EM wave propagation for space-borne systems – Prof. L. Luini,
June 26th, 2023

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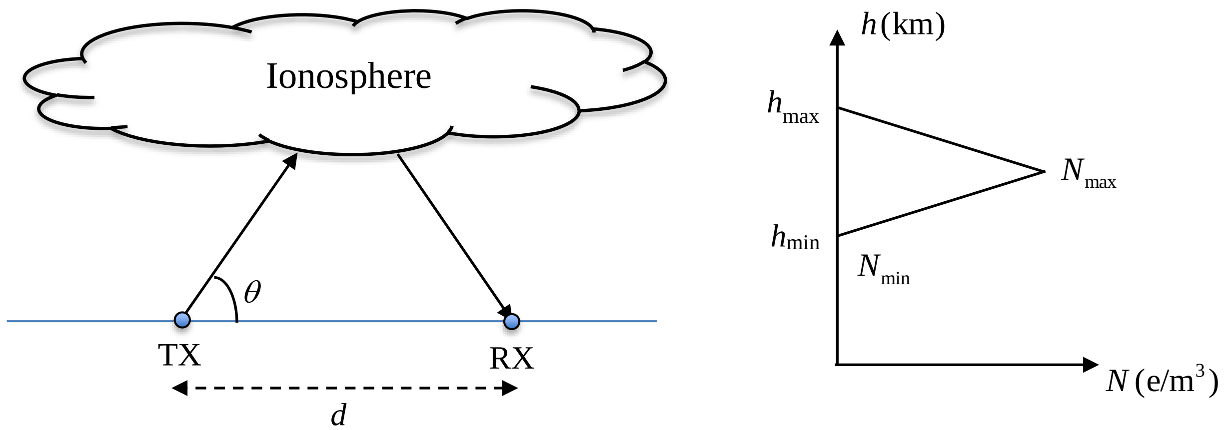
Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance d by exploiting the ionosphere (elevation angle θ = 60°). The ionosphere is modelled with the symmetric electron density profile (daytime) sketched in the figure (right side), where $N_{\max} = 6\times10^{12}$ e/m^3, $N_{\min} = 4\times10^{10}$ e/m^3, $h_{\min}$ = 100 km and $h_{\max}$ = 400 km.

1) Determine the maximum distance d achievable for the TX → RX link.
2) Determine the operational frequency f to achieve the conditions at point 1).
3) Indicate a reasonable margin on f found at point 2) to guarantee the TX → RX link notwithstanding the ionospheric variations.
4) Indicate the best polarization to be used for the TX → RX link.

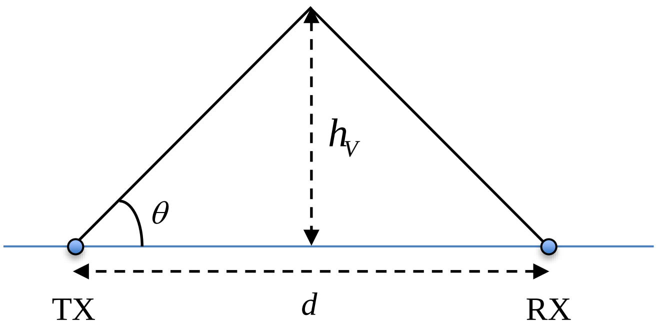
Assume: the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected; the Earth is flat; no tropospheric effects to be considered.



Solution

1) The distance d is maximized if the reflection occurs as high as possible in the ionosphere, i.e. at the height h_p = 250 km, correspondent to N_{max}. Considering the figure below, the distance can be found by inverting the following expression:

$h_V = 1.2\, h_p = {}^{d}/_{2} \tan\theta \rightarrow\ d = \frac{2.4\, h_p}{\tan\theta} = 346.4$ km



2) The link operational frequency *f* can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1 - \left(\frac{f_C}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f_m}\right)^2}$$

Solving for the frequency f_m, we obtain:

$$f_m = \sqrt{\frac{81 N_{\max}}{1 - [\cos(\theta)]^2}} = 25.5 \text{ MHz}$$

3) During the night, the peak values of the electron content will decrease: it is a good rule of thumb to use reduce by 10% the peak frequency to avoid that the wave crosses the ionosphere at nighttime. Therefore → $f' = 0.9f = 22.95$ MHz

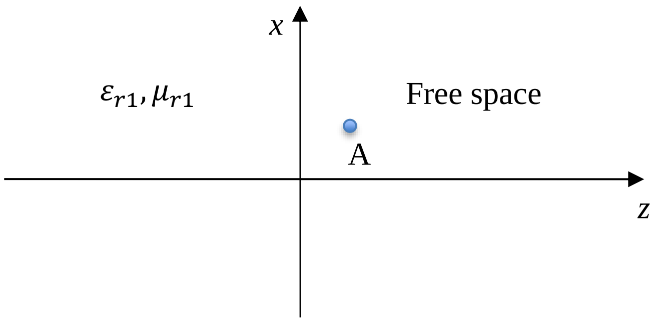
4) Depolarization in the ionosphere affects linearly polarized waves, but not circularly polarized ones. Therefore, the best polarization is LHCP or RHCP.

Problem 2

A uniform plane wave with horizontal polarization (frequency f = 300 MHz) propagates along z into free space from a medium with the following electromagnetic features: ε_{r1} = 3, μ_{r1} = 1 and σ_1 = 0.05 S/m. The incident electric field at the origin of the axis is $\vec{\boldsymbol{E}}_{\boldsymbol{i}}(\boldsymbol{z} = \mathbf{0}\ \mathbf{m}) = \boldsymbol{E}_{\mathbf{0}}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}}$ V/m.

For this scenario:

1) Calculate the expression of the incident magnetic field in the first medium (left side) in the time domain.
2) Calculate the wavelength in medium 1.
3) Calculate E_0 knowing that power density power at point A(1,1,λ_2) is $S(A)$ = 3 mW/m^2.



Solution

1) The propagation in medium 1 is regulated by the propagation constant. As no approximations are possible (the loss tangent is roughly 1):

$$\gamma_1 = \sqrt{j\omega\mu_1(\sigma_{1+}j\omega\varepsilon_1)} = 4.95 + j11.96\ \text{1/m}$$

Therefore, the expression of the electric field in medium 1 is:

$$\vec{\boldsymbol{E}}_{\boldsymbol{i}} = \boldsymbol{E}_{\mathbf{0}}\boldsymbol{e}^{-\gamma_1 z}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}} = \boldsymbol{E}_{\mathbf{0}}\boldsymbol{e}^{-\mathbf{4.95}\boldsymbol{z}}\boldsymbol{e}^{-\boldsymbol{j}\mathbf{11.96}\boldsymbol{z}}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}}\ \text{V/m.}$$

To find the magnetic field, we first need to calculate the intrinsic impedance of the medium:

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{(\sigma_{1+}j\omega\varepsilon_1)}} = 169.1 + j70\ \Omega$$

In the phasor domain, the incident magnetic field is:

$$\vec{\boldsymbol{H}}_{\boldsymbol{i}} = \frac{\boldsymbol{E}_{\mathbf{0}}}{\boldsymbol{\eta}_{\mathbf{1}}}\boldsymbol{e}^{-\mathbf{4.95}\boldsymbol{z}}\boldsymbol{e}^{-\boldsymbol{j}\mathbf{11.96}\boldsymbol{z}}\vec{\boldsymbol{\mu}}_{\boldsymbol{y}} = \frac{\boldsymbol{E}_{\mathbf{0}}}{|\boldsymbol{\eta}_{\mathbf{1}}|}\boldsymbol{e}^{-\mathbf{4.95}\boldsymbol{z}}\boldsymbol{e}^{-\boldsymbol{j}\mathbf{11.96}\boldsymbol{z}}\boldsymbol{e}^{-\boldsymbol{j}\measuredangle(\eta_1)}\vec{\boldsymbol{\mu}}_{\boldsymbol{y}}$$

where $|\eta_1| = 183\ \Omega$ and $\measuredangle\ (\eta_1) = 0.3924$ rad.

Therefore, the expression of the incident magnetic field in the time domain is:

$$\vec{\boldsymbol{H}}_{\boldsymbol{i}}(\boldsymbol{t}) = \frac{\boldsymbol{E}_{\mathbf{0}}}{|\boldsymbol{\eta}_{\mathbf{1}}|}\boldsymbol{e}^{-\mathbf{4.95}\boldsymbol{z}}\mathbf{cos}\ (\mathbf{2}\boldsymbol{\pi}\boldsymbol{f}\boldsymbol{t} - \mathbf{11.96}\boldsymbol{z} - \mathbf{0.3924})\vec{\boldsymbol{\mu}}_{\boldsymbol{y}}$$

2) $\lambda_1 = \frac{2\pi}{\beta_1} = 0.525$ m

3) First, it is necessary to calculate the reflection coefficient, given by:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.359\ - j\ 0.174$$

where

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

The transmitted electric field, at z = 0 m, is:

$$\vec{E}_t(z = 0\text{ m}) = \boldsymbol{E_0} T \vec{\boldsymbol{\mu}}_{\boldsymbol{x}} = \boldsymbol{E_0}(1 + \Gamma)\vec{\boldsymbol{\mu}}_{\boldsymbol{x}} = \boldsymbol{E_0}(1.359\ - j\ 0.174)\vec{\boldsymbol{\mu}}_{\boldsymbol{x}}\ \text{V/m.}$$

The power density reaching point A is (as there are no losses in the second medium, the power density does not change along z):

$$S(A) = \frac{1}{2}\frac{\left|\vec{E}_t(z = 0)\right|^2}{\eta_2} = \frac{1}{2}\frac{(E_0)^2|T|^2}{\eta_2} = 0.0025\ (E_0)^2 = 3\ \text{mW/m}^2$$

Therefore:

$$E_0 = 1.1\ \text{V/m}$$

Problem 3

We need to design a link to a deep-space probe orbiting Mars and operating at Ka-band, specifically at f = 26 GHz. The ground station is equipped with a steerable antenna to track the probe. Calculate the reflector antenna diameter of the ground station (Gregorian configuration with efficiency η = 0.5) necessary to guarantee that the probe can be correctly tracked down to an elevation angle θ = 30° for 99.9% of the time in a year, i.e. that the minimum SNR is 5 dB. To this aim, assume:

- that the atmosphere is stratified;
- that the ground station LNA noise temperature is T_R = 50 K;
- to neglect the cosmic background temperature;
- that the mean radiating temperature of the atmosphere is T_{mr} = 10 °C;
- that the probe makes use of a parabolic antenna with gain G_T = 45 dB;
- the transmit power is P_T = 110 W;
- the probe antenna always points at the ground station;
- the distance between the probe satellite and the ground station is L = 225000000 km;
- the receiver bandwidth is B = 1 kHz;
- that the CCDF of the zenithal atmospheric attenuation A_T is modelled by:

$$P\left(A_Z^{dB}\right) = 100e^{-0.69A_Z^{dB}} \quad (A_Z \text{ in dB and } P \text{ in \%})$$

Solution

The zenithal attenuation A_Z^{dB} is determined using the CCDF model. 99.9% availability corresponds to P = 0.1% exceedance. Inverting the CCDF formula:

$$A_Z^{dB} = -\frac{1}{0.69}\ln\left(\frac{0.1}{100}\right) \approx 10 \text{ dB}$$

Scaling the zenithal attenuation to the target elevation angle:

$$A_S^{dB} = \frac{A_Z^{dB}}{\sin(\theta)} \approx 20 \text{ dB}$$

which, in linear scale, corresponds to:

$$A_L = 10^{\frac{A_S^{dB}}{10}} \approx 0.01$$

The system noise temperature is (for the Gregorian configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}(1 - A_L) = 330.3 \text{ K}$$

The SNR is given by:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_L}{kT_{sys}B}$$

where f_R = 1 and f_T = 1.

Inverting the expression above to solve for G_R (by setting SNR = SNR_{min} = 5 dB):

$G_R \approx$ 74 dB

Recalling that:

$$\frac{\eta A_g}{G_R} = \frac{\lambda^2}{4\pi}$$

where A_g is the geometrical area of the antenna:

$$A_g = \left(\frac{D_R}{2}\right)^2 \pi$$

the antenna diameter D_R is obtained:

$D_R \approx 26$ m

This is indeed the dimension of Ka-band deep-space antennas installed at NASA Deep Space Network (DSN) sites (Goldstone, Madrid, Camberra).

Problem 4

A GNSS receiver, whose height is H = 700 m a.m.s.l., tracks the signal of a GPS satellite at θ = 30°. The vertical profile of the tropospheric refractivity can be modeled as:

$$N = N_0 e^{-h/h_0} = 700e^{-h/25000} \quad \text{(h in m)}$$

Making reference to this scenario, determine the tropospheric error affecting the measured pseudorange for a single frequency and a dual frequency GNSS receiver.

Solution

The pseudorange measurement is affected by different error sources, including the tropospheric delay d due to the presence of gases, which can be calculated from the knowledge of the refractivity profile. In fact, the tropospheric delay τ, in seconds, can be calculated as:

$$\tau = T - T_0 = \int_H^{H_S} \frac{dh}{v(h)} - \int_H^{H_S} \frac{dh}{c} = \int_H^{H_S} \frac{dh}{c} n - \int_H^{H_S} \frac{dh}{c} = \frac{1}{c}\left[\int_H^{H_S} (n-1)dh\right] =$$

$$= \frac{10^{-6}}{c}\left[\int_H^{H_S} N dh\right] = \frac{N_0 10^{-6}}{c} \int_H^{H_S} e^{-\frac{h}{h_0}} dh = -\frac{h_0 N_0 10^{-6}}{c}\left[e^{-\frac{h}{h_0}}\right]_H^{H_S}$$

Assuming $H_S >> h_0$:

$$\tau = -\frac{h_0 N_0 10^{-6}}{c}\left(0 - e^{-\frac{H}{h_0}}\right) = \frac{h_0 N_0 10^{-6}}{c} e^{-\frac{H}{h_0}} = 56.7 \text{ ns}$$

As a result the error on the pseudorange due to the tropospheric delay is:

$$d = \frac{c\tau}{\sin(\theta)} = 34 \text{ m}$$

As the tropospheric delay is not frequency dependent, there is no benefit in using a dual frequency receiver.