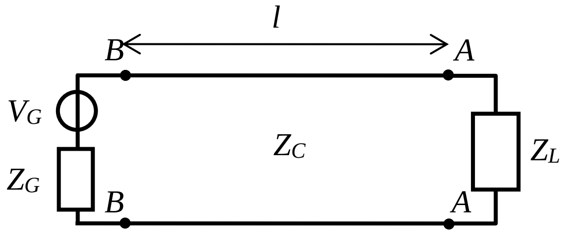
$$\eta^2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} = 188.5\ \Omega = \eta^1$$

Therefore, there is no reflected field in medium 1 and the whole power density crosses the interface.

Problem 2

A source with voltage V_G = 100 V and internal impedance Z_G = 100 Ω is connected to a load Z_L = 200 Ω by a transmission line with characteristic impedance Z_C = 50 Ω. The line length is l = 10.2 m, the frequency is f = 300 MHz.
For this circuit:

1) Calculate the power absorbed by Z_L.
2) Calculate the absolute value of the voltage at section, $|V_{AA}|$.
3) Calculate the trend in time of the voltage at section, $V_{AA}(t)$.



Solution

1) First, let us calculate the reflection coefficient at AA:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = 0.6$$

The reflection coefficient at BB is:

$$\Gamma_{BB} = \Gamma_L e^{-j2\beta l} = -0.485 - j0.353$$

where $\beta = 6.283\ \mathrm{rad/m}$

The equivalent input impedance is:

$$\mathrm{Z}_{BB} = 13.7 - j15.1\ \Omega$$

Let us now calculate:

$$\Gamma_G = -0.73 - j0.23$$

The power absorbed by the load Z_L is:

$$P_L = P_{AV}(1 - |\Gamma_G|^2) = 5.2\ \mathrm{W}$$

with

$$P_{AV} = \frac{|V_G|^2}{8Z_G} = 12.5\ \mathrm{W}$$

2) This value can be easily calculated by inverting the following equation:

$$P_L = \frac{1}{2}\frac{|V_{AA}|^2}{Z_L} \rightarrow\ |V_{AA}| = 45.67\ \mathrm{V}$$

3) To answer this question, it is necessary to know V_{AA} in amplitude and phase. Starting from section BB:

$$V_{BB} = V_G \frac{Z_{BB}}{Z_G + Z_{BB}} = 13.6\ - j11.5\ \mathrm{V}$$

Then, the progressive wave at BB is:

$$V_{BB}^{+} = \frac{V_{BB}}{1 + \Gamma_{BB}} = 28.4\ - j2.9\ \mathrm{V}$$

Finally:

$V_{AA} = V_{BB}^{+} e^{-j\beta l}(1 + \Gamma_L) = 9.7\ - j44.6\ \mathrm{V}$ → $|V_{AA}| = 45.67\ \mathrm{V}$ (obviously) and $\measuredangle V_{AA} = -1.36\ \mathrm{rad}$

Therefore:
$V_{AA}(t) = 45.67\cos(1.88 \times 10^9 t - 1.36)\ \text{V}$

Problem 3

An airborne Radar transmits the signal as $s_{tx}(t) = g(t)cos(2\pi f_0 t)$, with $g(t)$ a real-valued base-band signal with a two-sided bandwidth of B = 100 MHz and f_0 = 1 GHz.
The received signal is composed by the sum of two signals: one is the direct return from a target at a distance of 4000 m; the other arises from a reflection of the wave onto the wing, which determines a further delay $\Delta\tau$. The model of the received signal is:

$$s_{rx}(t) = s_{tx}(t-\tau) + s_{tx}(t-\tau-\Delta\tau)$$

With τ the delay from the target at 4000 m and $\Delta\tau = 10$ ns.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Find the expression of the Fourier Transform of the complex-envelope of the received signal and draw the graph of its magnitude.
3. Discuss if it is possible to cancel the signal associated with the reflection on the wing, and propose a procedure to do that.
4. A discrete-time signal s_n is obtained by sampling the real-valued received signal $s_{rx}(t)$ with sampling frequency $f_s = 450\ MHz$. Write the expressions of the Fourier Transform of s_n and draw a graph of its magnitude.
5. Describe a procedure to recover the complex envelope of the received signal from the sequence s_n.

Solution

1) Complex signals

$$s_{tx}(t) = g(t)exp(j2\pi f_0 t)$$

$$s_{rx}(t) = g(t-\tau)exp\big(j2\pi f_0(t-\tau)\big) + g(t-\tau-\Delta\tau)exp\big(j2\pi f_0(t-\tau-\Delta\tau)\big)$$

Complex envelope

$$s_{ce}(t) = s_{rx}(t)exp(-j2\pi f_0 t) = g(t-\tau)exp(-j2\pi f_0\tau) + g(t-\tau-\Delta\tau)exp\big(-j2\pi f_0(\tau+\Delta\tau)\big)$$

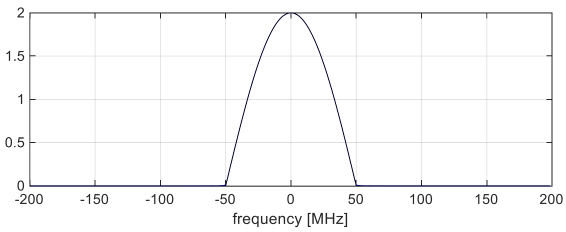
2) Fourier Transform

$$S_{ce}(f) = G(f)exp(-j2\pi(f+f_0)\tau) + G(f)exp\big(-j2\pi(f+f_0)(\tau+\Delta\tau)\big)$$
$$= G(f)exp(-j2\pi(f+f_0)\tau)\{1 + exp(-j2\pi(f+f_0)\Delta\tau)\}$$
$$= G(f)exp(-j2\pi(f+f_0)\tau)\{1 + exp(-j2\pi f\Delta\tau)exp(-j2\pi f_0\Delta\tau)\}$$

Noting that $2\pi f_0\Delta\tau$ is an integer multiple of 2π, we have $exp(-j2\pi f_0\Delta\tau) = 1$, hence:

$$|S_{ce}(f)| = |G(f)||1 + exp(-j2\pi f\Delta\tau)|$$

Which peaks at f=0 and assumes a null value at $f = \frac{1}{2\Delta\tau} = 50\ MHz$, as shown in the graph below



3) It is not possible to fully restore the signal due to the presence of zeroes at +- 50 MHz. However, large part of the signal can be recovered by applying an inverse filter like:

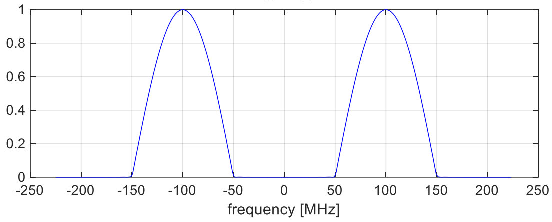
$$Hi(f) = \frac{1}{1 + 0.99exp(-j2\pi f\Delta\tau)}$$

Where the 0.99 factor is used to avoid infinite values (other solutions are possible)

4) The Fourier Transform of the real valued signal consists of two replicas of the complex envelope above at the frequency of + and – 1 GHz. When the signal is sampled, each replicas is shifted by an integer multiple of 450 MHz. The position of each replica is:

- +1 GHz (original signal) => +550 MHz => +100 MHz => -350 MHz
- -1 GHz (original signal) => -550 MHz => -100 MHz => +350 MHz

Hence, in the interval between + and –half the sampling frequency (225 MHz) we have only two replicas at frequency + and – 100 MHz, as in the graph below:



5) The two replicas are well separated, so it is possible to retrieve the complex envelope simply by shifting the signal by 100 MHz and applying a low-pass filter

$$s_{ce}(n) = \{s_n \cdot exp(-j2\pi f_1 nT)\} * h_n$$

Where f_1 = 100 MHz, T is the sampling time (inverse of the sampling frequency), and h_n a low-pass filter with bandwidth of 100 MHz.

Problem 4

An antenna array operating at the frequency $f_0 = 1$ GHz is used to transmit a beam with beamwidth $\Delta\psi = 10°$. Let $s(x_n)$ denote the signal transmitted by the antenna at position x_n.

1. Assuming that $s(x_n) = 1$, determine the required array length and number of antenna elements to transmit a **single** beam at boresight (that is, in the direction orthogonal to the array).
2. Determine how to modify the signal $s(x_n)$ to steer the beam at 30° w.r.t. boresight.
3. Starting from the result at point 2, determines the position of the secondary beam that arises by removing every second antenna (i.e.: use only antenna number 1,3,5,7, and so on)

Solution

1) the beamwidth (in radians) is $\Delta\psi = \frac{\lambda}{L}$ => L = 1.72 m. For the transmission of a single beam we require the array to be sampled at half a wavelength => 11 elements

2) We remember that the spatial frequency is related to beam pointing angle via $f_x = \frac{sin(\psi)}{\lambda}$. Accordingly, the beam must be steered at a spatial frequency $f_{x0} = \frac{sin(30°)}{\lambda} = \frac{1}{2\lambda} = 1.67$ m^{-1}, which is obtained by exciting the antennas with the signal

$$s(x_n) = exp(j2\pi f_{x0}x_n)$$

, with x_n the position of the n-th antenna.

3) array length doesn't change, but the spacing between neighboring antennas is now equal to one wavelength. This determines replicas in the spatial frequency domain at positions $f_x = f_{x0} + \frac{k}{\lambda}$, with k any integer. The corresponding angles are obtained as:

$$\psi = asin(\lambda f_x) = asin(\lambda f_{x0} + k) = asin\left(\frac{1}{2} + k\right)$$

The only physical solution is found for k=-1, corresponding to $\psi = -30°$