EM wave propagation for space-borne systems – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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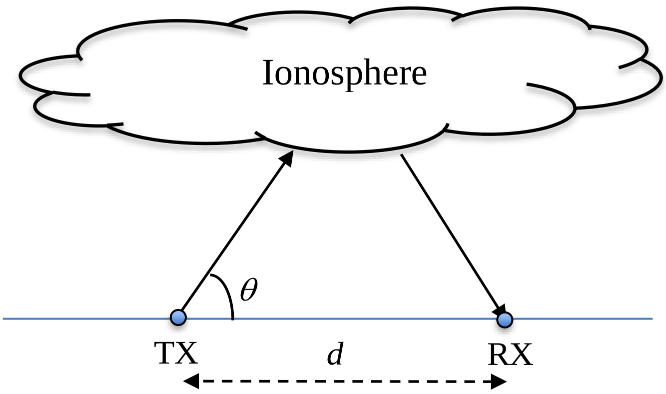
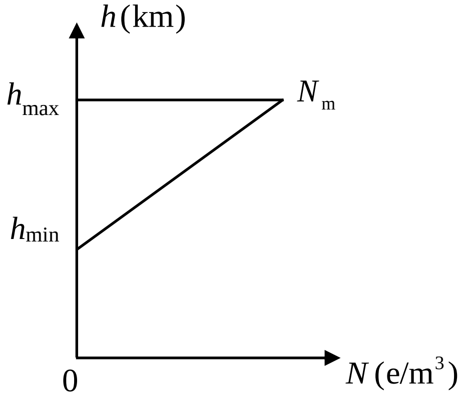
Problem 1

Making reference to the figure below, the ionosphere is modelled with the sketched electron density profile, where $N_m = 4\times10^{12}$ e/m^3, h_{min} = 100 km and h_{max} = 400 km. The distance between TX and RX is d = 600 km and the elevation angle $\theta = 50°$.

1) Calculate the link frequency f.
2) Keeping the same operational frequency f found at point 1), what happens if the elevation angle is increased to $\theta_2 = 60°$?

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.





Solution

1) The distance d depends on the virtual reflection height as follows:

$$d = \frac{2h_V}{\tan\theta}$$

Therefore:

$$h_V = d\frac{\tan\theta}{2} = 357.5 \text{ km}$$

The real reflection height is:

$$h_R = \frac{h_V}{1.2} = 297.9 \text{ km}$$

The trend of the electron content with height is:

$$N(h) = \frac{N_m}{h_{\max} - h_{\min}}(h - h_{\min}) \quad \text{with } h \text{ in km}$$

The N value at height hR is:

$$N(h_R) = 2.6 \times 10^{12} \text{ e/m}^3$$

The link frequency is:

$$f = \sqrt{\frac{81N(h_R)}{1-[\cos(\theta)]^2}} \approx 19.1 \text{ MHz}$$

2) The maximum frequency guaranteeing reflection for the new elevation angle θ_2 is:

$$f_{\max} = \sqrt{\frac{81N_m}{1-[\cos(\theta_2)]^2}} \approx 20.8 \text{ MHz}$$

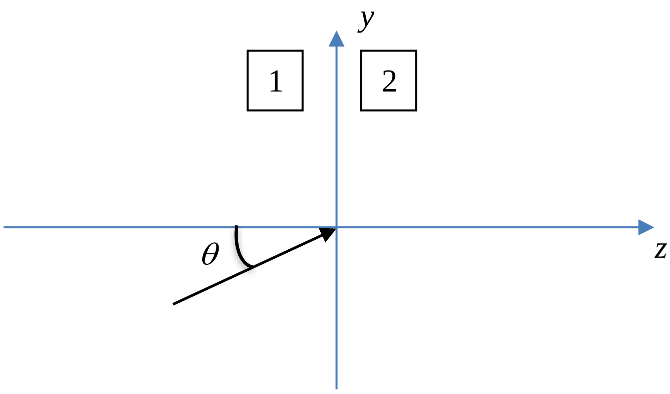
As $f < f_{\max}$, the wave is still completely reflected by the ionosphere.

Problem 2

A plane sinusoidal EM wave (f = 9 GHz) propagates from free space into a medium with electric permittivity $\varepsilon_{r2} = 4$ (assume $\mu_r = 1$ for both media). The expression of the incident electric field is:

$$\vec{E}_i(z,y) = \left(\frac{\sqrt{2}}{2}\vec{\mu}_y - \frac{\sqrt{2}}{2}\vec{\mu}_z + j\vec{\mu}_x\right) e^{-j\beta\frac{\sqrt{2}}{2}z} e^{-j\beta\frac{\sqrt{2}}{2}y} \quad \text{V/m}$$

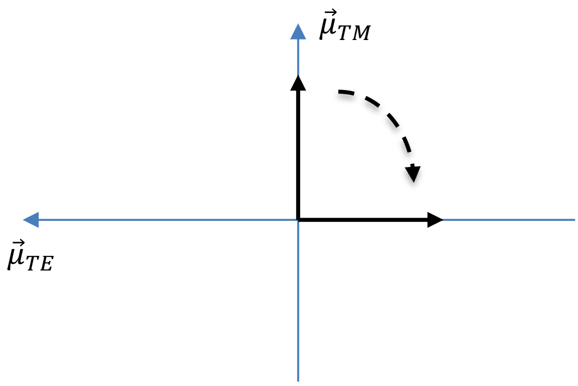
1) Determine the incidence angle θ.
2) Calculate the polarization of the incident field.
3) Write the expression of the reflected field $\vec{E}_r$.
4) OPTIONAL: calculate the power density of the reflected field $\vec{E}_r$.



Solution

1) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta\sin(\theta) = \beta\sqrt{2}/2 \rightarrow \sin(\theta) = \sqrt{2}/2 \rightarrow \theta = 45°$

2) The wave has two components: a TE one, along x, and a TM one, given by the combination of the two fields along y and z. The absolute value of the components is 1 V/m (TM) and 1 V/m (TE), and the differential phase shift is $-\pi/2$. The wave is therefore circularly polarized. To understand the rotation direction, it is worth writing the field in the time domain for (z = 0, y = 0):
$\vec{E}_i(t,0,0) = \cos(\omega t)\vec{\mu}_{TM} + \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE}$ V/m
Looking from the back of the incident wave, in the direction of propagation:



Setting $t = 0 \rightarrow \vec{E}_i(t,0,0) = \vec{\mu}_{TM}$ V/m
Setting $\omega t = \pi/2 \rightarrow \vec{E}_i(t,0,0) = -\vec{\mu}_{TE}$ V/m
As a result, the wave has RHCP.

3) To find the reflected field, it is first necessary to calculate the transmission angle is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 20.7°$$

The TE reflection coefficient is:

$$\Gamma_{TE} = \frac{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) - \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})}{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) + \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})} = -0.4514$$

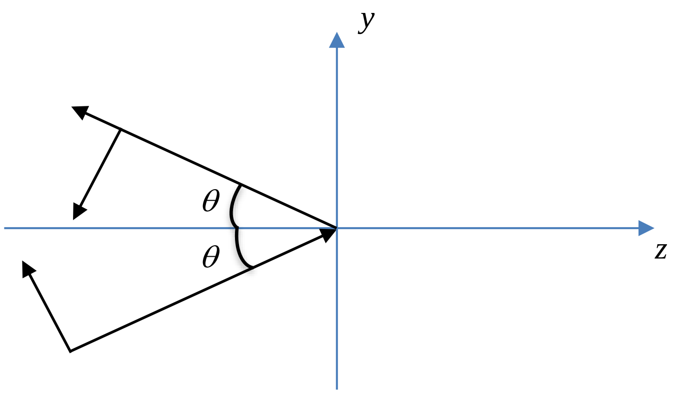
The TM reflection coefficient is:

$$\Gamma_{TM} = \frac{\eta_0\cos(\theta_2)/\sqrt{\varepsilon_{r2}} - \eta_0\cos(\theta)/\sqrt{\varepsilon_{r1}}}{\eta_0\cos(\theta_2)/\sqrt{\varepsilon_{r2}} + \eta_0\cos(\theta)/\sqrt{\varepsilon_{r1}}} = -0.2038$$

The reflected field is therefore given by:

$$\vec{E}_r(z,y) = \left(\frac{\sqrt{2}}{2}\Gamma_{TM}\vec{\mu}_y - |\Gamma_{TM}|\frac{\sqrt{2}}{2}\vec{\mu}_z + j\Gamma_{TE}\vec{\mu}_x\right)e^{j\beta\frac{\sqrt{2}}{2}z}e^{-j\beta\frac{\sqrt{2}}{2}y}\ \mathrm{V/m}$$

It is worth pointing out that the reflection of the TM component is more complex. In fact, the TM reflection coefficients needs to be first applied to the *y* component of the incident field, from which the *z* component can be then derived by taking into account the reflection angle (obviously 45°) and the direction of the *z* component itself (see sketch below): as Γ_{TM} is negative, the reflected *y* component points at -*y*, and so the *z* component will have to be negative; this is possible only by applying the absolute value of Γ_{TM} to the incident *z* component.



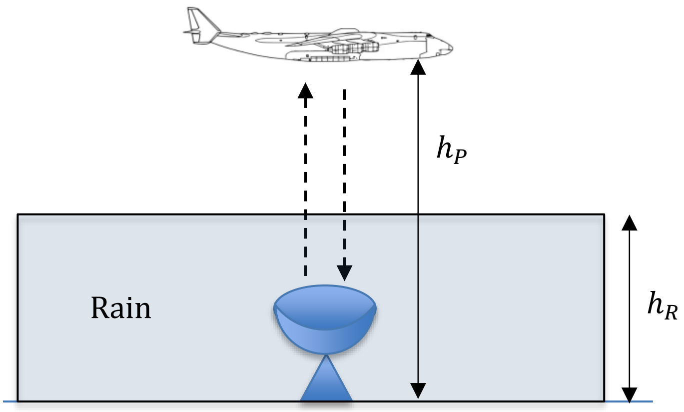
4) The power density of the reflected field is:

$$S_r = \frac{1}{2}\frac{|\vec{E}_r|^2}{\left(\eta_0/\sqrt{\varepsilon_{r1}}\right)} = 3.25 \times 10^{-4}\ \mathrm{W/m^2}$$

Problem 3

A radar with zenithal pointing, working at f = 10 GHz, illuminates an aircraft flying at altitude h_P = 10000 m. As depicted in the figure below, a rain layer (propagation constant in the rain layer $\gamma_R = 1.7\times10^{-4}+j\ 1.886\times10^{3}$ 1/m; rain height h_R = 5000 m) affects the radar. Calculate:

1) The time for the pulse to reach back the radar after reflection on the aircraft.
2) The backscatter section of the airplane, knowing that: the radar transmit power is P_T = 1 kW; the received power is P_R = 0.15 nW; the antenna gain is G = 50 dB.



Solution

1) The wave propagates with different phase velocity in the rain layer and in air. For the former:

$$v_R = \frac{\omega}{\beta_R} = 0.33 \times 10^8 \text{ m/s}$$

while, for the latter, we can assume:

$$v_A = c = 3 \times 10^8 \text{ m/s}$$

The two-way propagation time is given by:

$$\tau = 2\left(\int_0^{h_R} \frac{ds}{v_R} + \int_{h_R}^{h_P} \frac{ds}{v_A}\right) = 2\left(\frac{h_R}{v_R} + \frac{h_P - h_R}{v_A}\right) = 3.33 \times 10^{-4} \text{ s}$$

2) First, the rain attenuation is given by:

$$A_R = e^{-2\alpha_R h_R} = 0.1813$$

The power density reaching the aircraft is:

$$S_A = \frac{P_T}{4\pi h_P^2} G A_R = 0.0144 \text{ W/m}^2$$

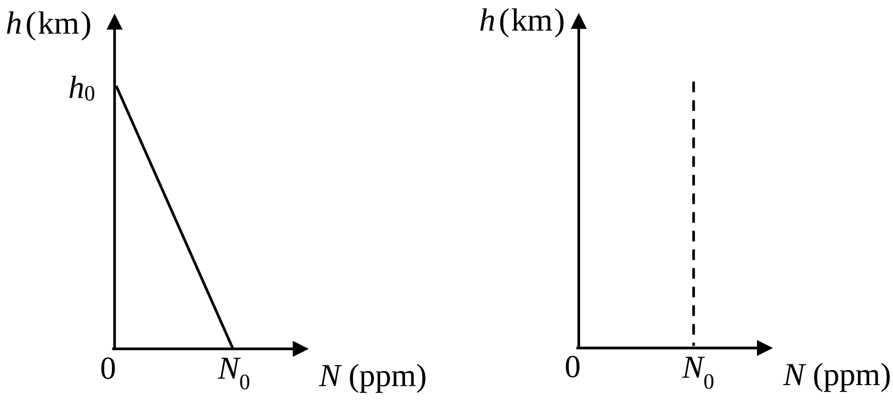
The power received back by the radar is:

$$P_R = \frac{S_A \sigma}{4\pi h_P^2} A_R G \frac{\lambda^2}{4\pi}$$

Solving for the backscatter section → $\sigma \approx 10$ m^2.

Problem 4

Two ground stations operating at f = 15 GHz and f = 30 GHz aim at communicating with a GEO satellite at elevation angle θ = 45°. For each ground station, and for each refractivity profiles depicted below, state if the elevation angle to be used needs to be higher or lower that 45° to correctly point at the satellite (N_0 = 1000 ppm, h_0 = 8 km).



Solution

The vertical gradient of the refractivity will induce a change in the propagation direction (ray bending). This effect is frequency independent, therefore the results will be the same for both ground stations.

The curvature radius of the EM wave depends on the refractivity gradient as follows:

$$\rho = -\frac{10^6}{dN/dh}$$

<u>Profile on the left</u>

The refractivity profile is:

$$N(h) = -\frac{N_0}{h_0}h + N_0 \quad \rightarrow \quad \frac{dN}{dh} = -\frac{N_0}{h_0} = -125\ \text{km}^{-1}$$

Therefore, the curvature radius is:

$$\rho_1 = 8000\ \text{km}$$

ρ_1 is positive, so the ray is bent towards the ground: the actual elevation angle will need to be higher than 45°.

<u>Profile on the right</u>

The refractivity profile is:

$$N(h) = N_0 \quad \rightarrow \quad \frac{dN}{dh} = 0\ \text{km}^{-1}$$

Therefore, the curvature radius is:

$$\rho_2 \rightarrow \infty$$

ρ_2 indicates that the EM ray will not be curved, so the actual elevation angle will need to be exactly 45°.