EM wave propagation for space-borne systems – Prof. L. Luini,
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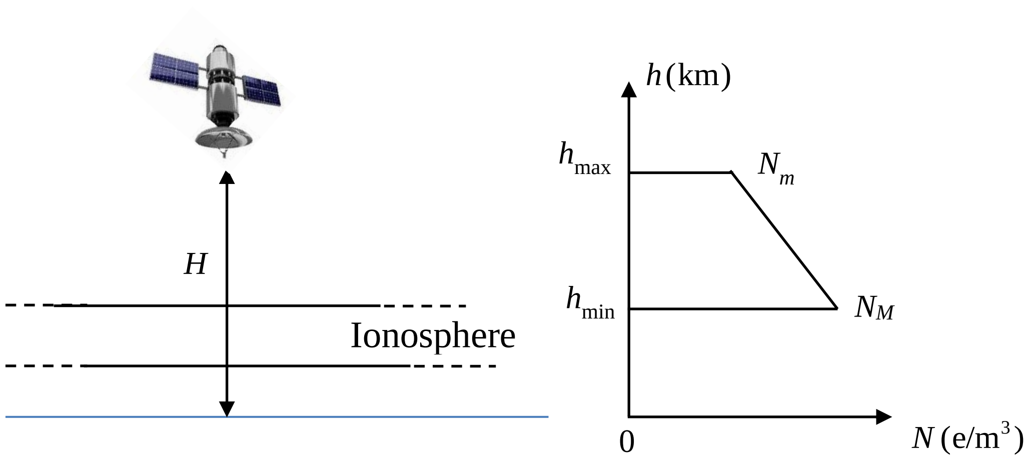
Problem 1

The figure below shows a monostatic radar onboard a LEO satellite, with height above the ground H = 700 km (left side). The radar extracts information on the ionospheric electron content by using concurrent pulses emitted at two carrier frequencies, f_1 and f_2. Making reference to the electron content profile on the right side ($N_m = 10^{12}$ e/m^3, $N_M = 4\times10^{12}$ e/m^3):

1) Determine the maximum frequency $f_{\max}$ for the associated pulse to be reflected by the ionosphere at $h_{\max}$. Set $f_1 = 0.9f_{\max}$.
2) Determine the minimum frequency $f_{\min}$ for the associated pulse to be reflected by the ground. Set $f_2 = 2f_{\min}$.
3) Determine $h_{\max}$ knowing that the one-way travel time of the pulse at f_1 is t_1 = 1 ms.
4) Determine $h_{\min}$ knowing also that the one-way travel time of the pulse at f_2 is t_2 = 2.411072 ms.

Assumption: neglect tropospheric effects.



Solution

1) To determine the carrier frequency f_1 for the pulse to be reflected at $h_{\max}$, the following expression can be used:

$$\cos(90) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f_\mathrm{max}}\right)^2} \quad \Rightarrow \quad f_\mathrm{max} = 9\sqrt{N_\mathrm{m}} = 9\ \mathrm{MHz} \quad \Rightarrow \quad f_1 = 0.9 f_\mathrm{max} = 8.1\ \mathrm{MHz}$$

2) To determine the carrier frequency f_2 for the pulse to be reflected at the ground, i.e. crossing the bottom part of the ionosphere at the highest electron content value (at h_min), the following expression can be used again:

$$\cos(90) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{M}}}{f_\mathrm{min}}\right)^2} \quad \Rightarrow \quad f_\mathrm{min} = 9\sqrt{N_\mathrm{M}} = 18\ \mathrm{MHz} \quad \Rightarrow \quad f_2 = 2 f_\mathrm{min} = 36\ \mathrm{MHz}$$

3) The pulse at f_1 travels through free space, so its single-way travel time is simply:

$$t_1 = \frac{H - h_\mathrm{max}}{c} = 1\ \mathrm{ms} \quad \Rightarrow \quad h_\mathrm{max} = 400\ \mathrm{km}$$

4) The pulse at f_2 travels partially through free space, and partially through the ionosphere. Its single-way travel time is therefore:

$$t_2 = \frac{H}{c} + \frac{40.3}{c f_2^2} TEC = 2.411072\ \mathrm{ms}$$

The total electron content is given by:

$$TEC = (h_\mathrm{max} - h_\mathrm{min}) N_\mathrm{m} + \frac{(h_\mathrm{max} - h_\mathrm{min})(N_\mathrm{M} - N_\mathrm{m})}{2}$$

Using both equations and solving for the only unknown h_{min}:

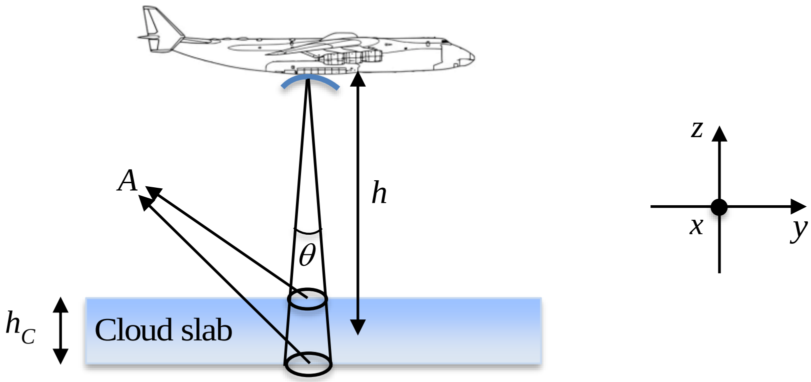
$$h_\mathrm{min} = 100\ \mathrm{km}$$

Problem 2

Making reference to the figure below, a pulsed radar onboard an airplane, operating with carrier frequency f = 80 GHz and pointed zenithally, is used to measure the cloud liquid water content. The wave has RHCP. The beam illuminates a volume V filled with rain at distance h = 5 km. The area of the volume is A = 200 m^2 and its height is h_C = 500 m. The cloud droplets have density N = 100000 drops/m^3 and they all have the same backscatter section, i.e. σ = 2 μm^2.

1) Calculate the power received by the radar P_R, when the transmit power is P_T = 500 W.
2) What is the polarization of the wave received by the radar?

Consider the following: radar antenna gain G = 50 dB; assume negligible cloud attenuation.



Solution

1) First, let us calculate the power density reaching the rain volume:

$$S = \frac{P_T}{4\pi h^2} G f = 0.1592 \text{ W/m}^2$$

where $G = 10^5$, f = 1 (radar pointing to the volume).

The power reirradiated by a single rain drop is (with gain = 1 according to the definition of backscatter section), is:

$$P_d = S\sigma$$

Considering all the drops in the volume and under the assumption of Wide Sense Stationary Uncorrelated Scatterers (based on which we can sum the power reirradiated by the single drops), we obtain:

$$P_t = NAh_c S\sigma = 0.0032 \text{ W}$$

Finally, the power received by the radar is:

$$P_R = \frac{P_t}{4\pi h^2} A_E = \frac{P_t}{4\pi h^2} G \frac{\lambda^2}{4\pi} = 1.133 \text{ pW}$$

2) The droplets are spherical, so they do not induce a change in the wave polarization. However, the reflection of the wave (backscatter) changes the wave polarization from RH to LH.

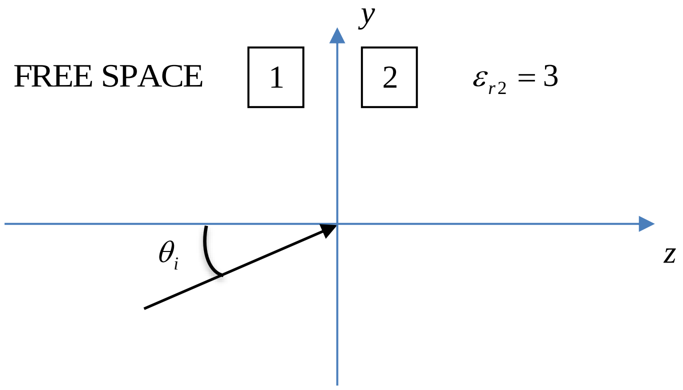
Problem 3

A plane sinusoidal EM wave propagates from free space into a medium with electric permittivity $\varepsilon_{r2} = 3$ ($\mu_{r2} = 1$), with incidence angle θ_i. The expression for the electric field in the first medium is:

$$\vec{E}_i(z,y) = \left[\left(\frac{1}{2}\vec{\mu}_y - \frac{\sqrt{3}}{2}\vec{\mu}_z\right) + j2\vec{\mu}_x\right] e^{-j104.72z} e^{-j181.38y} \text{ V/m}$$

For this wave:

1) Determine θ_i.
2) Determine the frequency f.
3) Determine the polarization of the incident and reflected EM wave.



Solution:

1) The incident angle can be obtained from the TM component of the field. For example:

$$\left|\vec{E}_{TM}\right| = \sqrt{\left(E_{TM}^y\right)^2 + \left(E_{TM}^z\right)^2} = 1$$

$$E_{TM}^y = \cos\theta_i \left|\vec{E}_{TM}\right| = \frac{1}{2} \rightarrow \theta_i = 60°$$

2) The frequency of the incident EM wave can be derived from the phase constant. For example:

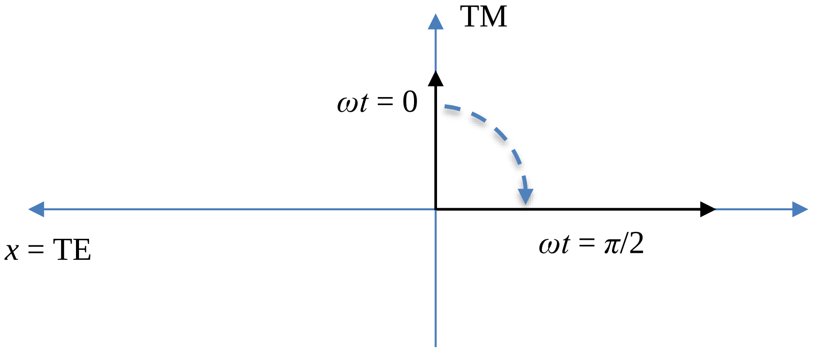
$$\beta_z = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}}\cos\theta \quad \Rightarrow \quad f = \frac{c\beta_z}{2\pi\cos\theta\sqrt{\varepsilon_{r1}}} = 10 \text{ GHz}$$

3) The polarization of the incident wave is RHEP (right-hand elliptical polarization) because the two TE and TM components have different amplitudes and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re}\left\{\left[\left(\frac{1}{2}\vec{\mu}_y - \frac{\sqrt{3}}{2}\vec{\mu}_z\right) + j2\vec{\mu}_x\right] e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_{TM} + 2\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} \text{ V/m}$$

Thus, making reference to the figure below that shows the reference system as seen from behind the wave, for $t = 0 \rightarrow \vec{E}(0,0)\Big|_{\omega t=0} = \vec{\mu}_{TM}$ V/m

Afterwards, for $\omega t = \pi/2 \rightarrow \vec{E}(0,0)\Big|_{\omega t=\pi/2} = -2\vec{\mu}_{TE}$ V/m



The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. As there is a TM component, it is worth checking the Brewster angle θ_B:

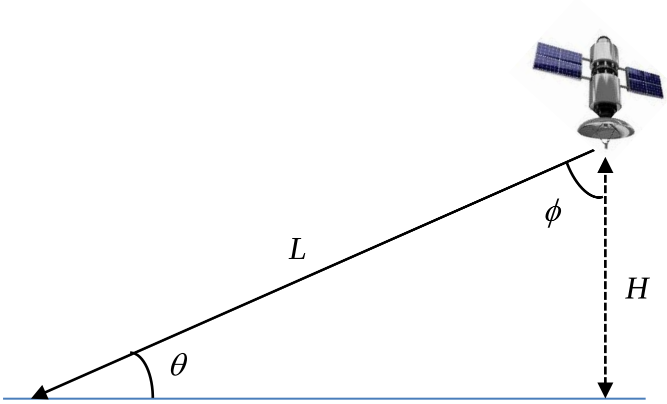
$$\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 60°$$

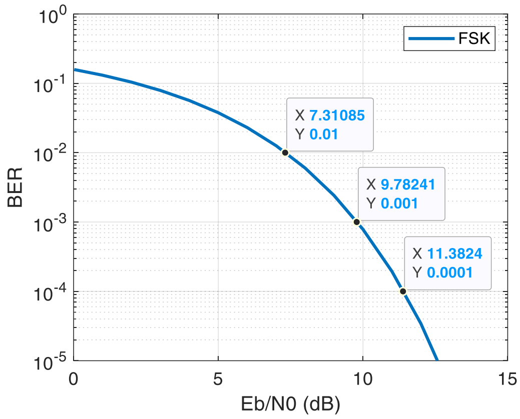
As $\theta_i = \theta_B$, the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

Problem 4

Consider a link from a LEO satellite to a ground station, operating at f = 26 GHz. For this system, determine the link availability, considering that the link elevation angle is $\theta = 30°$ and that the BER needs to be lower than 10^{-4} using the FSK modulation (see graph on the right). The CCDF of the zenithal tropospheric attenuation is given by:

$$P(A_T^{dB}) = 100e^{-1.1511 A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in } \%)$$





Additional assumptions and data:

- use the simplified geometry depicted above (left side)
- the specific attenuation of the troposphere is homogeneous vertically and horizontally
- ground station tracking the satellite optimally
- power transmitted by the satellite P_T = 160 W
- disregard the cosmic background contribution
- no additional losses at the transmitter and at the receiver
- LEO satellite pointing always to the centre of the Earth
- radiation patter of the LEO satellite antenna (circular symmetry): $f_T = [\cos(\phi)]^2$
- mean radiating temperature T_{mr} = 288 K
- gain of the antenna on board the satellite G_S = 13 dB
- gain of the antenna at the ground station G_G = 40 dB
- altitude of the LEO satellite: H = 700 km
- bandwidth of the receiver: B = 2 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution

For the BER to be lower than 10^{-4}, the target $E_b/N0$ (wih corresponds to the SNR in this case) is 11.38 dB = 13.74. The SNR is given by:

$$SNR = \frac{P_T G_S f_T (\lambda/4\pi L)^2 G_G f_R A}{k[T_R + T_{mr}(1 - A)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = [\cos(90°-\theta)]^2 = 0.25$, $f_R = 1$, A is the slant path tropospheric attenuation (in linear scale), and $L = H/\sin(\theta) = 1400$ km. Setting the SNR = 11.38 dB = 13.74, and solving for A:

$A = 0.063 \rightarrow A^{dB}$ = 12 dB

The attenuation scaled to the zenith is:

$A_T^{dB} = A^{dB}\sin(\theta)$ = 6 dB.

Using this value in the CCDF expression, we obtain $P\left(A_T^{dB}\right) = 0.1\%$, which is the link outage probability. The link availability is therefore 99.9%.