Electromagnetics and Signal Processing for Spaceborne Applications
August 29th, 2023

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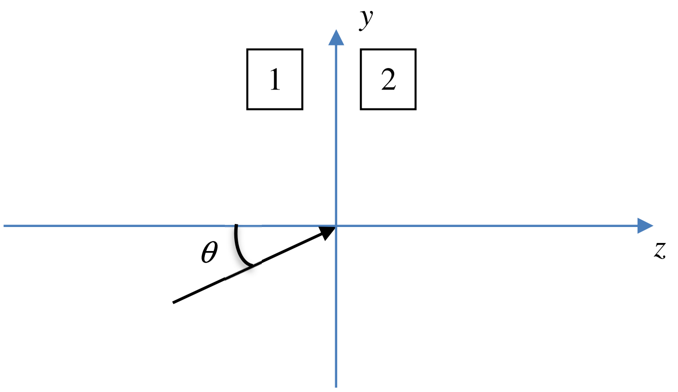
Problem 1

A plane sinusoidal EM wave (f = 9 GHz) propagates from free space into a medium with electric permittivity $\varepsilon_{r2} = 4$ (assume $\mu_r = 1$ for both media). The expression of the incident electric field is:

$$\vec{E}_i(z,y) = \left(\frac{\sqrt{2}}{2}\vec{\mu}_y - \frac{\sqrt{2}}{2}\vec{\mu}_z + j\vec{\mu}_x\right) e^{-j\beta\frac{\sqrt{2}}{2}z} e^{-j\beta\frac{\sqrt{2}}{2}y} \quad \text{V/m}$$

1) Determine the incidence angle θ.
2) Calculate the polarization of the incident field.
3) Write the expression of the reflected field $\vec{E}_r$.
4) OPTIONAL: calculate the power density of the reflected field $\vec{E}_r$.



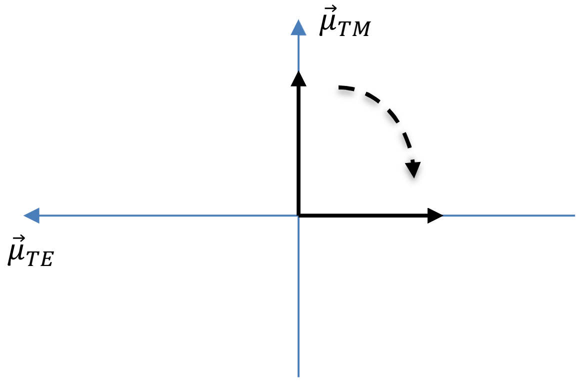
Solution

1) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta \sin(\theta) = \beta\sqrt{2}/2 \rightarrow \sin(\theta) = \sqrt{2}/2 \rightarrow \theta = 45°$

2) The wave has two components: a TE one, along x, and a TM one, given by the combination of the two fields along y and z. The absolute value of the components is 1 V/m (TM) and 1 V/m (TE), and the differential phase shift is $-\pi/2$. The wave is therefore circularly polarized. To understand the rotation direction, it is worth writing the field in the time domain for (z = 0, y = 0):

$\vec{E}_i(t,0,0) = \cos(\omega t)\vec{\mu}_{TM} + \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE}$ V/m

Looking from the back of the incident wave, in the direction of propagation:



Setting $t = 0$ → $\vec{E}_i(t,0,0) = \vec{\mu}_{TM}$ V/m

Setting $\omega t = \pi/2$ → $\vec{E}_i(t,0,0) = -\vec{\mu}_{TE}$ V/m

As a result, the wave has RHCP.

3) To find the reflected field, it is first necessary to calculate the transmission angle is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 20.7°$$

The TE reflection coefficient is:

$$\Gamma_{TE} = \frac{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) - \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})}{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) + \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})} = -0.4514$$

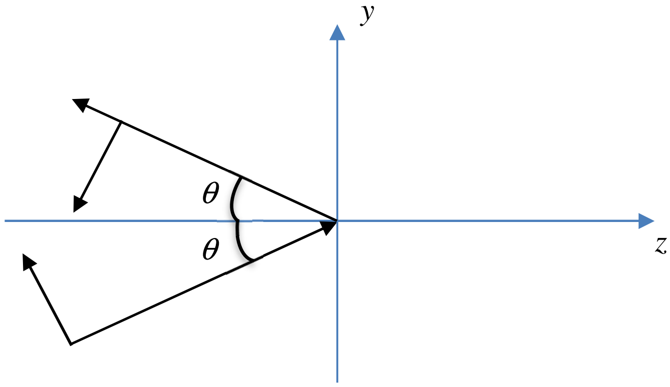
The TM reflection coefficient is:

$$\Gamma_{TM} = \frac{\eta_0\cos(\theta_2)/\sqrt{\varepsilon_{r2}} - \eta_0\cos(\theta)/\sqrt{\varepsilon_{r1}}}{\eta_0\cos(\theta_2)/\sqrt{\varepsilon_{r2}} + \eta_0\cos(\theta)/\sqrt{\varepsilon_{r1}}} = -0.2038$$

The reflected field is therefore given by:

$$\vec{E}_r(z,y) = \left(\frac{\sqrt{2}}{2}\Gamma_{TM}\vec{\mu}_y - |\Gamma_{TM}|\frac{\sqrt{2}}{2}\vec{\mu}_z + j\Gamma_{TE}\vec{\mu}_x\right)e^{j\beta\frac{\sqrt{2}}{2}z}e^{-j\beta\frac{\sqrt{2}}{2}y}\ \text{V/m}$$

It is worth pointing out that the reflection of the TM component is more complex. In fact, the TM reflection coefficients needs to be first applied to the *y* component of the incident field, from which the *z* component can be then derived by taking into account the reflection angle (obviously 45°) and the direction of the *z* component itself (see sketch below): as Γ_{TM} is negative, the reflected *y* component points at -*y*, and so the *z* component will have to be negative; this is possible only by applying the absolute value of Γ_{TM} to the incident *z* component.



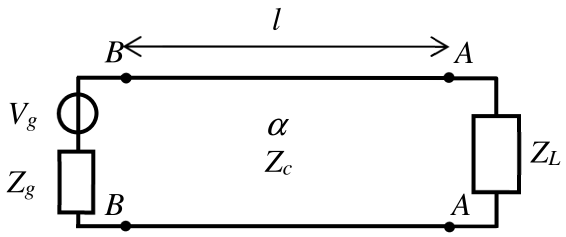
4) The power density of the reflected field is:

$$S_r = \frac{1}{2}\frac{\left|\vec{E}_r\right|^2}{\left(\eta_0/\sqrt{\varepsilon_{r1}}\right)} = 3.25 \times 10^{-4}\ \mathrm{W/m^2}$$

Problem 2

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a load Z_L = j100 Ω by a transmission line with characteristic impedance Z_c = 50 Ω. The line length is l = 20 m, the frequency is f = 300 MHz and the attenuation constant is α = 30 dB/km.
For this circuit:

1) What kind of load is Z_L?
2) Calculate the power absorbed by the load.
3) Calculate the power absorbed by the transmission line.



Solution

1) The load, being imaginary (positive), corresponds to an inductor.

2) As the load is imaginary, no power will be absorbed by Z_L.

3) The circuit is matched at the generator section, but not at the load one: there is only one reflection at the generator section, which allows to use the time domain approach. The power crossing section BB is:
$P_{BB}^{+} = P_{AV} = 0.25\text{ W}$
The power reaching section AA is:
$P_{AA}^{+} = P_{BB}^{+} e^{-2\alpha l} = 0.2177\text{ W}$
with α = 30/(8.686·1000) = 0.0035 Np/m.
The absolute value of the reflected field is obviously one; the reflected power is:
$P_{AA}^{-} = P_{AA}^{+} |\Gamma|^2 = P_{AA}^{+} = 0.2177\text{ W}$
The power reaching back section BB is:
$P_{BB}^{-} = P_{AA}^{-} e^{-2\alpha l} = 0.1896\text{ W}$
The power actually absorbed by the elements beyond section BB is:
$P_{BB} = P_{BB}^{+} - P_{BB}^{-} = 0.0604\text{ W}$
This is also the power absorbed by the line, as the load cannot absorb any power.

Problem 3

A sensor transmits a monochromatic signal at the frequency $f_0 = 100$ GHz over a temporal duration $T_o = 10$ milliseconds. The signal impinges on two moving targets, which modulate the frequency due to the Doppler effect. The received signal is to be modeled as:

$$s_{Rx} = rect\left(\frac{t}{T_o}\right)\{exp(j2\pi(f_0 + df_1)t) + exp(j2\pi(f_0 + df_2)t)\}$$

1. Write the expressions of the complex envelope of the received signal
2. Write the expression of the Fourier Transform of the received signal and draw the graph of its magnitude.
3. Discuss the frequency resolution allowed by the signal and state the condition upon which it is possible to distinguish the two sinusoids.
4. Assuming that target velocity cannot exceed 10 m/s in magnitude, discuss how to set the sampling time to correctly sample the complex envelope of the received signal at point 1. *Note: the frequency shift is related to target velocity via* $df = \frac{2v}{c} f_0$, *with v the target's velocity and c the speed of light.*
5. What is the velocity resolution allowed by the signal?

Solution

1) The complex envelope is obtained from the complex signal by multiplying by $exp(j2\pi f_0 t)$, hence:

$$s_{ce}(t) = rect\left(\frac{t}{T_o}\right)\{exp(j2\pi df_1 t) + exp(j2\pi df_2 t)\}$$

2) The Fourier transform is the sum of two cardinal sines centered about df_1 and df_2

$$s_{ce}(f) = T_o\{sinc((f - df_1)T_o) + sinc((f - df_2)T_o)\}$$

3) the frequency resolution is $\frac{1}{T_o}$, hence the two sinusoids are resolved (no significant interference between the two peaks) as long as $|df_1 - df_2| > \frac{1}{T_o}$

4) the maximum frequency shift is *via* $df_{max} = \frac{2v_{max}}{c} f_0$, hence the signal bandwidth is $B = \frac{4v_{max}}{c} f_0 = 13.3\, KHz$. It follows that the signal should be sampled at a rate $T < \frac{c}{4f_0 v_{max}}$. A more accurate statement could be achieved by considering the spread due to the observation time, i.e.:

$B = \frac{4v_{max}}{c} f_0 + \frac{1}{T_o}$. However, both solutions are acceptable for this exercise.

5) Just substitute $df_1 = \frac{2v_1}{c} f_0$ and $df_2 = \frac{2v_2}{c} f_0$ in the condition at point 3. The result is that the two sinusoids can be resolved as long as $|v_1 - v_2| > \frac{c}{2T_o f_0} = 0.15\, m/s$

Problem 4

Two signals $v(t)$ and $u(t)$ are received at two antennas with different delays:

$$s_1(t) = v(t - \tau_{v1}) + u(t - \tau_{u1})$$
$$s_2(t) = v(t - \tau_{v2}) + u(t - \tau_{u2})$$

where:

- The Fourier Transform of $v(t)$ and $u(t)$ can be modeled as being constant in the frequency interval $\left(-\frac{B}{2}, \frac{B}{2}\right)$, with B = 1 Hz.
- $\tau_{v1} = 0$ s, $\tau_{u1} = 1$ s
- $\tau_{v2} = 0.5$ s, $\tau_{u2} = 0.5$ s,

1. Write the expressions of the Fourier Transforms of $s_1(t)$ and $s_2(t)$ and draw the graphs of their magnitudes (two graphs are required).
2. Consider the new signal $x(t) = s_1(t + \tau_{v1}) - s_2(t + \tau_{v2})$. Write the expressions of the Fourier Transform of $x(t)$ and draw the graph of its magnitude.
3. Propose a procedure to recover the signal $u(t)$ based on the knowledge of $x(t)$. Is it possible to recover $u(t)$ with no error?
4. Write a short pseudo-code to derive $x(t)$ from $s_1(t)$ and $s_2(t)$ and $u(t)$ from $x(t)$.

Solution

1)

$$s_1(f) = v(f) + u(f)exp(-j2\pi f)$$
$$s_2(f) = v(f)exp(-j\pi f) + v(f)exp(-j\pi f)$$

Assuming $v(f) = u(f) = rect(f)$ we have:

$$|s_1(f)|^2 = 2rect(f)[1 + cos(2\pi f)]$$
$$|s_2(f)|^2 = 4rect(f)$$

So $s_1(f)$ exhibits a peak in f =0 and falls to 0 for $f = \pm\frac{1}{2}$, whereas $s_2(f)$ is constant in magnitude.

2) $x(t) = s_1(t + \tau_{v1}) - s_2(t + \tau_{v2})$:

$$s_1(t + \tau_{v1}) = v(t + \tau_{v1} - \tau_{v1}) + u(t + \tau_{v1} - \tau_{u1})$$
$$s_2(t) = v(t + \tau_{v2} - \tau_{v2}) + u(t + \tau_{v2} - \tau_{u2})$$

Hence: $x(t) = v(t) + u(t - 1) - v(t) - u(t) = u(t - 1) - u(t)$.

The Fourier transform is:

$$x(f) = u(f)\{exp(-j2\pi f) - 1\}.$$

So that $|x(f)|$ is 0 for f =0 and reaches 2 for $= \pm\frac{1}{2}$.

3) since there is a 0 in $x(f)$, it is clear that $u(f)$ cannot be perfectly recovered. For all the other values of frequency one can retrieve $u(f)$ as

$$u_{recovered}(f) = \frac{x(f)}{exp(-j2\pi f) - 0.99}$$

Where we write 0.99 instead of 1 to avoid the presence of a null value at the denominator (of course, other numbers realizing the same condition would do)

4) % x(t) from s1 and s2

x(t) = conv(s1,sinc(t-tv1)/T) - conv(s2,sinc(t-tv2)/T)

where T is the sampling time. The convolution with a shifted sinc is implemented to translate the signals by tv1 and tv2 seconds.

```
% recover u from x

X = fft(x); % Fourier Transform of x

Hi = 1./(exp(-1i*2*pi*f)-0.99);

U_rec = X.*Hi;

u = ifft(U)
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