FUNDAMENTALS OF ELECTROMAGNETIC FIELDS – February 20th, 2019

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Name and surname	
Identification number	
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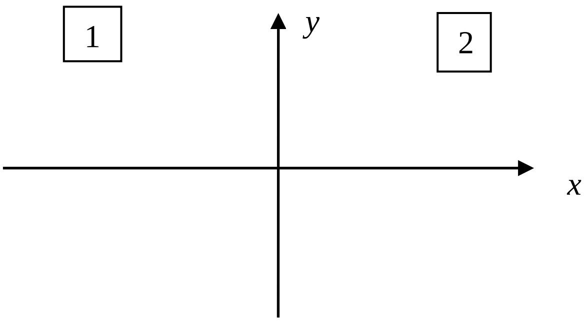
<u>Problem 1</u> - Making reference to the figure below depicting two perfect dielectrics, the first medium is characterized by ε_{r1} = 4 and μ_{r1} = 4, while the second material is characterized by ε_{r2} = 6 and μ_{r2} = 1. A uniform current density $\vec{J}_S = -5\vec{\mu}_z$ A/m flows along the *zy* plane, and a uniform charge density σ_S = -10^{-12} C/m^2 is embedded in such a plane as well. The electric field and the magnetic field in medium 1 are $\vec{E}_1 = 2\vec{\mu}_x$ V/m and $\vec{H}_1 = -4\vec{\mu}_y$ A/m, respectively. Calculate the electric field and the magnetic field in the second medium.



Solution

The electric field in the second medium can be calculated by resorting to the boundary condition for components orthogonal to the boundary between the two media. Such a condition derives from the Gauss' theorem:

$$\int_S \vec{D} \cdot d\vec{S} = \int_V \rho \, dV$$

For this case, using the proper volume (a cylinder crossing both media), the equation above becomes:

$$D_{1x} - D_{2x} = |\sigma_S| \rightarrow E_{2x} = \frac{\varepsilon_{r1}\varepsilon_0 E_{1x} - |\sigma_S|}{\varepsilon_{r2}\varepsilon_0} = 1.3 \text{ V/m}$$

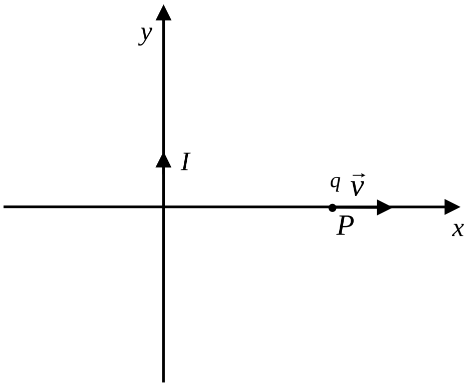
The magnetic field can be calculated by using the Ampère's law:

$$\oint_l \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$$

For this case, using a simply line for the circulation, the equation above becomes:

$$H_{1y} - H_{2y} = \left|\vec{J}_S\right| \rightarrow H_{2y} = H_{1y} - \left|\vec{J}_S\right| = -9 \text{ A/m}$$

Problem 2 - A constant current I = 0.05 A flows along the y axis. An electron with charge q = -1.6·10^{-19} C is in P(2,0), travelling with velocity $\vec{v} = c\,\vec{\mu}_x$ m/s, being c the speed of light. In addition, there is an electric field $\vec{E} = -3\vec{\mu}_y$ V/m, which is uniform across the xy plane. The magnetic permeability across the plane is μ_r = 2. Calculate the force which the electron in P is subject to (amplitude and direction).



Solution

The electron will be subject to two forces: one due to the electric field (Coulomb's force) and the other due to the magnetic field (Lorentz's force).

The first one can be easily calculated as:

$$\vec{F}_C = q\vec{E} = 4.8\cdot 10^{-19}\,\vec{\mu}_y \text{ N}$$

The second force depends on the magnetic field generated by the wire, whose value in P is:

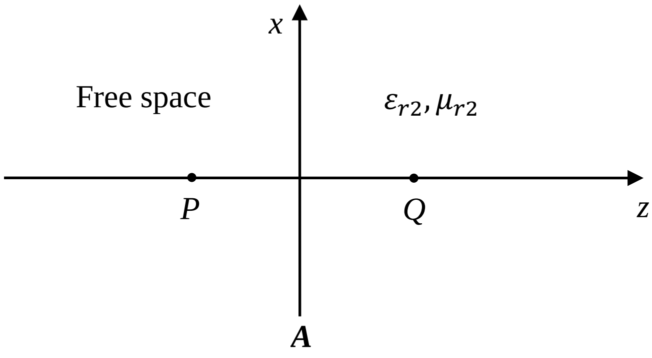
$$\vec{H}(P) = -\frac{I}{2\pi x_P}\vec{\mu}_z \text{ A/m}$$

The Lorentz's force is:

$$\vec{F}(P) = q\vec{v}\times\mu_r\mu_0\vec{H}(P) = q\mu_r\mu_0 c\frac{I}{2\pi x_P}\vec{\mu}_y = -4.8\cdot 10^{-19}\,\vec{\mu}_y \text{ N}$$

Therefore, the total force acting on the electron is zero.

Problem 3 - A uniform plane wave (frequency f = 200 MHz) propagates from free space into a medium with the following electromagnetic features: $\boldsymbol{\varepsilon_{r2} = 4, \mu_{r2} = 4}$. Calculate the positions of $P(0,0,-\lambda_1)$ and $Q(0,0,\lambda_2)$, being λ_1 and λ_2 the wavelengths in the first and second medium, respectively. In addition, calculate the absolute value of the electric field in $P(0,0,-\lambda_1)$ knowing that the power density carried by the wave in $Q(0,0,\lambda_2)$ is $S_2(Q)$ = 10 W/m^2.



Solution

The intrinsic impedance for the two media is the same. In fact:

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} \approx 377\ \Omega$$

As a result, the reflection coefficient is zero:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0$$

Therefore, there is no reflected wave and all the power density will go through the discontinuity. The wavelength in the two media is different though:

$$\lambda_1 = \frac{c}{f} = \lambda_0 = 1.5\ \text{m}$$

$$\lambda_2 = \frac{2\pi}{\beta} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}} = 0.375\ \text{m}$$

Therefore $P(0,0,-1.5\ \text{m})$ and $Q(0,0,0.375\ \text{m})$

As there is no attenuation and no reflection the power density in P and Q are equal:

$$S_1(P) = \frac{1}{2}\frac{\left|\vec{E}_1(P)\right|^2}{\eta_1} = S_2(Q)\ \text{W/m}^2$$

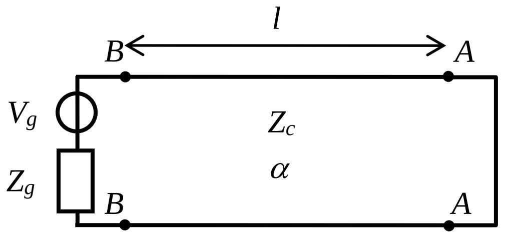
Therefore:

$$\left|\vec{E}_1(P)\right| = \sqrt{2\eta_1 S_2(Q)} = 86.8\ \text{V/m}$$

Problem 4 - A transmitter with voltage V_g = 10 V (sinusoidal regime) and internal impedance Z_g = 50 Ω is connected to a short circuit by a transmission line with characteristic impedance Z_C = 50 Ω and attenuation coefficient α = 30 dB/km . The line length is l = 20.2 m and the frequency is f = 600 MHz.

Calculate:

1) The power absorbed by the short circuit.
2) The voltage at section AA.
3) The power absorbed by the line.



Solution

1) As the load is short circuit (Z_L = 0 Ω), no power will be absorbed but it will all be reflected back to the generator. In fact:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -1$$

2) As the load is a short circuit, the total voltage at section AA is zero.

3) In order to find the power absorbed by the line, we first need to calculate the power going beyond section BB. For this, we need to calculate the reflection coefficient at the input section as:

$$\Gamma_{BB} = \Gamma_L e^{-2\alpha l} e^{-j2\beta l} = -0.269 - j0.827$$

In principle, we would need Z_{BB} to calculate Γ_g, but as $Z_C = Z_g$, therefore $\Gamma_g = \Gamma_{BB}$:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_{BB} - Z_C}{Z_{BB} + Z_C} = \Gamma_{BB}$$

As a result, the power given by the generator to the whole circuit beyond section BB is:

$$P_{BB} = P_{AV}(1 - |\Gamma_g|^2)$$

$$P_{AV} = \frac{|V_g|^2}{8\mathrm{Re}\{Z_g\}} = 0.25 \text{ W}$$

$$P_{BB} = 0.06 \text{ W}$$

As the short circuit cannot absorb any power, all P_{BB} is absorbed by the line.