Electromagnetic Wave Propagation for Space-borne Systems – Prof. L. Luini
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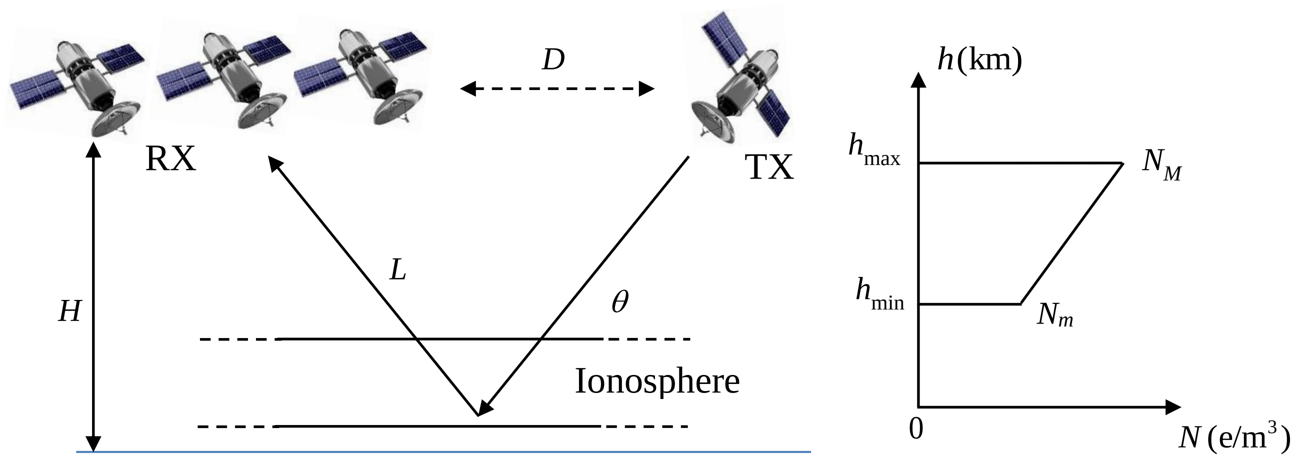
Problem 1

A space-borne pulsed radar system is designed to sound the ionosphere. The radar works at a fixed angle $\theta = 40°$, but it can change the carrier frequency f. An array of receiving satellites is deployed to catch the signal reflected by the ionosphere (see left side of the figure below). The electron content profile is depicted on the right side (N_M = 4×10^{12} e/m^3, N_m = 4×10^{11} e/m^3, h_{max} = 350 km, h_{min} = 100 km)

1. Determine the carrier frequency f_M to correctly identify N_M (i.e. reflection at h_{max}).
2. Determine the carrier frequency f_m to correctly identify N_m (i.e. reflection at h_{min}).
3. Calculate the propagation time τ between the TX and RX when $f = 0.9\, f_M$.
4. Calculate the distance D between the TX and RX at when $f = 1.1\, f_M$.

Assumptions: flat Earth, no tropospheric effects, real reflection height equal to the virtual one, altitude of the satellites H = 800 km.



Solution

1) f_M can be identified as follows:

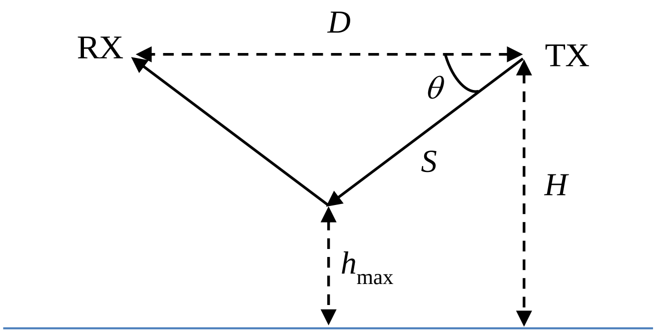
$$\cos(\theta) = \sqrt{1 - \left(\frac{9\sqrt{N_M}}{f}\right)^2} \quad \Rightarrow \quad f_M = 28 \text{ MHz}$$

2) Reflection at h_{min} is not possible: for frequencies below f_M, the wave will be reflected. For frequencies higher than f_M, the wave will cross the ionosphere and will be reflected at the ground.

3) Working at $f = 0.9\ f_M = 25.2$ MHz, the wave will be reflected at h_{max}, so the propagation will occur at light speed c. Therefore:

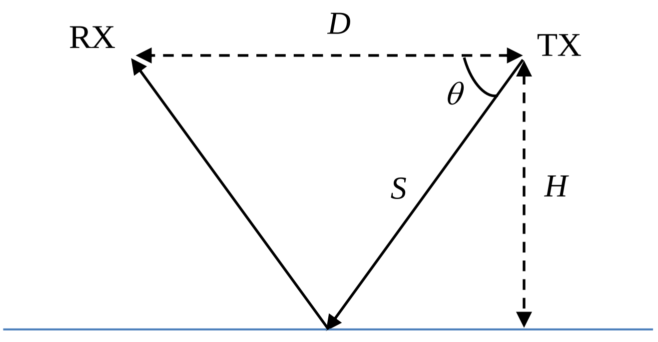
$$S = \frac{(H - h_{max})}{\sin(\theta)} = 700 \text{ km}$$

$$\tau = \frac{2S}{c} = 4.7 \text{ ms}$$



4) Working at $f = 1.1\ f_M = 30.8$ MHz, the wave will be reflected at the ground, so distance D can be calculated as:

$$D = \frac{2H}{\tan(\theta)} = 1907 \text{ km}$$



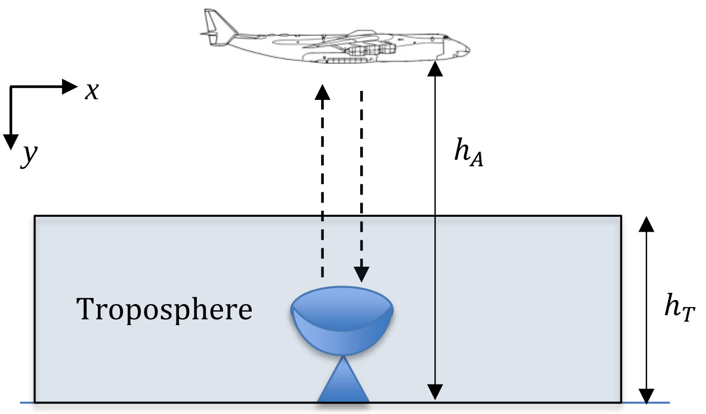
Problem 2

A pulsed radar with zenithal pointing (transmit power P_T = 1 kW), working with carrier frequency f = 80 GHz and with antenna gain G = 40 dB, illuminates an aircraft flying at altitude h_A = 10 km. As depicted in the figure below, the wave crosses an isotropic layer (precipitating particles), whose propagation constants are:

$\gamma_z = \alpha_z + j\,\beta_z = 4\times10^{-4} + j1675.5$ 1/m
$\gamma_x = \alpha_x + j\,\beta_x = 4\times10^{-4} + j15080$ 1/m

For this scenario:

1) Which polarization among the following ones allows improving the radar accuracy in estimating the aircraft distance and its backscatter section σ? Linear along x, linear along z, circular.
2) Working with the polarization chosen at point 1, calculate the radar backscatter σ and the troposphere height h_T, knowing that the power received by the radar in clear sky conditions and under precipitation is 2.126×10^{-13} W and 1.75×10^{-15} W, respectively.



Solution

1) From the propagation constants, it emerges that the two linear polarizations are subject to the same attenuation, but to a different delay: the z-component will travel at light speed c (in fact, $v_z = \omega/\beta_z = c$), while the x-component will travel at a lower speed → $v_x = \omega/\beta_x = 0.33\times10^8$ m/s. Therefore, considering that the radar estimates the aircraft distance from the propagation time (which is affected by the propagation velocity), it is better to use the linear polarization along z.

2) The backscatter σ can be obtained from the power received in clear sky conditions. In this case, the power density reaching the aircraft is:

$$S_A = \frac{P_T}{4\pi h_A^2} G$$

The power received back by the radar is:

$$P_R = \frac{S_A \sigma}{4\pi h_A^2} G \frac{\lambda^2}{4\pi}$$

Inverting the equation to solve for the backscatter yields $\sigma = 3$ m^2.
Under precipitation, the equations become:

$$S_A = \frac{P_T}{4\pi h_A^2} G A_R$$

and

$$P_R = \frac{S_A \sigma_C}{4\pi h_A^2} A_R G \frac{\lambda^2}{4\pi}$$

Solving for A_R yields A_R = 0.0907. Recalling that:

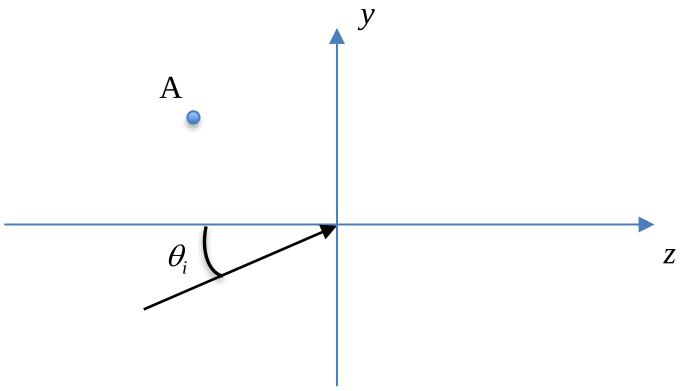
$A_R = e^{-2\alpha_z h_T}$,

we get → h_T = 3000 m.

Problem 3

Consider a sinusoidal plane EM wave (f = 1 GHz) with (left hand or right hand) circular polarization propagating into free space from a lossless dielectric material with relative magnetic permeability equal to 1. Consider the propagation vector to lie on the zy plane. In particular:

1. Define the relative electric permittivity of the first medium and the incident angle to obtain total reflection.
2. Write the equation of the incident wave.
3. Calculate the total power absorbed by a dipole antenna with gain G = 3 dB, positioned in A(-1,1), perpendicularly to the zy plane.



Solution

Here is a possible solution to the problem.

1) Let us first fix the relative electric permittivity of the first medium → ε_{r1} = 4. For total reflection to take place, we need the incidence angle to be the critical angle for evanescent waves:

$$\theta_i = \theta_C = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

2) A possible circularly polarized incident wave is:

$$\vec{E}_i(y,z) = \left(\frac{\sqrt{3}}{2}\vec{\mu}_y - \frac{1}{2}\vec{\mu}_z + j\vec{\mu}_x\right) e^{-j\frac{\beta}{2}y} e^{-j\frac{\sqrt{3}}{2}\beta z} \text{ V/m}$$

with β = 20.94 rad/m

3) The dipole can receive only the TE portion of the reflected wave. In case of evanescent waves, the absolute value of the reflection coefficient is 1. Therefore the received power is:

$$P_R = S_i|\Gamma|^2 A_E = \frac{1}{2}\frac{|E_{TE}|^2}{\eta_1}|\Gamma|^2 \frac{\lambda^2}{4\pi} G = 0.19 \text{ mW}$$

Problem 4

A geostationary satellite operates at a frequency of 19 GHz and provides a communication link to a ground station with altitude h_S = 1 km a.m.s.l (elevation angle ϕ = 60°). The slant path distance between the satellite and the ground station is D = 38000 km. The satellite transmits with an effective isotropic radiated power (EIRP) of 90 dBm.

Consider the following data:

- Zenithal tropospheric attenuation profile (α in dB/km and h in km a.m.s.l.):

  $\alpha = 3.65e^{-0.1h}$ for $0 \leq h \leq 5$ km
  $\alpha = 0$ for $h > 5$ km
- Mean radiating temperature T_{mr} = 290 K
- Ground station antenna gain G_G = 43 dB
- Receiver noise temperature T_R = 150 K
- Gound antenna optimally pointed to the satellite, but satellite mispointed by 16° with respect to the ground station
- Radiation pattern of the satellite antenna $f_T = [\cos(\theta)]^2$, being θ the boreside off angle

Determine the link maximum bandwidth B to guarantee a minimum SNR of 12 dB.

Solution

First, let us calculate the slant path total attenuation:

$$A_T^{dB} = \frac{\int_{h_S}^{5} \alpha(h)dh}{\sin(\phi)} = 12.57 \text{ dB} \rightarrow A_T = 0.0533$$

Let us now calculate the SNR:

$$SNR = \frac{EIRP f_T (\lambda/4\pi D)^2 G_G A_T}{k[T_R + T_{mr}(1 - A_T) + T_C A_T]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = [\cos(\theta)]^2 = 0.924$, T_C = 2.73 K, $EIRP = 10^6$ W. Setting SNR > 12 dB yields B < 8 MHz.