EM wave propagation for space-borne systems – Prof. L. Luini,
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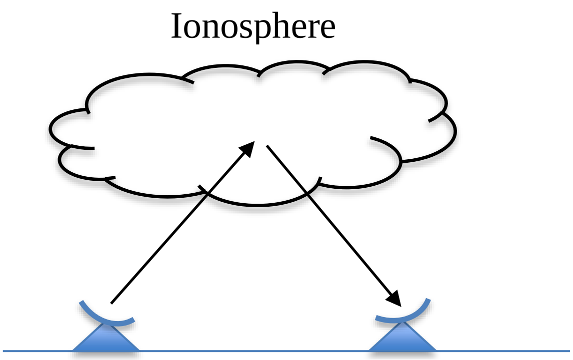
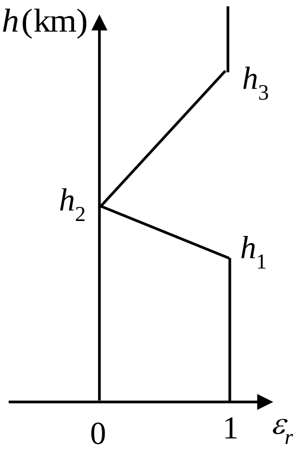
Problem 1

Making reference to the figure below, a communication system exploits the ionosphere during nighttime: it operates at f = 3.6 MHz and works with elevation angle θ = 30°. The trend of ε_r in the ionosphere is depicted in the figure below on the right side (h_1 = 100 km, h_2 = 200 km and h_3 = 400 km). Calculate the distance between the stations to enable the communication.

Assumptions: no tropospheric effects; flat Earth; virtual reflection height $h_V = 1.2\ h_R$ (h_R being the real reflection height).





Solution

The peak electron content value can be obtained from the following expression:

$$\sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2} = \sqrt{\min\left(\varepsilon_r\right)}$$

Setting ε_r = 0 as from the graph on the right, inverting the expression → $N_{\max} = 16\times10^{10}$ e/m^3. $N_{\max}$ obviously corresponds to the lowest value of ε_r, but, as the link is not zenithal, the reflection will occur somewhere below h_2. In fact, the reflection point depends on N through the following expression:

$$\sqrt{1-\left(\frac{9\sqrt{N}}{f}\right)^2} = \cos(\theta)$$

Inverting the expression → N = 4×10^{10} e/m^3. Given the trend of ε_r → $N(h_1)$ = 0 e/m^3 and $N(h_2) = N_{\max}$ = 16×10^{10} e/m^3. Therefore, the value of h (up to h_2) as a function of N is:

$$h = \frac{h_2 - h_1}{N_{\max}} N + h_1$$

Using N = 4×10^{10} e/m^3 → h = 125 km. The distance between the station is:

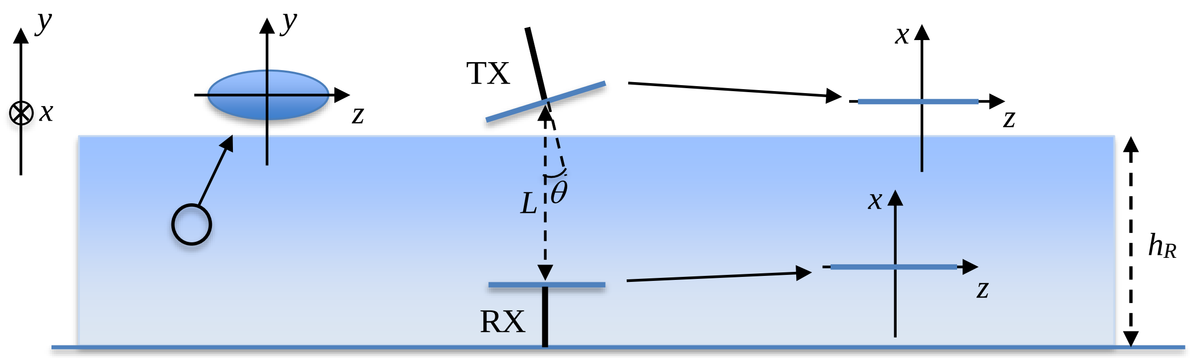
$$D = 2\frac{h_V}{\tan(\theta)} = 2\frac{1.2h_R}{\tan(\theta)} \approx 520 \text{ km}$$

Problem 2

An Earth-space link, with path length L = 500 km and operating at f = 900 MHz, crosses a rain layer with thickness h_R = 2 km (raindrops all aligned as shown in the sketch below). Both the transmitter (TX) and the receiver (RX) use linear horizontal antennas, but the TX antenna is tilted by θ = 30° as shown in the sketch below, due to issues in the satellite attitude control. For this link:

1) Determine the polarization of the wave in front of the receiver RX (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
2) Calculate the power received by RX.

Assume: transmit power P_T = 200 W; gain of the antennas G = 8 dB; radiation pattern of the antennas $f = [\cos(\theta)]^2$.



Solution

1) As the operational frequency is much lower than 10 GHz, the wave is unaffected by the presence of rain along the path, both in terms of attenuation and delay. This means that no depolarization effects can take place.

2) Given the operational frequency, tropospheric and ionospheric attenuation can be neglected. Thus, the power received by RX is simply:

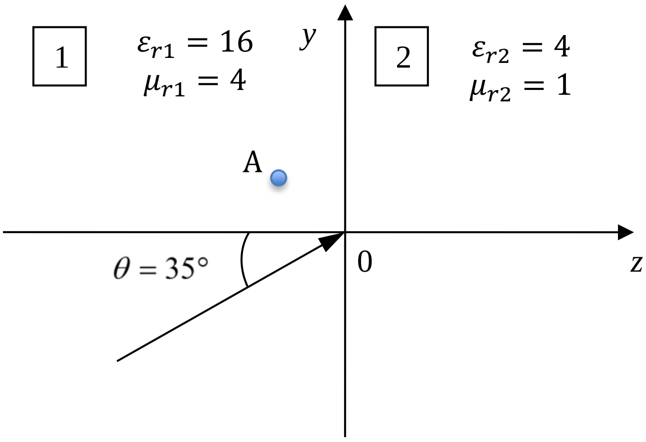
$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi L}\right)^2 f_R G_R \approx 16.8 \text{ pW}$$

where, f_T = 1, $f_R = [\cos(\theta)]^2 = 0.75$, $G_T = G_R$ = 6.3, λ = 0.334 m.

Problem 3

Consider the plane sinusoidal wave below, with f = 1 GHz and incident electric field given by:

$$\vec{E}_i(0,0,0) = j\mu_x \quad \text{(V/m)}$$



Calculate the electric field in point A(x = -5 m, y = 1 m, z = -1 m).

Solution

The total electric field is given by the summation between the incident field and the reflected field (if any). To determine the reflected field, it is first necessary to calculated the refraction angle:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\mu_{r1}\varepsilon_{r1}}{\mu_{r2}\varepsilon_{r2}}}\right) \approx \sin^{-1}(2.294)$$

This result indicates an evanescent wave, which means total reflection. For the complete expression of the reflected field though, it is necessary to calculate the reflection coefficient. To this aim, let us calculate β_{2z}:

$$\beta_{2z} = \beta_2\cos(\theta_2) = \beta_2\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{\mu_{r2}\varepsilon_{r2}}\sqrt{1-[\sin(\theta_2)]^2} = \pm j4.13\beta_0 = -j4.13\beta_0$$

It is necessary to pick the negative sign to obtain a physical solution. The reflection coefficient can now be calculated:

$$\eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}\frac{1}{\cos(\theta)} = 266.6\ \Omega$$

$$\eta_2 = \eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\frac{1}{\cos(\theta_2)} = \eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\frac{1}{\cos(\theta_2)} = j91.3\ \Omega$$

The choice of the negative sign for $\cos(\theta_2)$ in η_2 is consistent with the one in β_{2z}.

$$\Gamma = \frac{\eta_2 - \eta_2}{\eta_2 + \eta_1} = -0.728 + j0.686$$

The expression of the total electric field in medium 1 is therefore:

$$\vec{E}(y,z) = \vec{E}_i(y,z) + \vec{E}_r(y,z) = j\mu_x e^{-j\beta_1\cos(\theta)z}e^{-j\beta_1\sin(\theta)y} + \Gamma j\mu_x e^{j\beta_1\cos(\theta)z}e^{-j\beta_1\sin(\theta)y} \text{ V/m}$$

The value of the electric field in A is:

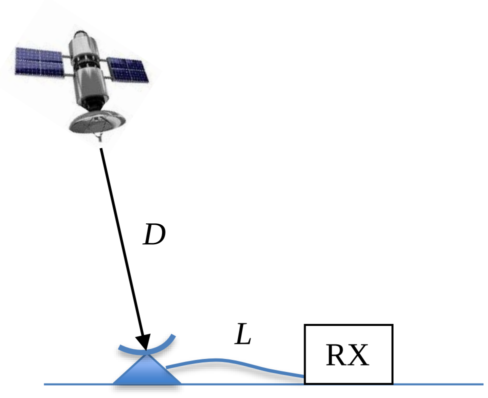
$\vec{E}(A) = (-0.697 - j0.893)\mu_x$ V/m

with $\beta_1 = \beta_0\sqrt{\mu_{r1}\varepsilon_{r1}} = \frac{2\pi}{\lambda_0}\sqrt{\mu_{r1}\varepsilon_{r1}} = 167.5$ rad/m

Problem 4

Consider the downlink from a GEO satellite to a ground station consisting of a VSAT (Very Small Aperture Terminal), operating at f = 25 GHz. The VSAT consists of a typical reflector antenna with a feed and a transmission line (specific attenuation α_{TL} = 1 dB/m) guiding the signal from the feed to the receiver RX. Determine the maximum length L of the transmission line to guarantee a minimum signal-to-noise ratio (SNR) of 6 dB at the receiver. The link undergoes tropospheric attenuation; the zenithal trend of the specific attenuation α is given by:

$$\alpha(h) = 5e^{-0.5h} \quad (\alpha \text{ is in dB/km and } h \text{ is the height in km})$$



Additional assumptions and data:

- elevation angle $\theta = 30°$
- power transmitted by the satellite P_T = 400 W
- antennas optimally pointed
- mean radiating temperature T_{mr} = 280 K
- gain of the antennas: G_T = 40 dB, G_R = 20 dB
- distance to the satellite D = 37000 km
- bandwidth of the receiver B = 1 MHz
- internal noise temperature of the receiver T_R = 250 K
- physical temperature of the transmission line T_P = 295 K
- no additional losses in the transmitter and the receiver
- troposphere: horizontally homogeneous

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi D)^2 G_R f_R A_S}{k[T_A + T_{TL} + T_R/A_{TL}]B}$$

where A_S is the slant path attenuation in linear scale, T_{TL} is the equivalent noise temperature of the transmission line, A_{TL} is the transmission line attenuation in linear scale, $f_T = f_R = 1$, k is the Boltzmann's constant (1.38×10^{-23} J/K) and T_A is the equivalent antenna noise temperature. The latter is calculated as:

$$T_A = A_S T_C + T_{mr}(1 - A_S)$$

with T_C = 2.73 K. Let us calculate the zenithal tropospheric attenuation as:

$$A_Z = \int_0^{30\text{ km}} \alpha(h)dh = \int_0^{30\text{ km}} 5e^{-0.5h}dh \approx \int_0^{\infty} 5e^{-0.5h}dh = \frac{5}{0.5} = 10\text{ dB}$$

The slant attenuation, in linear scale, is:

$$A_S = 10^{-\frac{A_Z}{10\sin(\theta)}} = 0.01$$

Numerically → T_A = 277 K.

As for the transmission line equivalent noise temperature:

$$T_{TL} = T_P(1 - A_{TL})$$

A_{TL} is defined as:

$$A_{TL} = 10^{-\frac{\alpha_{TL}L}{10}}$$

where L is in m.

Therefore, imposing SNR_{min} = 6 dB = 3.981:

$$SNR = \frac{P_T G_T (\lambda/4\pi D)^2 G_R A_S}{k[T_A + T_P(1 - A_{TL}) + T_R/A_{TL}]B} = 3.981$$

Inverting the equation above to solve for L in A_{TL} (one solution is an acceptable real number, the other one is a complex number)

$$L \approx 12.35\text{ m}$$