EM wave propagation for space-borne systems – Prof. L. Luini,
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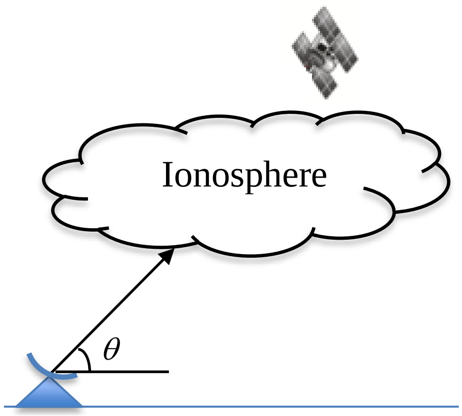
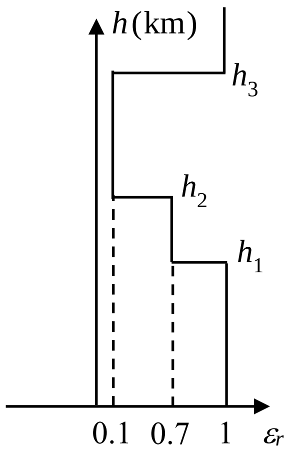
Problem 1

Making reference to the figure below, a ground station transmits an EM signal to a satellite with elevation angle $\theta = 60°$. The figure also reports the vertical profile of the relative permittivity ε_r in the ionosphere (h_1 = 100 km, h_2 = 150 km and h_3 = 200 km), associated to the link frequency f = 12 MHz. For this scenario:

1) Draw the vertical profile of the total electron content N.
2) Determine if the wave reaches the satellite.
3) Determine the change in the elevation angle necessary to modify the condition at point 2 (from crossing to reflection; from reflection to crossing).





Solution

1) Inverting the usual relationship and using the values in the profile:

$$\sqrt{\varepsilon_r} = \sqrt{1-\left(\frac{f_P}{f}\right)^2} = \sqrt{1-\frac{81N}{f^2}}$$

we obtain → $N_1 = 5.3\times10^{11}$ e/m^3 (value between h_1 and h_2) and $N_2 = 1.6\times10^{12}$ e/m^3 (value between h_2 and h_3).

2) The condition to obtain total reflection is

$\cos(\theta) = \sqrt{\varepsilon_r}$

For the given elevation angle, $\cos(\theta) = 0.5$. As this value lies between $\sqrt{\varepsilon_{r1}} = 0.3162$ and $\sqrt{\varepsilon_{r2}} = 0.8367$, the wave is totally reflected at h_2.

3) For the wave to cross the ionosphere, it is necessary to increase the elevation angle:

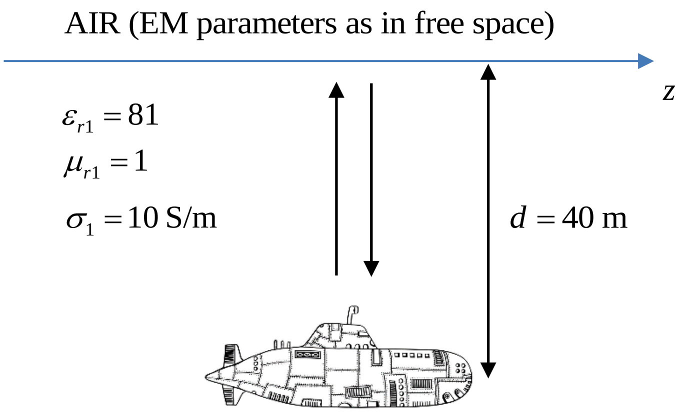
$\cos(\theta') = \sqrt{0.1} = 0.3162 \rightarrow \theta' = 71.57°$

For any elevation angle higher that θ', the wave will cross the ionosphere.

Problem 2

A submarine transmits electromagnetic signals towards the sea surface to keep track of its depth. Assuming that the submarine antenna emits plane waves with electric field $|\vec{E}_{out}| = 5\ \mathbf{V/m}$ at frequency $f = 1$ kHz, and that its depth is $d = 40$ m, calculate:

1) The wavelength underwater.
2) The propagation velocity underwater.
3) The power density reaching back the submarine after the reflection on the sea surface.



Solution

1) First we need to characterize the electromagnetically the first medium (sea water). In this case, the loss tangent is $\frac{\sigma}{\omega\varepsilon} = 2.2\times10^6 >> 1$. Therefore the second medium can be well approximated as a good conductor. Accordingly, the attenuation and propagation constants are:

$$\alpha_1 = \beta_1 = \sqrt{\pi f \mu_1 \sigma_1} = 0.1987\ \text{1/m}$$

As for the intrinsic impedance, we obtain:

$$\eta_1 = \sqrt{\frac{\pi f \mu_1}{\sigma_1}}(1+j) = 0.0199(1+j)\ \Omega$$

The wavelength is:

$$\lambda_1 = \frac{2\pi}{\beta_1} = 31.62\ \text{m}$$

2) The propagation velocity is:

$$v_1 = \frac{\omega}{\beta_1} = 3.162\times10^4\ \text{m/s}$$

3) For the second medium (air/free space), $\eta_2 = 377\ \Omega$. The reflection coefficient is therefore:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx 1$$

The power density emitted by the submarine antenna is:

$$S_{out} = \frac{1}{2}\frac{|\vec{E}_{out}|^2}{|\eta_1|}\cos(\measuredangle \eta_1) = 314.6\ \text{W/m}^2$$

The power density reaching the submarine after reflection is: 4.9 pW/m^2

Problem 3

Making reference to the figure below, a satellite, equipped with a linear antenna oriented along *x*, transmits a signal to a ground station, whose receiving antenna is linear and oriented along *z*. The distance between the satellite and the ground station is d = 600 km and the link operates at f = 20 GHz. The signal crosses the layer consisting of anisotropic particles (h = 400 m, h_S = 598 km), all oriented as indicated in the figure below. The propagation constants associated to the two principal axes of the particles are:

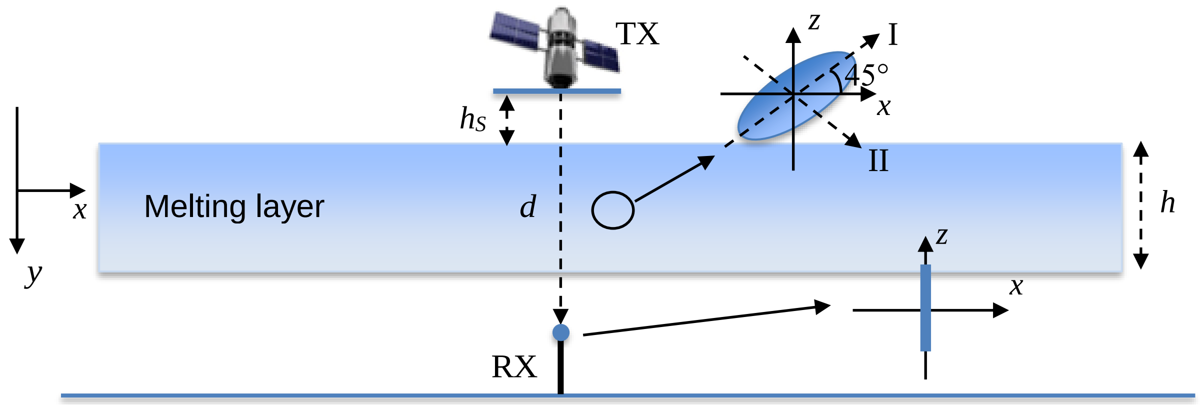
γ_I =3.4×10^{-4}+j1623 1/m

γ_{II} =2.4×10^{-4}+j1082 1/m

The absolute value of the electric field emitted by the satellite antenna is E_0 = 100 V/m.

For this link, calculate the power received at RX.

Additional data: disregard additional attenuation due to the atmosphere; the effective area of the receiver antenna A_E = 0.004 m^2. Assume plane wave propagation. Assume free space propagation outside the melting layer.



Solution

The power received by the RX antenna will depend on the *z* electric field component at RX. The propagation along the two principal axes of the particle will be different, according to the associated propagation constants. First, the *x* component of the electric field is projected along directions I and II:

$$E_I(y=0) = E_0(0)\cos(45°) = E_0(0)/\sqrt{2}$$
$$E_{II}(y=0) = E_0(0)\sin(45°) = E_0(0)/\sqrt{2}$$

Assuming plane wave propagation, these two components will propagate in the same way from the satellite to the upper limit of the melting layer and from its lower limit to RX, while they will propagate differently in the melting layer. Thus, the two components reaching RX will be:

$$E_I(y=d) = E_0(0)/\sqrt{2}\ e^{-j\beta_0 h_S} e^{-\gamma_I h} e^{-j\beta_0(d-h_S-h)}$$
$$E_{II}(y=d) = E_0(0)/\sqrt{2}\ e^{-j\beta_0 h_S} e^{-\gamma_{II} h} e^{-j\beta_0(d-h_S-h)}$$

Finally, the field along *z* is obtained as:

$$E_z(y=d) = E_I(y=d)\sin(45°) - E_{II}(y=d)\cos(45°) = -64 + j9.5 \quad \text{V/m} \rightarrow |E_z(y=d)| = 26.4\ \text{V/m}$$

Note that, for the calculation of the absolute value of E_z, in turn needed to calculate the power density reaching the RX antenna, the propagation terms in "free space" involving β_0 can be neglected (no additional depolarization is introduced).

The power density reaching RX is therefore:

$$S_{RX} = \frac{1}{2}\frac{|E_z(y = d)|^2}{\eta_0} \approx 0.9249 \text{ W}$$

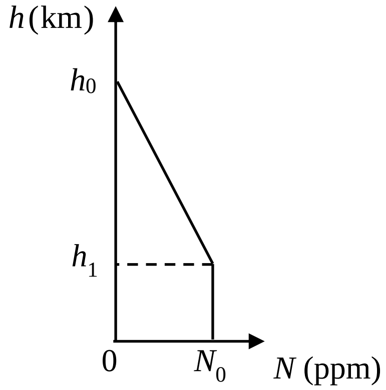
Finally, the received power is:

$$P_{RX} = S_{RX}A_E = 3.7 \text{ mW}$$

Problem 4

A dual frequency GNSS receiver, tracking a GPS satellite at 20° elevation angle and lying at altitude h_S = 500 m, measures the ionospheric delay at L1 band (f_1 = 1575.42 MHz) and L2 band (f_2 = 1227.60 MHz), which are d_{L1}^I = 32.5 ns and d_{L2}^I = 53.5 ns, respectively.

1) Calculate the TEC along the path to the satellite.
2) Considering the refractivity profile below (N_0 = 700 ppm, h_1 = 1 km and h_0 = 9.5 km), calculate the total atmospheric error affecting the measured pseudorange for a such dual frequency receiver and for a single frequency one.



Solution

1) Starting from the ionospheric delays at both bands, the TEC along the path can be calculated as:

$$TEC = \frac{2c(d_{L2}^I - d_{L1}^I)}{81}\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right)^{-1} = 60 \text{ TECU}$$

1) The total atmospheric error affecting the pseudorange is:

$$d_{Li}^{TOT} = d_{Li}^I + d^T$$

The slant tropospheric delay can be calculated as:

$$d^T = \frac{10^{-6}}{\sin(\theta)}\int_{h_S}^{h_0} N\,dh = \frac{10^{-6}}{\sin(\theta)}\left[(h_1 - h_S)N_0 + \frac{(h_0 - h_1)N_0}{2}\right] = 9.72 \text{ m}$$

This delay is frequency independent. For the dual frequency receiver, d^I can be neglected, so $d_{L1/L2}^{TOT} = 9.72$ m. For the single frequency one, which operates at L1, $d_{L1}^{TOT} = d_{L1}^I + d^T = 19.5$ m