EM wave propagation for space-borne systems – Prof. L. Luini,
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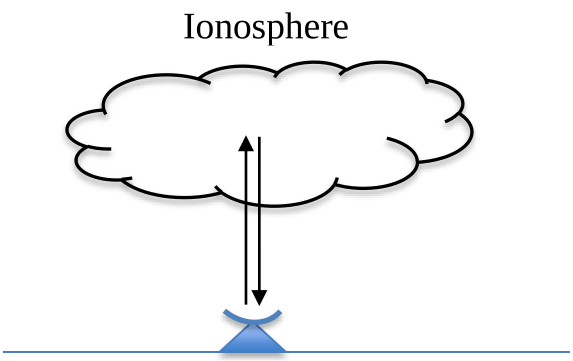
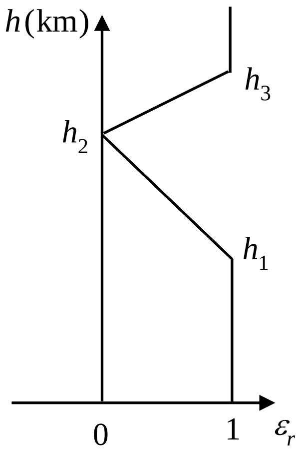
Problem 1

Making reference to the figure below, a monostatic ground-based pulsed radar system, operating at f = 18 MHz and pointing zenithally, aims at monitoring the ionosphere, i.e. the peak electron content and its value. The trend of ε_r in the ionosphere is depicted in the figure below on the right side (h_1 = 100 km). Knowing that the radar pulse round trip time is τ = 2.33169 ms, determine the value and position of peak electron content N_{max}.

Assumption: neglect tropospheric effects.





Solution

The peak electron content value can be obtained from the following expression:

$$\cos(\theta) = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2} = \sqrt{\varepsilon_r}$$

Setting θ = 90° and inverting the expression → N_{max} = 4×10^{12} e/m^3. N_{max} obviously corresponds to the lowest value of ε_r; also, being ε_r = 0 (for zenithal pointing) indicates that the wave is totally reflected exactly where N_{max} lies, i.e. at h_2.

The time required for the radar pulse to reach h_2 from the ground is $t = \tau/2 = 1.165845$ ms. Such time can be expressed as:

$$t = \frac{h_2}{c} + \frac{40.3}{cf^2}\mathrm{TEC} = \frac{h_2}{c} + \frac{40.3}{cf^2}\frac{(h_2 - h_1)N_{\max}}{2}$$

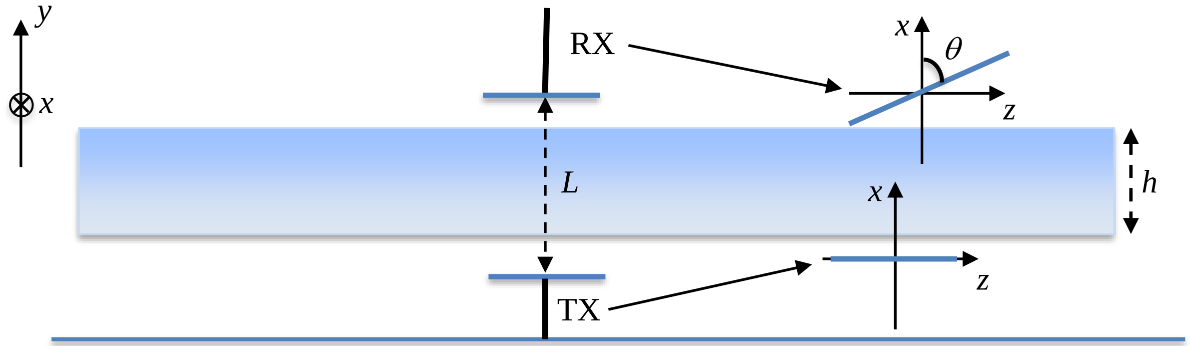
Solving for the only unknown, i.e. $h_2 \rightarrow h_2 = 300$ km.

Problem 2

An Earth-space link, with path length L = 400 km and operating at f = 90 GHz, crosses a homogeneous cloud with thickness h = 3 km. The transmitter (TX) uses a linear horizontal antenna, while the receiver (RX) linear horizontal antenna is tilted by an angle of θ = 60° (see sketch below) due to problems with the satellite attitude control. For this link:

1) Determine the polarization of the wave in front of the receiver RX (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
2) Calculate the power density in front of the RX antenna.
3) Calculate the power received by RX.

Assume: EIRP = 36 dBW; effective area of the RX antenna, A_{RX} = 2 m^2; specific attenuation due to clouds, α_c = 2 dB/km; RX pointing at the Earth center and TX pointing at zenith; no additional losses at TX and RX.



Solution

1) As clouds consist of small spherical droplets, which are isotropic, the polarization transmitted by TX is unaffected by the presence of clouds.

2) The power density in front of the RX antenna is given by:

$$S_{RX} = \frac{EIRP}{4\pi L^2} f_T L_{TX} A_C \approx 0.5 \text{ nW/m}^2$$

where, given the assumptions: $f_T = L_{TX} = 1$. A_C is the cloud attenuation in linear scale. Such value in dB is obtained as: $A = \alpha_C h = 6$ dB → $A_C = 0.2512$.

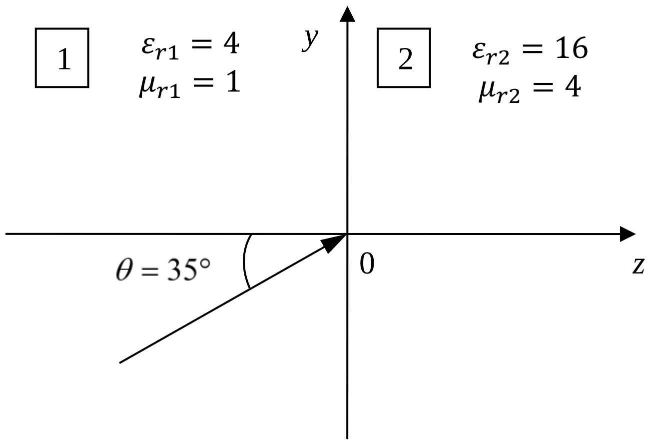
3) The power received by RX needs to account for the RX antenna tilt:

$$P_{RX} = S_{RX} A_{RX} f_R L_{RX} [\cos(90 - \theta)]^2 \approx 0.75 \text{ nW}$$

Problem 3

Consider the plane sinusoidal wave below, with f = 1 GHz and incident electric field given by:

$$\vec{E}_i(0,0,0) = j\mu_x \quad \text{(V/m)}$$



Calculate:

1) The total electric field in the first medium.
2) The power density in the second medium in the *z* direction.
3) What happens if the incidence angle becomes 0°?

Solution:

1) The wave is TE. For the total electric field in medium 1, the reflection coefficient is needed. The refraction angle is:

$\sqrt{\mu_{r1}\varepsilon_{r1}}\sin(\theta) = \sqrt{\mu_{r2}\varepsilon_{r2}}\sin(\theta_t) \rightarrow \theta_t = 8.24°$

$$\eta_{TE}^1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}\frac{1}{\cos(\theta)} = 230.1\ \Omega$$

$$\eta_{TE}^2 = \eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\frac{1}{\cos(\theta_t)} = 190.5\ \Omega$$

$$\Gamma = \frac{\eta_{TE}^2 - \eta_{TE}^1}{\eta_{TE}^2 + \eta_{TE}^1} = -0.094$$

$$\vec{E}_t(x,y,x) = \vec{E}_i(x,y,x) + \vec{E}_r(x,y,x) = j\mu_x e^{-j\frac{2\pi}{\lambda_1}[\cos(\theta)z+\sin(\theta)y]} + \Gamma j\mu_x e^{-j\frac{2\pi}{\lambda_1}[-\cos(\theta)z+\sin(\theta)y]}$$
V/m

where

$$\lambda_1 = \frac{c}{f\sqrt{\mu_{r1}\varepsilon_{r1}}} \approx 0.15\ \text{m}$$

2) The power density propagating in the second medium in the *z* direction is:

$$S_z^2 = S_z^1(1 - |\Gamma|^2) = \frac{1}{2}\frac{|\vec{E}|^2}{\eta^1}\cos(\theta)\,(1 - |\Gamma|^2) = 2.2\ \text{mW/m}^2$$

where

$$\eta^1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 188.5\ \Omega$$

3) For orthogonal incidence:

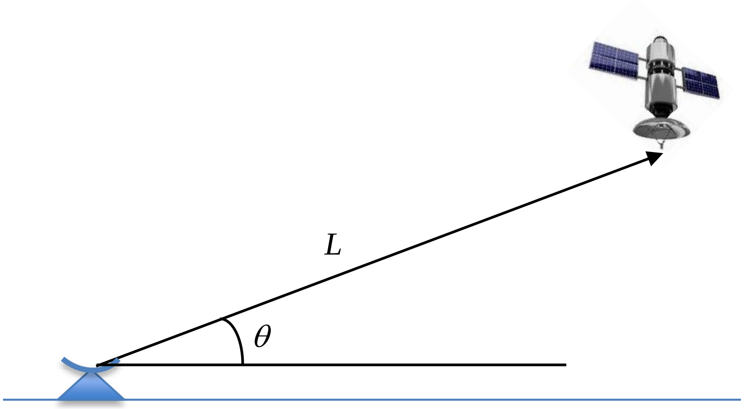
$$\eta^2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} = 188.5\ \Omega = \eta^1$$

Therefore, there is no reflected field in medium 1 and the whole power density crosses the interface.

Problem 4

Consider the uplink to a LEO satellite (pointing at the Earth center) from a ground station, operating at f = 30 GHz, whose antenna is pointed zenithally. Determine the uplink yearly availability to guarantee a minimum signal-to-noise ratio (SNR) of 6 dB at the satellite. The CCDF of the zenithal tropospheric attenuation is given by:

$$P\left(A_T^{dB}\right) = 100e^{-1.15A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in \%})$$



Additional assumptions and data:

- elevation angle θ = 30°
- power transmitted from the ground P_T = 378 W
- radiation patter of the antennas (circular symmetry): $f = [\cos(\phi)]^2$
- equivalent noise temperature emitted by the ground T_B = 200 K
- mean radiating temperature T_{mr} = 285 K
- gain of the antennas (on board the satellite and on the ground) $G_T = G_R$ = 30 dB
- distance to the satellite L = 2400 km
- bandwidth of the receiver B = 2 MHz
- internal noise temperature of the receiver T_R = 310 K
- no additional losses in the transmitter and the receiver

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_S}{k[T_R + T_A]B}$$

where A_S is the slant path attenuation in linear scale, $f_T = f_R = [\cos(90°-\theta)]^2 = 0.25$, k is the Boltzmann's constant (1.38×10^{-23} J/K) and T_A is the equivalent antenna noise temperature. For this scenario, as the satellite points at the ground, T_A is therefore calculated as:

$$T_A = A_S T_B + T_{mr}(1 - A_S)$$

Therefore, imposing $SNR_{\min}$ = 6 dB = 3.981:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_S}{k[T_R + A_S T_B + T_{mr}(1 - A_S)]B} = 3.981$$

Inverting the equation above to solve for A_S:

$$A_S = \frac{SNR_{min}(BkT_R + T_{mr}Bk)}{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R - SNR_{min}(BkT_B - Bk)} = 0.0254$$

In dB:

$$A_S^{dB} = -10\log_{10}(A_S) = 15.948 \text{ dB}$$

The zenithal attenuation is:

$$A_T^{dB} = A_S^{dB} \sin(\theta) = 7.974 \text{ dB}$$

Using the CCDF expression, such an attenuation corresponds to an outage probability of roughly 0.01% in a year, i.e. to a yearly availability of 99.99%.